ÜBUNGEN ZUR VORLESUNG TOPOLOGIE II

Aufgabenblatt 6
Abgabe: Mittwoch, 27.5.2009 VOR DER BIBLIOTHEK[1]!!.

Exercise 6.1. The *Möbius band* is the topological space $M := [-1,1] \times [-1,1]/\sim$, where $\sim$ is the equivalence relation $(-1,t) \sim (1,-t)$ for all $t \in [-1,1]$. The *boundary* is $\partial M := \{(t,\pm 1) | t \in [-1,1]\}/\sim$. Compute the long exact homology sequence of the pair $(M; \partial M)$.

Exercise 6.2. Compute the singular homology of the 2-dimensional torus $T = \mathbb{S}^1 \times \mathbb{S}^1$. Let $x_1, \ldots x_r \in T$ be pairwise distinct points. Compute the singular homology of $T \setminus \{x_1, \ldots x_r\}$.

Exercise 6.3. Let $\mathbb{S}^n \subset \mathbb{R}^{n+1}$ be the standard sphere and $\mathbb{S}^k \subset \mathbb{S}^n$ be the unit sphere in $\mathbb{R}^{k+1}$. Compute the singular homology of $\mathbb{S}^n \setminus \mathbb{S}^k$ without using Theorem 2.3 from the lecture.

Exercise 6.4. Let $A \subset U \subset X$ be topological spaces, where A is closed and U is open. Assume further that A is a strong deformation retract of U (reminder: this means that there exists a map $P : [0,1] \times U \to U$ such that $P(0,u) = u$, $P(1,u) \in A$ for all $u \in U$ and $P(t,a) = a$ for all $(t,a) \in [0,1] \times A$.) Show that the natural map of pairs $(X;A) \to (X/A, *)$, where $* = A/A$ induced an isomorphism $H_n(X;A) \to H_n(X/A, *)$ for all $n \in \mathbb{N}$. Hint: you have to show the following steps:

(1) Show that there is an open neighborhood $V \subset X/A$ of $*$ such that $*$ is a strong deformation retract of V.
(2) Show that $H_n(X;U) \cong H_n(X;A)$ and $H_n(X/A;V) \cong H_n(X/A;*)$.
(3) Use (1) and (2) and the excision theorem to get the conclusion.

Exercise 6.5. The group $SU(2)$ of unitary 2×2 matrices with complex entries and determinant 1 inherits a toplogy from the vector space of all matrices. With that topology, $SU(2)$ becomes a topological group that is homeomorphic to the sphere $\mathbb{S}^3$. Let $f_k : SU(2) \to SU(2)$ be the map that takes $A \in SU(2)$ to A^k; $k \in \mathbb{Z}$. Show that the mapping degree of f is k. Hint: first find an element $A \in SU(2)$ such that $f_k^{-1}(A)$ has precisely $|k|$ points.

[1]There will be no lecture on that day. There is a cart in the foyer in front of the library. On this cart you find the folders for the exercise sheets. The tutors will collect the exercises at 12 p.m.