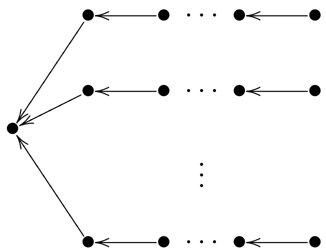
Abstract

The s-tame dimension vectors for stars

A *quiver* $Q = (Q_0, Q_1, s, t)$ is given by a set of vertices Q_0 and a set of arrows Q_1 and two maps $s, t : Q_1 \to Q_0$ which assign to every arrow $\alpha \in Q_1$ its *starting point* $s(\alpha)$ and its *terminating point* $t(\alpha)$. A *star* Q is a quiver of the following shape:



A *subspace representation* of a star Q is a collection of vector spaces V_i, $i \in Q_0$, over a particular field K together with K-linear maps $V_\alpha : V_{s(\alpha)} \to V_{t(\alpha)}$, $\alpha \in Q_1$, which are all injective. A *dimension vector* for a representation $(V_i, V_\alpha)_{i \in Q_0, \alpha \in Q_1}$ is given by $(d_i)_{i \in Q_0}$, where d_i is the dimension of the vector space V_i, $i \in Q_0$.

The *Tits form* $q : \mathbb{Z}^{Q_0} \to \mathbb{Z}$ for a quiver Q is given by

$$q((d_i)_{i \in Q_0}) = \sum_{i \in Q_0} d_i^2 - \sum_{\alpha \in Q_1} d_{s(\alpha)} d_{t(\alpha)}.$$

In 1999, Magyar, Weyman and Zelevinsky classified all dimension vectors, for which there are only finitely many isomorphism classes of subspace representations.

A dimension vector of a star is called *s-tame*, if there is at least one one-parameter family, but no n-parameter family of subspace representations with $n \geq 2$, and it is called *s-hypercritical*, if there is an n-parameter family of subspace representations with $n \geq 2$, but there is no proper decomposition as a sum of dimension vectors of subspace representations with an m-parameter family of subspace representations with $m \geq 2$ for one of the summands.

In the first part of this thesis, the complete lists of all s-tame and s-hypercritical dimension vectors of stars are given, provided the underlying field is algebraically closed. In particular, the following is shown:

Theorem 1. A dimension vector $\mathbf{d}$ of a star is s-tame, if and only if the following conditions are satisfied:

1. $q(\mathbf{d}) = 0$, and
2. $q(\mathbf{d}') \geq 0$ for all $\mathbf{d}' \leq \mathbf{d}$.

Theorem 2. A dimension vector $\mathbf{d}$ of a star is s-hypercritical, if and only if the following conditions are satisfied:

1. $q(\mathbf{d}) < 0$, and
2. $q(\mathbf{d}') \geq 0$ for all $\mathbf{d}' < \mathbf{d}$.

In the second part of the thesis, the isomorphism classes of the families of indecomposable subspace representations for the s-tame and s-hypercritical dimension vectors are constructed explicitly.