

## ÜBUNGEN ZUR VORLESUNG TOPOLOGIE II

### Aufgabenblatt 11

Abgabe: Mittwoch, 8.7., in der Vorlesung

**Exercise 11.1.** Let  $f : \mathbb{S}^n \rightarrow \mathbb{S}^n$  be a map of degree  $k$ . From the lecture, you know that  $H^n(\mathbb{S}^n; A) \cong A$  for an arbitrary abelian group. What is the map  $f^* : H^n(\mathbb{S}^n; A) \rightarrow H^n(\mathbb{S}^n; A)$ ?

**Exercise 11.2.** Let  $F_g$  be a closed orientable surface of genus  $g$ . Construct a CW structure on  $F_g$  and use this CW structure to compute the cohomology of  $F_g$  (with arbitrary coefficients).

**Exercise 11.3.** Compute the cohomology of  $\mathbb{R}P^n$  using the CW structure given in the lecture with coefficients in  $\mathbb{Z}/2$  and  $\mathbb{Z}$  and compute the Bockstein sequence for the short exact coefficient sequence  $0 \rightarrow \mathbb{Z} \xrightarrow{2} \mathbb{Z} \rightarrow \mathbb{Z}/2 \rightarrow 0$ .

**Exercise 11.4.** Let  $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$  be a short exact sequence of abelian groups. Show that the induced sequence  $0 \rightarrow A \otimes \mathbb{Q} \rightarrow B \otimes \mathbb{Q} \rightarrow C \otimes \mathbb{Q} \rightarrow 0$  is also exact. Hint: to show the injectivity of  $A \otimes \mathbb{Q} \rightarrow B \otimes \mathbb{Q}$ , you have to use the classification theorem for finitely generated abelian groups.

**Exercise 11.5.** Let  $A$  be a finitely generated abelian group and let  $M(A, 1)$  be the Moore space from exercise 9.2. Compute  $H^*(M(B, 1); B)$  for an arbitrary abelian group  $B$ .