

Degeneration of variations of Hodge structure in the Baily–Borel compactification

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Abstract

We prove an analogue for Hodge modules of Pink’s theorem on the degeneration of ℓ -adic sheaves (Math. Ann. 292). Let j be the open immersion of a Shimura variety M into its Baily–Borel compactification M^* . The boundary $M^* - M$ has a natural stratification into locally closed subsets, each of which is itself a Shimura variety (up to taking the quotient by the action of a finite group). Let i be the inclusion of an individual such stratum M' . Saito’s formalism gives a functor i^*j_* from the bounded derived category of Hodge modules on M to that of Hodge modules on M' . Our result gives a formula for the effect of i^*j_* on *automorphic* Hodge modules, i.e., variations of Hodge structure coming from algebraic representations of the group associated to M . This formula is of a purely representation theoretical nature.