Research Workshop on KK-Theory and its Applicaations

July 20 - July 24, 2009

Equivariant K-homology, Paul Baum

K-homology is the dual theory to K-theory. There are three points of view on K-homology : homotopy theory, BD (Baum-Douglas) , and AK (Atiyah-Kasparov). The BD approach defines K-homology via geometric cycles and homologies of these cycles. The resulting theory in a certain sense is simpler and more direct than classical homology. For example, K-homology and K-theory are made into equivariant theories in an utterly immediate and canonical way. For classical (co) homology there is an ambiguity abut what is the "correct" definition of equivariant (co)homology. In the twisted case, the geometric cycles of the BD theory are the D-branes of string theory. This talk will give the definition of equivariant BD theory and its extension to a bivariant theory. An application to the BC (Baum-Connes) conjecture will be indicated.

A Mayer-Vietoris sequence for metric spaces, Jacek Brodzki

This talk will present an approach for computing K-theory of a certain class of C*-algebras, called partial translation algebras, associated with discrete metric spaces. A partial translation algebra is set up to capture information provided by the "partial symmetries" of the space which are provided by a chosen class of partially defined bijections on the space, known as a partial translation structure. A very interesting class of examples of such spaces is provided by metric subspaces of discrete groups and in this case we shall derive a Mayer-Vietoris type sequence that links the C*-algebra of the space with the reduced C*-algebra of the group. This provides a unifying framework for a number of classical C*-algebra extensions, including the Toeplitz extension, the Cuntz extension, etc. We shall present an example K-theoretic applications of this result.

E-theory for C*-algebras over a space, Marius Dadarlat

(Joint work with Ralf Meyer) We introduce E-theory for separable C*-algebras over possibly non-Hausdorff topological spaces and establish its basic properties, such as six-term exact sequences in each variable and a universality property. We also plan to discuss certain applications.

Equivariant correspondences and applications, Heath Emerson

We describe two applications of the theory of equivariant correspondences developed by the author and Ralf Meyer. The first is an apparently new proof of the shift in parameters occurring in the Borel-Bott-Weil theorem. The second is a generalisation of the Lefschetz fixed-point formula. The Borel-Bott-Weil theorem describes a specific relationship between the holomorphic representations of a complex semisimple Lie group induced by varying the parabolic subgroup used to define them. This shift can be explained by first invoking the Atiyah-Singer index theorem to lift the objects concerned to appropriate equivariant correspondences, and then by computing a certain intersection product in topological KK-theory. For the second application, to Lefschetz formulas, the general framework of duality, the theory of correspondences, and some some homological algebra techniques developed by Ralf Meyer, are combined to yield an analogue of this basic theorem of topology which works both equivariantly with respect to a proper groupoid, and applies to equivariant correspondences, not just maps. In the resulting formula we use \(\K\)-oriented coincidence spaces of equivariant correspondences on the geometric side, and the Hattori-Stallings trace on the homological side. The first application is joint work with Robert Yuncken, and the second is joint work with Ralf Meyer.

C*-algebras and Harish-Chandra theory, Pierre Julg

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Cone-depending KK-theory, Eberhard Kirchberg

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The K-groups of certain ring C*-algebras and a duality theorem for global fields, Xin Li

I will report on joint work with Joachim Cuntz. Recently, there has been a growing interest in links between operator algebras and number theory. Motivated by these developments, we study the following C*-algebras:

Take any (discrete) ring R and consider the Hilbert space of square-summable functions on R. The ring structure of R gives rise to unitaries and isometries corresponding to addition and multiplication, respectively. These operators generate a C*-algebra, the (reduced) ring C*-algebra of R.

In particular, we can apply this construction to the ring of integers in a number field. An important step in understanding the associated ring C*-algebra is to determine its K-theory. In my talk, I will explain how to achieve this goal (at least for a first class of number fields). The main ingredient is a duality theorem which holds true for arbitrary global fields. It identifies the crossed products arising from affine transformations on the finite adeles and the infinite ones

Asymptotic homomorphisms and tensor products of C*-algebras, Vladimir Manuilov

Replacing representations by asymptotic representations in the definition of the minimal tensor product norm of C*-algebras, one gets a new tensor product C*-norm. We discuss this norm and its variants. For one variant of this tensor C*-norm, we show that the corresponding tensor product is not associative.

Non-Hausdorff symmetries of C*-algebras, Ralf Meyer

Badly behaved quotient spaces are commonly described by groupoids and their convolution algebras. The symmetry groups of such quotient spaces may be non-Hausdorff as well. Higher category theory provides generalisations of groups and group actions that are appropriate to study such non-Hausdorff symmetries of algebras and C*-algebras.

A 2-category has three levels of data: objects, arrows between objects, and arrows between arrows. The first example is the 2-category of categories, which comprises categories, functors between categories, and natural transformations between functors. We enrich the category of C*-algebras by viewing unitary intertwiners between *-homomorphisms as arrows between morphisms. This 2-category allows us to formalise in which sense inner automorphisms are trivial, and it provides a good notion of a group action that is defined up to inner automorphisms. The subtle point is that we must require the unitaries that implement inner automorphisms to satisfy a certain cocycle condition. Such twisted group actions were already studied by Busby and Smith. General 2-category theory explains the cocycle conditions that appear for twisted actions and for outer equivalence of actions.

Non-Hausdorff symmetry groups of C*-algebras manifest themselves by actions of 2-categories on C*-algebras. I will discuss some examples of this such as the gauge symmetry of rotation algebras.

Spectral flow formulas and KMS states, Ryszard Nest

We will describe a general construction of K-theoretic invariants associated with KMS states with (quasi)periodic action of modular group and some explicit residue type formulas for their computations.

Algebraic K-theory of properly infinite C*-algebras, N. Christopher Phillips

Let A be a properly infinite unital C*-algebra. Then the comparison map from the algebraic K-theory of A to the topological K-theory of A (what C*-algebraists think of as the K-theory of A) is an isomorphism. This is joint work with Guillermo Cortinas.

Base change and K-theory for GL(n,R), Roger Plymen

In the general theory of automorphic forms, an important role is played by base change. Base change has a global aspect and a local aspect. In this talk, we focus on the archimedean case of base change for the general linear group GL(n,R), and we investigate base change for this group at the level of K-theory.

We investigate the interaction of base change with the Baum-Connes correspondence for GL(n,R) and GL(n,C). If there is time, we will touch on the corresponding result for nonarchimedean local fields.

Joint work with Sergio Mendes, see arXiv:math/0607522v2 [math.KT] and Journal of Noncommutative Geometry 1 (2007) 311--331.
[pdf-version]

New examples of holomorphically closed subalgebras of group C*-algebra, Michael Puschnigg

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Extensions of the reduced group C*-algebra of a free product of amenable groups, Klaus Thomsen

All extensions of a C*-algebra as in the title by a separable stable C'-algebra are semi-invertible. In the talk I will explain what this means, put the result into perspective and sketch the proof.

An index theorem in the gauge-equivariant K-theory, Evgenij V. Troitsky

(joint work with Victor Nistor)

We consider a family of elliptic operators invariant with the respect to an action of a family of compact Lie groups. Under some natural assumptions analytic and topological indices are defined with values in the appropriate version of twisted K-theory and the corresponding index theorem is proved.

On the coarse Baum-Connes conjecture with coefficients, Jean-Louis Tu

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The Baum-Connes Conjecture for KK-theory, Otgonbayar Uuye

The Baum-Connes Conjecture (BCC) as formulated by Baum-Connes-Higson has a straightforward generalization to KK-theory. This allows one to treat, for instance, K-theory with coefficients. While this generalization behaves nicely for groups such as connected Lie groups, it fails for trivial reasons if the group has a big universal proper space. The remedy is to work with the formulation of the BCC given by Meyer-Nest.

Towards a bivariant Cuntz semigroup, Wilhelm Winter

The Cuntz semigroup has played an increasingly important role for the structure theory and the classification of nuclear C*-algebras in recent years. As an invariant, it is much finer than K-theory (and in general much harder to compute). The two are nontheless closely related (in many cases the Cuntz semigroup is entirely determined by the Elliott invariant), and both may certainly be regarded as homological invariants. It is tempting to ask whether it is possible to use ideas from KK-theory to define a bivariant version of the Cuntz semigroup.

In the talk, we discuss what one should (and should not) expect from such a bivariant theory. We propose a working definition, based on the concept of c.p.c. order zero maps, and explain a number of promising results along these lines.

This is work in progress, partially joint with Andrew Toms, and with Joachim Zacharias.

Constructing spectral triples on C*-algebras, Joachim Zacharias

Spectral triples are a refinement of K-homology cycles modeled on the Dirac operator of a compact spin manifold. There are various regularity properties of spectral triples connected to dimension and Rieffel's notion of noncommutative or quantum metric space. We review various examples of spectral triples which have been constructed (e.g. on group C*-algebras, irrational rotation algebras and AF-algebras) and present some new examples and open problems.

This is joint work in progress with Adam Skalski and Stuart White.