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SS 2012

3.Übung zur Vorlesung Gebäude

Please hand in your solutions on the morning of Friday 27 April before the lecture.

Aufgabe 3.1 (1. Coxeter Systems)

Let (W, I) be a Coxeter system.
(a) (2 marks) Show that if $w \in W$ and $i \in I$ then $l(iw) = l(w) \pm 1$.
(b) (4 marks) Suppose that W is finite. Show that the Coxeter group W has an element w_0 of maximal length and that

$$l(w_0) = l(w) + l(w^{-1}w_0) \text{ for all } w \in W.$$

Aufgabe 3.2 (2. Parabolic Subgroups)

Let (W, I) be a Coxeter system. Let $J, K \subseteq I$ be two sets, and let $W_J = \langle J \rangle$ and $W_K = \langle K \rangle$ denote the subgroups generated by J and K, respectively. Show that:

(a) (2 marks) The length of a reduced decomposition of an element in W_J using elements of J is equal to the length of a reduced decomposition of the same element using elements of I.
(b) (2 marks) Let $i \in I \setminus J$ and $w \in W_J$, show that $l(iw) = l(w) + 1$.
(c) (2 marks) Show that $W_J \cap W_K = W_{J \cap K} = \langle J \cap K \rangle$. (Hint: use induction on the word length and (b)).

Aufgabe 3.3 (3. Folding Condition)

(4 marks) Suppose that W is a group with a generating set I of elements of order 2. The folding condition (**F**) says:

(**F**) Let $w \in W$, and $i, j \in I$ be such that $l(iw) = l(w) + 1$ and $l(wj) = l(w) + 1$, then either $l(iwj) = l(w) + 2$ or $iwj = w$.

Show that (**E**) $\Rightarrow$ (**F**) and (**F**) $\Rightarrow$ (**D**), in particular (**F**) is satisfied by a Coxeter system.

(Therefore the three conditions (**D**), (**E**), and (**F**) are equivalent. A group with a generating set of involutions which satisfies any one of these three equivalent conditions is in fact a Coxeter group.)