

# Towards a mod. $p$ local Langlands correspondence

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## Abstract

Let  $F$  be a local field with finite residue field of characteristic  $p > 0$ . A correspondence has been established between supercuspidal  $\mathbf{C}$ -representations of  $GL_n(F)$  and irreducible  $n$ -dimensional  $\mathbf{C}$ -representations of the Weil group of  $F$  (Laumon-Rapoport-Stuhler if  $\text{char}(F) = p$ ; Harris-Taylor/Henniart if  $\text{char}(F) = 0$ ). Vigneras has shown how to deduce from these results an analogous correspondence between  $\overline{\mathbf{F}}_\ell$ -representations under the crucial assumption  $\ell \neq p$ . When  $\ell = p$ , the question is entirely open. I will speak about recent partial and quite elementary results that entertain the hope there should also be a mod.  $p$  correspondence.