8. Übungszettel zur Vorlesung „Räume nichtpositiver Krümmung“

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WWU Münster

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Each question is worth 4 points.

Aufgabe 8.1 (Finitely generated subgroups)
Suppose that G and H are groups and that $f : G \to H$ is an epimorphism. Show that G is finitely generated if H and $\mathrm{Ker}(f)$ are finitely generated.

Aufgabe 8.2 (Isomorphisms of $\mathbb{Z}^m$)
Let $f = (f_1, \dots, f_m)$ be any generating set for $\mathbb{Z}^m$. Show that the canonical epimorphism $\phi_f : \mathbb{Z}^m \to \mathbb{Z}^m$ that maps $(z_1, ..., z_m)$ to $\sum_{j=1}^m z_j f_j$ is injective and hence an isomorphism.

Hint: Proceed similarly as in 4.18 Lemma C from class notes.

Aufgabe 8.3 (General linear group over integers)
Let $\mathrm{GL}(m, \mathbb{Z})$ denote set of all $m \times m$ matrices with integer coefficients and determinant ± 1.

a) Show that $\mathrm{GL}(m, \mathbb{Z})$ is a group.

b) Show that $\mathrm{GL}(m, \mathbb{Z})$ is infinite and non-abelian if $m > 1$.

c) Show that $\mathrm{GL}(m, \mathbb{Z})$ is the automorphism group of $\mathbb{Z}^m$.

Aufgabe 8.4 (Finite order elements)
Give an example of a group G for which the subset of finite order elements $H = \{g \in G \mid \text{the order of } g \text{ is finite}\}$ is not a subgroup.

8.*-Aufgabe (Subgroups of finitely generated group)
Consider the multiplicative group G generated by the real matrices $a = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Let H be the set of all matrices in G whose diagonal entries are 1. Show that H is a subgroup that is not finitely generated.

Abgabe bis: Donnerstag, den 14.1.2021, 8 Uhr online im Learnwebkurs.