Reduced Basis Method for Parametrized Evolution Equations

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Many transport phenomena in physical systems can be modelled by parametrized partial differential equations (P^2DEs). Examples are the fuel, oxygen and water transport in the different components of a fuel cell parametrized by material, geometry or control parameters. In simulation of such problems, repeated computation runs for varying parameters and fast simulation response can be required. Such scenarios occur in design optimization, optimal control with PDE-constraints, online-simulation, parameter identification or state estimation. Model order reduction techniques must be applied to satisfy these time demands and are already applied successfully in fuel cell simulation [2]. The model order reduction methodology of *reduced basis (RB) methods* [3] offers efficient treatment of P^2DEs by providing both approximate solution procedures and efficient error estimates. RB-methods have so far mainly been applied to finite elements schemes for elliptic and parabolic problems. In the current study we extend the methodology to general evolution schemes such as finite volume schemes for parabolic and hyperbolic transport equations [1]. The RB-methodology becomes applicable by requiring a linear numerical scheme and affine parameter dependence in the data functions. This results in an offline-online decomposition which is the basis for a computationally efficient realization of both the approximation scheme and the rigorous a-posteriori error estimators for a wide range of parameters. Numerical experiments on an advection-diffusion problem demonstrate the applicability of the approach and the accuracy of the error estimates.

References

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