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# Acronyms

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cdf	(cumulative) distribution function
chf	characteristic function
CLT	central limit theorem
FT	Fourier transform
iff	if, and only if
iid	independent and identically distributed
mgf	moment generating function
MRW	Markov random walk
RP	renewal process
RW	random walk
SLLN	strong law of large numbers
WLLN	weak law of large numbers