Ultrasound mammography with a mirror

Frank Natterer
Department of Mathematics
University of Münster

Abstract—It is shown that a simple metal plate serving as mirror decisively improves ultrasound mammography. A suitable reconstruction algorithm is described. A numerical example based on computer simulations is given.

I. INTRODUCTION

In present day’s ultrasound mammography, only reflections are measured and an image is constructed simply by recording travel times and arranging them into a B-scan image. It is clear from the available theory that reflections alone can’t provide quantitative images; see e.g. [5]. In [7] it was suggested to complement the reflection measurements by an acoustic mirror, in this case a simple metal plate, which is placed beneath the object; see Fig. 1. This technique is referred to as CARI. The purpose of the present note is to demonstrate that this simple arrangement produces images comparable to those generated my much more complicated scanners taking transmission scans, such as those described in [2], [1].

II. THEORY OF CARI

It belongs to the folklore of seismic imaging that reflectors, be they known or unknown, and irrespectively of their geometrical shape, improve reflection imaging [3], [4]. We exploit this well-known fact for medical imaging. We consider a mammography scanner of the following type: It consists of two horizontal plates (or lines for n=2), the breast being situated in between. The top plate is covered with transducers that can act as sources and as receivers. The transducers are fired consequitively, and all the other transducers are listening. The bottom plate is just a metal plate that serves as acoustic mirror. Thus this scanner does not need mechanical movement, as opposed to those which are described in [2] and [1]. Obviously it is much cheaper to produce and much easier to handle.

To be more specific we assume that the propagation of sound obeys the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 (\Delta u + q(t)\delta(x-s)), \quad 0 < x_n < D,$$

(1)

were n is either 2 or 3, c = c(x), x ∈ R^n is the local speed of sound, and D is the distance between the two plates. For x ∈ R^n we write x = (x’, x_n), x’ ∈ R^n−1. We assume that c^2 = c_0(x)^2/(1 + f) where f vanishes outside the slab 0 ≤ x_n ≤ D and c_0 is the known background velocity. This form of c is convenient for the computations to follow. δ is the Dirac function modelling the source, s the source location, s_n = D, and q is the source wavelet. It is important that the spectrum of q does not contain small frequencies. Otherwise the object can be reconstructed from reflection data only [4], and a mirror is not necessary. The receivers r are sitting on the source plane x_n = D as well. We always assume u = 0 for t ≤ 0. The mirror is modelled by the boundary condition ∂u/∂x_n = 0 on x_n = 0.

We do the analysis in frequency domain. Inverse Fourier transforming (1.1) with respect to time leads to

$$\Delta \hat{u} + k^2(1 + f)\hat{u} = \hat{q}(\omega)\delta(x - s), \quad 0 < x_n < D,$$

$$\partial \hat{u} / \partial x_n = 0 \text{ on } x_n = 0$$

Let \( \hat{u}_0 \) be the solution for f = 0 with the Neuman boundary condition on x_n = 0. With \( \hat{u} = \hat{u}_0 + \nu \) we obtain

$$-\Delta \nu - k^2 \nu = k^2 \hat{q}(\omega)f \hat{u}$$

(2)

We now make the Born approximation, i.e. we replace \( \hat{u} \) by \( \hat{u}_0 \), obtaining

$$-\Delta \nu - k^2 \nu = k^2 \hat{q}(\omega)f \hat{u}_0$$

(3)

This is a linear equation for \( f \). After some algebra (see section 2 of [4]) and putting \( g(r’, \sigma’) = \nu(r’, \sigma’) \) we obtain

$$\hat{g}(\rho’, \sigma’) = (2\pi)^{(n-1)/2} A \int \hat{f}(\rho’ + \sigma’, a(\rho’) + a(\sigma’))$$

$$+ \hat{f}(\rho’ + \sigma’, -a(\rho’) - a(\sigma’))$$

$$+ \hat{f}(\rho’ + \sigma’, a(\rho’) - a(\sigma’))$$

$$+ \hat{f}(\rho’ + \sigma’, -a(\rho’) + a(\sigma’))$$

$$A = -4c_n^2(2\pi)^{-n-1} k^2 \hat{q}(\omega) e^{A(a(\rho’)+a(\sigma’))}$$

$$a(\rho’) = \sqrt{k^2 - |\rho’|^2}, \quad c_2 = 1/4\pi, \quad c_3 = 1/8\pi^2.$$

If the mirror is deleted, only the first term in the bracket is present. In that case the equation is readily solved for \( \hat{f}(\rho’ + \sigma’, a(\rho’) + a(\sigma’)) \). However, due to the plus sign in the second argument, mostly high frequency parts of \( f \) are provided. The frequency domain that is covered by the first term is depicted in Fig. 2. \( k_{\min} \) and \( k_{\max} \) are the wave numbers corresponding to the lower and upper frequency limits in the source wavelet \( \hat{q} \), respectively.
The circles with radius $k_{\text{min}}$, which are not covered cause serious problems for the reconstruction of $f$. They make it impossible to determine $f$ quantitatively.

If the mirror is present, as in CARI, only a linear combination of the values of $f$ for different arguments is determined. However these values contain low frequencies, as can be seen from the alternating signs in the second argument of $f$. It is shown in [4] that $f$ can be determined from this linear combination also in the two circles of radius $k_{\text{min}}$. Thus, in the presence of a mirror, $f$ can be determined for all frequencies inside the ball of radius $2k_{\text{max}}$, i.e. $f$ can be reconstructed with resolution $\pi/k_{\text{max}}$, which is one half of the smallest wavelength contained in the source wavelet $q$.

### III. THE RECONSTRUCTION ALGORITHM

So far we treated the problem within the Born approximation. We have shown that the Born approximation can be determined uniquely. Now we give up the Born approximation and treat the fully nonlinear problem. It will turn out that the mirror is equally useful for the nonlinear case, even though the analysis has been done only for the linear case only. We use the Kaczmarz method in time domain. Kaczmarz' method is widely known in CT under the name of ART. For nonlinear imaging problems such as the fully nonlinear problem. It will turn out that the mirror is equally useful. Now we give up the Born approximation and treat the wavelet $q$.

For each source position $s$ we define $R_s(f)$ to be the detector readings, i.e. $(R_s(f))(r,t) = u(r,t)$ for each receiver $r = (s', D)$ on the top plate. We have to solve the equations $R_s(f) = g_s$ where $g_s$ is the detector reading for each $s = (s', D)$ on the top plane. The Kaczmarz method is an iterative method whith the update

$$f \leftarrow f - \alpha (R'_s(f))^\ast(R_s(f) - g_s).$$

This has to be done consecutively for each source. $\alpha$ is the relaxation parameter and $R'_s(f)$ the adjoint of the derivative of $R_s(f)$. The evaluation of $R'_s(f)$ can be done by a finite difference time domain method, while the evaluation of $R'_s(f)$ requires time reversal which also can be done by finite differences. In both cases nonreflecting boundary conditions have to be used on the vertical boundaries. See chapter 7 of [6] for details.

For our numerical experiments we used the breast phantom created in [1]; see Fig. 2. It has a diameter of 12 cm. The five elliptic patches imbedded in the glandular tissue have diameter 16, 10, 8, 6 and 4 mm; they represent either tumors or cysts. In order to make the phantom more realistic we roughened the background, with oscillations almost as strong as the tumors. We reconstructed this phantom using 11 sources and 120 detectors on the top plane. The source wavelet $q$ had frequencies between 25 and 250 kHz.

Fig. 2. Left: Breast phantom. Diameter 12cm, the smallest tumor in the center is 4 mm in diameter. Right: Reconstruction from CARI data. On the top boundary we have 11 sources, the bottom boundary serves as mirror. The smallest wavelength is 6mm.

The ambient speed of sound is 1500 m/s. Thus the smallest wavelength is 6mm, leading to a theoretical resolution of 3mm. After 40 steps of the Kaczmarz methods with $\alpha = 3 \times 10^7$ we obtained the reconstruction in Fig. 2. In spite of the rough background, all the details are clearly visible, including the small tumor in the center, although not with the correct speed of sound. If we had done the reconstruction without the mirror the image would be totally unacceptable.

### REFERENCES


