Accurate EM-TV Algorithm in PET with Low SNR

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Introduction

PET measurements of tracers with a lower dose rate or short radioactive half life suffer from extremely low SNRs. In these cases standard reconstruction methods (OSEM, EM, filtered backprojection) deliver unsatisfactory and noisy results, cf. Fig. 1. Therefore, we propose to introduce nonlinear variational methods into the reconstruction process to make an efficient use of a-priori information and to attain improved quality of results. We illustrate our technique by evaluating cardiac $H_2{}^{15}O$ measurements [1]. The general approch can also be used for other specific goals allowing to incorporate a-priori information about the solution.

Objectives

The aim of this work is the improvement of reconstructions in case of bad data statistics. An interesting approach is the EM-TV algorithm. This method combines the classical EM algorithm with a relatively new and effective denoising technique based on total variation (TV) minimization. In the past such methods have been suggested [2, 3] but they all have the disadvantage that edges are blurred. These artefacts result from the approximation of TV by differentiable functionals. Due to an additional parameter dependence these algorithms are not very robust. Here, we propose a robust approach without smoothing approximation of TV which allows to realize cartoon reconstructions with sharp edges.

Methods

Classical EM algorithm:

- Minimize the negative log-likelihood functional, fitting data f with Poisson noise,

$$\min_{u\geq 0} \int_\Sigma (Ku - f \log Ku)\, d\mu \qquad \text{computed by} \qquad u_{k+1} \;=\; u_k \frac{K^*}{K^*1}\left(\frac{f}{Ku_k}\right) \qquad (1)$$

Total variation (TV):

- TV is defined by

$$\textit{formal:}\;\; |u|_{BV} = \int_\Omega |\nabla u| \qquad\qquad \textit{exact:}\;\; |u|_{BV} = \sup_{\substack{g\in C_0^\infty(\Omega,R^d)\\ ||g||_\infty\leq 1}} \int_\Omega u \,\mathrm{div} g$$

EM-TV approach:

- Modify negative log-likelihood functional in (1) by adding a weighted TV term [4],

$$\min_{\substack{u\in BV(\Omega)\\ u\geq 0}} \int_\Sigma (Ku - f\log Ku)\, d\mu \;+\; \alpha\, |u|_{BV} \;, \qquad \alpha > 0$$

- In effect, images with smaller total variation are preferred in the minimization
- Optimality condition: $0 \in K^*1 - K^*\left(\frac{f}{Ku}\right) \;+\; \alpha\, \partial |u|_{BV}$, $\quad \partial$ is subdifferential
- Algorithm is implemented by a nested two step iteration

$$\left\{ \begin{array}{lll} u_{k+\frac{1}{2}} & = & u_k \dfrac{K^*}{K^*1}\left(\dfrac{f}{Ku_k}\right) \\ u_{k+1} & = & u_{k+\frac{1}{2}} - \tilde{\alpha}\, u_k\, p_{k+1}, \qquad p_{k+1} \in \partial |u_{k+1}|_{BV} \;, \qquad \tilde{\alpha} := \dfrac{\alpha}{K^*1} \end{array} \right\} \qquad (2)$$

- First step corresponds to the classical EM step
- Second step (TV step) can be realized by a nonlinear and non-differentiable variational problem (weighted ROF model) which suppresses noise effectively,

$$u_{k+1} \;=\; \arg\min_{u\in BV(\Omega)} \left\{ \frac{1}{2}\int_\Omega \frac{(u-u_{k+\frac{1}{2}})^2}{u_k} \;+\; \tilde{\alpha}\; |u|_{BV} \right\} \qquad (3)$$

Numerical realization (dual approach):

- Use exact definition for TV and solve latter problem by use of duality [5]

$$u_{k+1} = u_{k+\frac{1}{2}} - \tilde{\alpha}\, u_k \,\mathrm{div}\tilde{g} \qquad \text{with} \qquad \tilde{g} = \arg\min_{g\in C_0^\infty(\Omega,R^d)} \int_\Omega \frac{(\tilde{\alpha}\, u_k\, \mathrm{div} g - u_{k+\frac{1}{2}})^2}{u_k} \quad \text{s.t. } ||g||_\infty - 1 \leq 0$$

- Algorithm obtains sharp edges very well and is very robust
- Compute the optimization problem for $\tilde{g}$ by a semi-implicit gradient descent algorithm,

$$g^{n+1} \;=\; \frac{g^n + \tau\; \nabla(\tilde{\alpha}\, u_k\, \mathrm{div} g^n - u_{k+\frac{1}{2}})}{1 + \tau\; |\nabla(\tilde{\alpha}\, u_k\, \mathrm{div} g^n - u_{k+\frac{1}{2}})|} \;, \qquad 0 \;<\; \tau \;<\; \frac{1}{4\,\tilde{\alpha}\, u_k}$$

Results

We illustrate our technique by evaluating cardiac $H_2{}^{15}O$ measurements. To illustrate the SNR problem we present reconstructions with the classical EM algorithm (A). As expected, the results suffer from unsatisfactory quality. We take EM reconstructions with Gauss smoothing (B) as reference. The next results (C - F) show different application strategies of TV smoothing with (weighted) ROF model (3). The classical approach is a complete EM algorithm run with a following single denoising step. The result C demonstrates this approach with the standard ROF model, i.e. $u_k = 1$ and $u_{k+1/2}$ = EM reconstruction in (3). The result D is generated similarly to C but with weighted ROF model, i.e. $u_k = u_{k+1/2}$ = EM reconstruction in (3). Our approach with nested EM-TV algorithm (2) is presented in F. The reconstruction E is the same result as in D where the values are scaled to the maximum intensity of F.

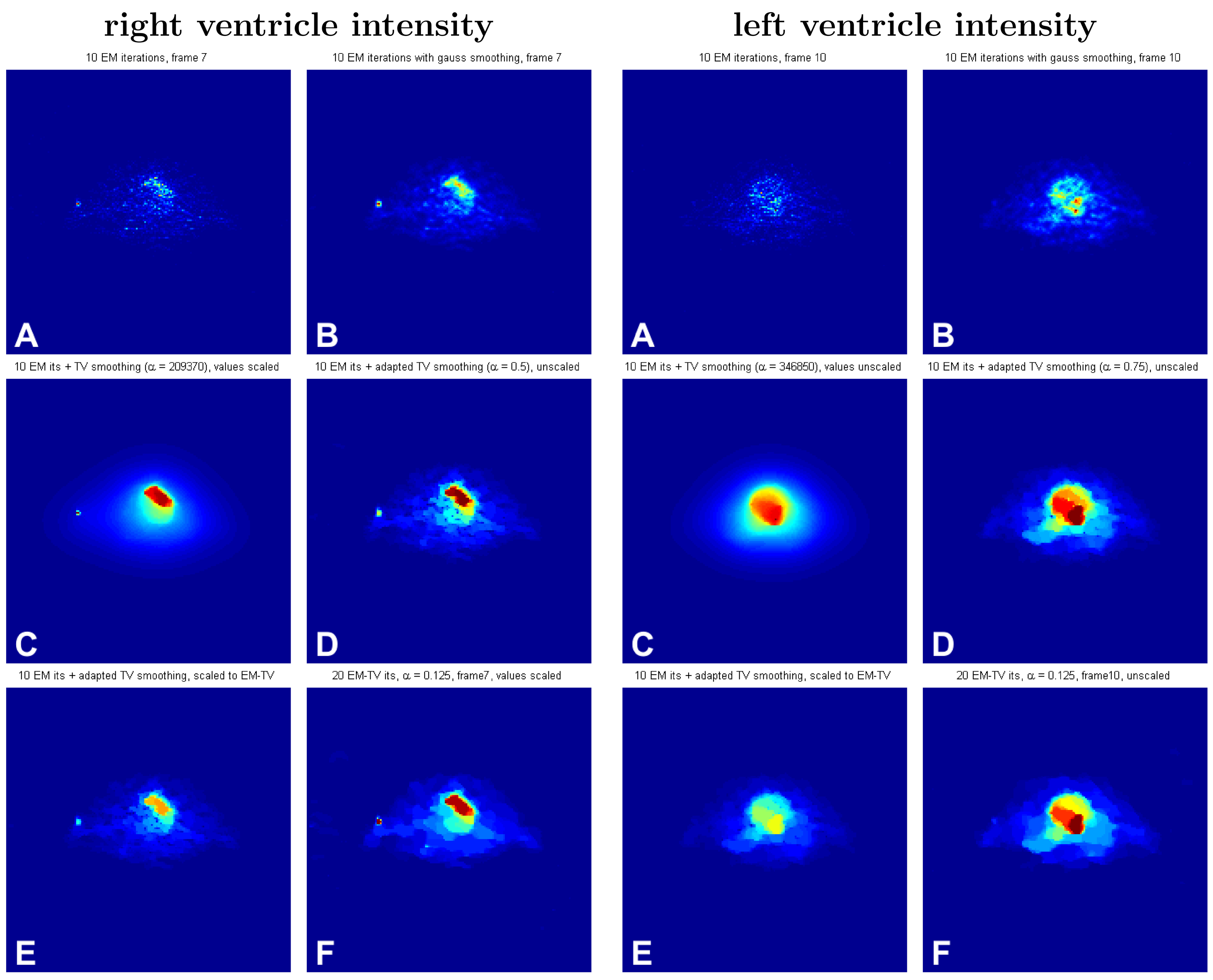

FIGURE 1: *Cardiac $H_2{}^{15}O$ PET measurements: results of different reconstruction methods in two different time frames.* ***A:*** *classical EM algorithm* (1). ***B:*** *EM with Gauss smoothing.* ***C:*** *EM with following TV smoothing* (3) *where $u_{k+1/2}$ = EM reconstruction and $u_k = 1$.* ***D:*** *As C but with $u_{k+1/2} = u_k$ = EM reconstruction in* (3). ***E:*** *The values of D are scaled to maximum intensity of F.* ***F:*** *Nested EM-TV algorithm* (2).

Discussion

The study demonstrates that denoising techniques based on total variation minimization deliver better results than the standard Gauss smoothing approach. The advantage is that cartoon reconstruction with TV preserves desired structures, e.g. homogeneous regions with sharp edges, very well. Furthermore, the reconstructions in D and F with weighted ROF model have a qualitative better resolution than the standard model. The reason is the adaptation of the regularization parameter with u_k. For the further use, e.g. segmentation for quantification of myocardial blood flow [6], the reconstructions with a single denoising step (D) are satisfactory. For quantitative conclusions the reconstructions with nested EM-TV algorithm deliver better interpretations, see Fig. 1 (E and F). Another advantage of EM-TV algorithm (2) is the robust convergence, in contrast to EM with following smoothing step.

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