Blockseminar Geometrische Gruppentheorie
Karlsruhe-Münster-Regensburg

24.–27. September 2012

The aim of the seminar is to understand the structure and theory of the isometry groups of CAT(0)-spaces and their lattices. The papers [6, 7] by Caprace and Monod provide the fundamental theory. Our background reference for CAT(0)-spaces is the book by Bridson and Haefliger [4]. Further papers are taken as a reference in suitable cases and if needed.

Program

Timetable

Background and classical results

Talk 1 (CAT(0) geometry I). Elementary introductory talk which discusses the definition and basic properties of CAT(0) geometry
Speaker: Francesca
Literature: [2–4]

Talk 2 (CAT(0) geometry II). Discuss main properties of CAT(0) geometry such as the Tits boundary, types of isometries,...
Speaker: Werner
Literature: [2–4, 7]

Talk 3 (lattices in products of trees). Overview with examples of lattices in products of trees (such as constructed in [9] for example)
Speaker: Malte
Literature: [9, 10]

Classes of examples of CAT(0)-spaces

Talk 4 (Symmetric spaces). Definition and overview with examples (such as $SL_n(\mathbb{R})/SO(n)$) of symmetric spaces of non-compact type.
Speaker: Markus
Literature: [2, 3]

Talk 5 (Euclidean buildings - an introduction from a metric viewpoint). Discuss metric definitions and characterizations of Euclidean buildings. (In particular the (easy to state) characterizations of Euclidean buildings of Kleiner (for a proof see [12]) and Charney-Lytchak.)
Speaker: Daniel
Literature: [11, 12]

Talk 6 (Characterizations of symmetric spaces and Euclidean buildings). Discuss results of mentioned literature which are used in the proof of in [7, Theoren 1.3].
Speaker: Matthias
Literature: [14, 15]

Talk 7 (Non-classical examples). Discussion of [1]. Refer to [8] for a general discussion of non-classical CAT(0)-examples.
Speaker: Lukas
Literature: [1, 8]

Structure theory of CAT(0)-spaces

We want to understand the generalized de Rham decomposition of isometry groups of CAT(0)-spaces and of the CAT(0)-spaces themselves as proved in [7].

Talk 8 (Decomposing CAT(0) spaces I). Discussion of Theorem 1.1 and 1.6 and 1.9. Explain chapter 5, which includes the proof of Theorem 1.9.
Speaker: Stefan
Literature: [7]

Talk 9 (Decomposing CAT(0) spaces II). Conclusion of the proof of Theorems 1.1 and 1.6 [7].
Speaker: Petra
Literature: [6, 7]

Lattices in the isometry groups of CAT(0)-spaces

The broad topic of this part are generalizations of classic theorems about lattices in Lie groups to CAT(0)-lattices.

Talk 10 (The Borel density theorem). Theorem 1.1 and 2.4 in [6]. Explain why it generalizes the classical Borel density theorem.
Speaker: Matthew
Literature: [6]

Talk 11 (Arithmeticity I). This and the following talk should give an overview of Theorem 1.9, which generalizes Margulis' arithmeticity. See also [7, section 8] for associated superrigidity statements.
Speaker: Roman
Literature: [6]

Talk 12 (Arithmeticity II). Continuation of the previous talk.
Speaker:
Literature: [6]

References

[1]   W. Ballmann and M. Brin, *Polygonal complexes and combinatorial group theory*, Geom. Dedicata **50** (1994), no. 2, 165–191.

[2]   Werner Ballmann, *Lectures on spaces of nonpositive curvature*, DMV Seminar, vol. 25, Birkhäuser Verlag, Basel, 1995. With an appendix by Misha Brin.

[3]   Werner Ballmann, Mikhael Gromov, and Viktor Schroeder, *Manifolds of nonpositive curvature*, Progress in Mathematics, vol. 61, Birkhäuser Boston Inc., Boston, MA, 1985.

[4]   Martin R. Bridson and André Haefliger, *Metric spaces of non-positive curvature*, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 319, Springer-Verlag, Berlin, 1999.

[5]   Pierre-Emmanuel Caprace and Nicolas Monod, *Decomposing locally compact groups into simple pieces*, Math. Proc. Cambridge Philos. Soc. **150** (2011), no. 1, 97–128.

[6] _____, *Isometry groups of non-positively curved spaces: discrete subgroups*, J. Topol. **2** (2009), no. 4, 701–746.

[7] _____, *Isometry groups of non-positively curved spaces: structure theory*, J. Topol. **2** (2009), no. 4, 661–700.

[8] Benson Farb, Chris Hruska, and Anne Thomas, *Problems on automorphism groups of nonpositively curved polyhedral complexes and their lattices*, Geometry, rigidity, and group actions, Chicago Lectures in Math., Univ. Chicago Press, Chicago, IL, 2011.

[9] Hyman Bass and Alexander Lubotzky, *Tree lattices* (2001).

[10] Marc Burger and Shahar Mozes, *Lattices in product of trees* (2001).

[11] Ruth Charney and Alexander Lytchak, *Metric characterization of spherical and Euclidean buildings*.

[12] Andreas Balser and Alexander Lytchak, *Building-like spaces*.

[13] Bruce Kleiner and Bernhard Leeb, *Rigidity of symmetric spaces and Euclidean buildings* (1997).

[14] Bernhard Leeb, *A characterization of ireducible symmetric spaces and Euclidean buildings of higher rank by their asymptotic geometry* (2000).

[15] Alexander Lytchak, *Rigidity of spherical buildings and joins* (1997).