Local currents in the continuous spin representations of the Poincaré group

Christian Köhler

Uni Göttingen

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Poincaré symmetry in Quantum Mechanics

- Physical system described by *Hilbert space* $(\mathcal{H}, \langle \cdot, \cdot \rangle)$, its states are *rays*

$$\Psi = \mathbb{C} \, |\psi\rangle \in \mathbb{P}(\mathcal{H}) \text{ where } |\psi\rangle \in \mathcal{H}$$

- Poincaré symmetry: *projective repn* of the group $\mathcal{P}_+^\uparrow$

$$\begin{aligned} \hat{U} : \; \mathcal{P}_+^\uparrow = \mathcal{L}_+^\uparrow \ltimes M &\to \operatorname{Aut}(\mathbb{P}(\mathcal{H})) \\ L &\mapsto \hat{U}(L) \end{aligned}$$

($\mathcal{L}_+^\uparrow$: proper orthochronous Lorentz group, M: 4-dim Minkowski space)
preserves transition amplitudes, i.e.

$$P(\Psi \to \Phi) := \left| \left\langle \frac{\phi}{||\phi||}, \frac{\psi}{||\psi||} \right\rangle \right|^2$$

$$P(\hat{U}(L)\Psi \to \hat{U}(L)\Phi) = P(\Psi \to \Phi)$$

$$\forall \Phi, \Psi \in P(\mathcal{H}), L \in \mathcal{P}_+^\uparrow$$

- ray repn property: $\hat{U}(L_1L_2) = \hat{U}(L_1)\hat{U}(L_2)\ \forall L_1, L_2 \in \mathcal{P}_+^\uparrow$
- $\Rightarrow$(Wigner's Theorem) $\forall L \in \mathcal{P}_+^\uparrow\ \exists\ U(L) \in \mathrm{Aut}(\mathcal{H})$ real, isometric, (anti)unitary, unique up to a factor:

$$\mathbb{C}U(L)\psi = \hat{U}(L)\Psi$$

- Remark: $\mathcal{P}_+^\uparrow$ is the identity component of $\mathcal{P} = \mathcal{L} \ltimes M$: $\forall L \in \mathcal{P}_+^\uparrow = \mathcal{L}_+^\uparrow \ltimes M\ \exists\sqrt{L}$, thus $U(\mathcal{P}_+^\uparrow)$ consists of unitaries
- ray repn property $\Rightarrow$ composition picks up phase factors

$$U(L_1)U(L_2) = \mathrm{e}^{\mathrm{i}\omega(L_1,L_2)}U(L_1L_2)$$

- *Question:* Can one absorb them into $U(L)$? $\rightarrow$*group cohomology of* $\mathcal{P}_+^\uparrow$

- If $\hat{U}$ is weakly continuous, i.e. $\langle \hat{U}(L)\Phi, \Psi \rangle \overset{L \to (\mathbf{1}, \vec{0})}{\to} \langle \Phi, \Psi \rangle$
$\Rightarrow$(Wigner-Bargmann) For $\mathcal{P}^c$, the twofold cover of $\mathcal{P}_+^\uparrow \overset{\Lambda}{\leftarrow} \mathcal{P}^c$, there is a strongly continuous unitary repn U on $\mathcal{H}$:

$$\mathbb{C}U(L)\psi = \hat{U}(\Lambda(L))\Psi$$

- Construction of the (homogeneous) covering homomorphism $\Lambda : \mathrm{SL}(2, \mathbb{C}) \to \mathcal{L}_+^\uparrow$:
For $x \in M$ one puts

$$\underset{\sim}{x} := x^0 + \vec{\sigma} \cdot \vec{x} = \begin{pmatrix} x^0 + x^3 & x^1 - \mathrm{i}x^2 \\ x^1 + \mathrm{i}x^2 & x^0 - x^3 \end{pmatrix}$$

and for $A \in \mathrm{SL}(2, \mathbb{C})$ defines $\Lambda(A) \in \mathcal{L}$ (check!) implicitly by requesting

$$\underset{\sim}{(\Lambda(A)x)} = A\underset{\sim}{x}A^*$$

Since $\Lambda(A) = \Lambda(B) \Rightarrow A = \pm B$, one has a twofold cover

irreducible representations of $\mathcal{P}^c$

- Let U a [...] repn of $\mathcal{P}^c$ and P_μ the *generators of the translations* $(\mathbf{1}, a)$:

$$U((\mathbf{1}, a)) = \mathrm{e}^{\mathrm{i}Pa}$$

- in the action of $\mathcal{L}_+^\uparrow$ on M the covering map Λ enters:
  $\mathcal{P}$: $(\Lambda_1, a_1)(\Lambda_2, a_2) = (\Lambda_1\Lambda_2, \Lambda_1 a_2 + a_1)$
  $\mathcal{P}^c$: $(A_1, a_1)(A_2, a_2) = (A_1 A_2, \Lambda(A_1)a_2 + a_1)$
  in particular:
  $(A, 0)(\mathbf{1}, a)(A^{-1}, 0) = (A, 0)(A^{-1}, a) = (\mathbf{1}, \Lambda(A)a)$
- corresponding statement for the generators P_μ:

$$U(A)PU(A)^{-1} = P\Lambda(A)$$

- $\Rightarrow \mathrm{sp}P$ is invariant under $\Lambda(A)$ since

$$P\psi = p\psi \Rightarrow PU(A)^{-1}\psi = U(A)^{-1} \underbrace{P\Lambda(A)\psi}_{=p\Lambda(A)\psi}$$

$$= p\Lambda(A)U(A)^{-1}\psi$$

- p^2 is a Lorentz scalar $\Rightarrow [P^2, U(A)] = 0$
  $\Rightarrow$(Schur's Lemma) $P^2 = m^2\mathbf{1}$ in an irreducible rep
- w.r.t. the equiv. relation $p \sim p' :\Leftrightarrow \exists L \in \mathcal{L}_+^\uparrow : \; pL = p'$, $\mathrm{sp}P$ decomposes into Lorentz-orbits
- Orbits O of physical interest:

$$\begin{aligned} &\text{upper mass shell (massive particles)} \\ H_m^+ =& \{p \in M | p^2 = m^2, p_0 > 0\} \\ &\text{forward light cone (photons \& worse)} \\ \partial V_+ =& \{p \in M | p^2 = 0, p_0 > 0\} \to \text{later!} \end{aligned}$$

- In the following: Assume U s.th. P are diagonal!

- Construction of momentum-space wavefunctions: Define

$$\langle \phi, \psi \rangle_{\vec{p}} := \frac{1}{(2\pi)^3} \int \mathrm{d}^3x \, \langle \phi, U(\vec{x})\psi \rangle \mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}$$

  (Hepp/Jost: falloff of the integrand faster than polynomial)
- *Almost* a scalar-product, but semidefinite! Put $N_{\vec{p}} := \{\psi \in \mathcal{H} | \langle \psi, \psi \rangle_{\vec{p}} = 0\}$ and instead use

$$\mathcal{H}_p := \overline{\mathcal{H}/N_{\vec{p}}}^{||\cdot||_{\vec{p}}}$$

  $\rightarrow$bundle over momentum space - denote sections by $\psi(p)$
- Hilbert spaces $\mathcal{H}_p$ with sharp momentum $p = (\omega(\vec{p}), -\vec{p})$ are related via

$$\begin{aligned} \underline{U}(A): \ \mathcal{H}_{p\Lambda(A)} &\rightarrow \mathcal{H}_p \\ \psi(p\Lambda(A)) &\mapsto (U(A)\psi)(p) \end{aligned}$$

  using the repn U *before* projecting on $\mathcal{H}_p$.

the little group G_p

- Useful observation: Stabilizer subgroups G_p of $\mathrm{SL}(2,\mathbb{C})$ for any momentum $p \in O$

  $$G_p = \{A \in \mathrm{SL}(2,\mathbb{C}) | p\Lambda(A) = p\}$$

  (*little groups*) are conjugate within O

  $$G_p = A G_{p\Lambda(A)} A^{-1}$$

  (attention: read from left to right, since p transforms with a lower index!)
- repn's $\underline{U}|_{\mathcal{H}_p}$ are related likewise, since for $R \in G_{p\Lambda(A)}$

  $$\underline{U}(\underbrace{ARA^{-1}}_{\in G_p}) = \underline{U}(A)\underline{U}(R)\underline{U}(A)^{-1}$$

- Construction simplified: One only needs G_q for fixed q and *Wigner boosts* $A_p \in \mathrm{SL}(2,\mathbb{C}):\ q\Lambda(A_p) = p$.
- in the following: Implement this fallback to G_q on $\mathcal{H}$

- Use an *intertwiner* $V: \mathcal{H} \to L^2(H_m^+, \widetilde{\mathrm{d}p}) \otimes \mathcal{H}_q$:

$$
\begin{aligned}
(V\psi)(p) &= \underline{U}(A_p)\psi(p) \\
&= (U(A_p)\psi)(p\Lambda(A_p^{-1})) \in \mathcal{H}_q
\end{aligned}
$$

little Hilbert space-valued *wavefunctions*

- Conjugating the original repn U with the intertwiner V gives an equivalent repn

$$U' = VU(\cdot)V^{-1}$$

whose application to a vector $\psi \in \mathcal{H}$ now reads

$$
\begin{aligned}
(U'(A,a)\psi)(p) =& \mathrm{e}^{\mathrm{i}pa}\underline{U}(R(p,A))\psi(p\Lambda(A)) \\
R(p,A) =& A_p A A_{p\Lambda(A)}^{-1} \in G_q
\end{aligned}
$$

Wigner rotation

- As demanded, one only needs $\underline{U}(G_q)$.

massive particles

- Important example: Let $O = H_m^+$. Then one can pick $q = (m, \vec{0})$ as the reference vector.
- $\underset{\sim}{(m, \vec{0})} = m\mathbf{1}$, that is $(m, \vec{0})\Lambda(R) = (m, \vec{0}) \Leftrightarrow R^*R = 1$, hence

$$G_{(m,\vec{0})} = \mathrm{SU}(2, \mathbb{C})$$

- irreducible repn's of $\mathrm{SU}(2, \mathbb{C})$ are characterized by spin $s \in \frac{1}{2}\mathbb{N}_0$
- *Majorana description*: little Hilbert space

$$\mathcal{H}^{(s)} := (\mathbb{C}^2)^{\otimes 2s}_{\mathsf{sym}} = \mathrm{span}\xi^{\otimes 2s}, \xi \in \mathbb{C}^2$$

action of $\underline{U}$

$$\underline{U}(A)\xi^{\otimes 2s} := (A\xi)^{\otimes 2s}$$

for $A \in \mathrm{SU}(2, \mathbb{C})$ *or* $\mathrm{SL}(2, \mathbb{C})$.

$$|\uparrow\uparrow \cdots \uparrow\rangle$$
$$|\downarrow\uparrow \cdots \uparrow\rangle$$
$$|\downarrow\downarrow \cdots \uparrow\rangle$$
$$\vdots$$
$$|\downarrow\downarrow \cdots \downarrow\rangle$$

- Transformation law for a covariant vector determines the Wigner boost

$$\widetilde{p\Lambda(A)} = A^*\widetilde{p}A \Rightarrow A_p = \sqrt{\widetilde{p}/m} \text{ (pos. def.)}$$

- Consider the map

$$\begin{array}{rcl} u: \; H_m^+ \times (\mathbb{C}^2)^{\otimes 2s}_{\text{sym}} & \to & \mathcal{H}_q \\ (p, \xi^{\otimes 2s}) & \mapsto & \underline{U}(A_p)\xi^{\otimes 2s} \end{array}$$

which satisfies the *intertwiner property*

$$D(R(\Lambda(A), p))u(p\Lambda(A), \xi^{\otimes 2s}) = u(p, A^{\otimes 2s}\xi^{\otimes 2s})$$

- One can define a single particle state vector $\psi(f, v)$ for each $f \in \mathcal{S}(M)$, $v \in (\mathbb{C}^2)^{\otimes 2s}_{\text{sym}}$ by

$$\psi(f, v)(p) = \widetilde{f}(p)u(p, v)$$

- For $(\Lambda(A), a)$ one checks the transformation law

$$U((\Lambda(A), a))\psi(f, v) = \psi((\Lambda(A), a)_* f, A^{\otimes 2s}v)$$

1 Wigner particle classification

- Poincaré symmetry in Quantum Mechanics
- irreps of $\mathcal{P}^c$
- The little group G_p
- massive particles

2 (Faithful) repn's for massless particles

- rigid motions $E(2)$
- No-Go Theorem

3 Construction of String-localized fields

- single particle states
- general construction of intertwiners
- the continuous spin case
- pointlike localized currents

4 Results/Outlook

- Summary
- More warped convolutions (?)

rigid motions $E(2)$

- Now consider the case $O = \partial V_+$ and pick $q = (1/2, 1/2\vec{e}_z)$ as the reference vector.
- $\widetilde{(1/2, 1/2\vec{e}_z)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, that is $(1/2, 1/2\vec{e}_z)\Lambda(R) = (1/2, 1/2\vec{e}_z)$

$$\Leftrightarrow R^* \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} R = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \text{ hence}$$

$$G_{(1/2,1/2\vec{e}_z)} = \left\{ \underbrace{\begin{pmatrix} e^{i\varphi} & 0 \\ a & e^{-i\varphi} \end{pmatrix}}_{=:[\varphi,a]} \in \mathrm{SL}(2,\mathbb{C}) | \varphi \in \mathbb{R}, a \in \mathbb{C} \right\}$$

$$= \widetilde{E(2)}$$

the double cover of $E(2) \overset{\lambda}{\leftarrow} \widetilde{E(2)}$ (rigid motions in the Euclidean plane)

- little group is not compact!→more obstacles than in the $\mathrm{SU}(2,\mathbb{C})$-case!
- already known: only repn $\underline{U}$ of $G_{(1/2,1/2\vec{e}_z)}$ + *Wigner boost* needed, then proceed analogously to the case $O = H_m^+$
- Let $\underline{U}$ a repn of $\widetilde{E(2)}$ and G_i the *generators of the translations* $[0,a]$:

$$\underline{U}([0,a_1+\mathrm{i}a_2]) = \mathrm{e}^{\mathrm{i}Ga}$$

- like for $\mathcal{P}$ itself: homogeneous transformation of the generators G_i:

$$\underline{U}([\varphi,0])G\underline{U}([\varphi,0])^{-1} = G\lambda([\varphi,0])$$

- ⇒(as before) $\mathrm{sp}G$ is invariant under $\lambda([\varphi,0])$
- $g^2 = g_1^2 + g_2^2$ is a Euclidean scalar ⇒ $[G^2, \underline{U}([\varphi,0])] = 0$ ⇒(Schur's Lemma) $G^2 = \kappa^2\mathbf{1}$ in an irreducible rep

- For $m > 0$ repn's of $\mathrm{SU}(2,\mathbb{C})$ are classified by spin s.
- For $m = 0$ repn's of $\widetilde{E(2)}$ are classified by the *Pauli-Lubanksi parameter* κ^2
- w.r.t. the equiv. relation $x \sim x' :\Leftrightarrow \exists \lambda \in \mathrm{SO}(2,\mathbb{R}) : \; x\lambda = x'$, $\mathrm{sp}G$ decomposes into Rotation-orbits
- two types of Orbits O:

$$\begin{aligned} &\text{trivial repn of translations (helicity)} \\ O_0 =& \{\vec{0}\} \subset \mathbb{R}^2 \\ &\text{faithful repn of } \widetilde{E(2)} \\ O_\kappa =& \{x \in \mathbb{R}^2 | x^2 = \kappa^2\} \end{aligned}$$

No-Go Theorem

- *Question:* Can one construct Wightman fields for the case $m = 0, \kappa > 0$?
- Wightman axioms
  1. fields=operator valued distributions $\Phi(x, v)$ on $M \times K$ (K=space for inner degrees of freedom such as spin) mapping a dense subspace $\mathcal{H}_D$ of $\mathcal{H}$ into itself
     $A(f, v)^\dagger = A(\overline{f}, v^*)$
  2. $\mathcal{P}_+^\uparrow$ represented continuously by U
     shifts $U(\mathbf{1}, a) = \int \mathrm{d}E_p \mathrm{e}^{\mathrm{i}pa}$ satisfy the spectrum condition $\operatorname{supp} \mathrm{d}E_p \in \partial V_+$
     $U(\mathcal{P}_+^\uparrow)\mathcal{H}_D \subset \mathcal{H}_D$ and $\exists! \Omega \in \mathcal{H}_D : \; U(\mathcal{P}_+^\uparrow)\Omega = \{\Omega\}$
  3. locality:
     $[A(x, v), A(x', v')] = 0$ if $x >< x'$ and any $v, v' \in K$
  4. covariance:
     $\exists$ repn D of $\mathrm{SL}(2, \mathbb{C})$ on K s.th.
     $U(\Lambda, a)A(x, v)U(\Lambda, a)^{-1} = A(\Lambda x + a, D(\Lambda)v)$

- If one considers the subspace $\mathcal{H}'$ of vectors obtained by appplying $A(f,v)$ once to the vacuum Ω

$$\mathcal{H}' = \overline{\left\{ \sum_{i=1}^{n} A(f_i, v_i)\Omega | n < \infty, f_i \in \mathcal{D}(M), v_i \in K \right\}}$$

  $\Rightarrow$(Yngvason '69)

- There is no subspace $\mathcal{H}_{\mathrm{irr}} \subset \mathcal{H}$ such that
  1. $E_{\mathrm{irr}}\mathcal{H} \neq 0$, where E_{irr} is the projection on $\mathcal{H}_{\mathrm{irr}}$.
  2. $U(\mathcal{P}_+^\uparrow)\mathcal{H}_{\mathrm{irr}} \subset \mathcal{H}_{\mathrm{irr}}$ and $U|\mathcal{H}_{\mathrm{irr}}$ is an irreducible rep with $m = 0$ and $\kappa > 0$.

- But: If one gives up *pointlike* localization on M, irreducible representations of $\mathcal{P}_+^\uparrow$ with covariant transformation behaviour can be constructed.

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2 (Faithful) repn's for massless particles

- rigid motions $E(2)$
- No-Go Theorem

3 Construction of String-localized fields

- single particle states
- general construction of intertwiners
- the continuous spin case
- pointlike localized currents

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single particle states

- The following construction is due to Yngvason, Mund and Schroer
- Let $H = \{e \in M | e^2 = -1\}$ the submanifold of spacelike directions and for a chosen reference vector $q \in O$ consider a map

$$u:\ O \times H \ \ \rightarrow \ \ \mathcal{H}_q$$

(cf V above) satisfying the *intertwiner property*

$$\underline{U}(R(p,A))u(\Lambda(A)^{-1}p, \Lambda(A)^{-1}e) = u(p,e)$$

- One can define a single particle state vector $\psi(f,e)$ for each $f \in \mathcal{S}(M)$, $e \in H$ by

$$\psi(f,e)(p) = \widetilde{f}(p)u(p,e)$$

- For (Λ, a) one checks the transformation law

$$U((\Lambda,a))\psi(f,e) = \psi((\Lambda,a)_* f, \Lambda e)$$

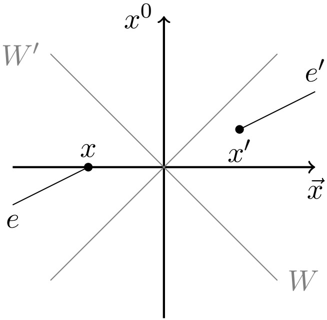
general construction of string-intertwiners

- The intertwiner functions u for the little group G_q can be constructed with the follwing recipe:
  1. One defines the G_q-orbit $\Gamma_q := \{p \in O | pq = 1\}$ and uses the *pullback repn* $\widetilde{U}$ on $L^2(\Gamma_q)$:
  $$(\widetilde{U}(R)v)(\xi) = v(R^{-1}\xi) \ \forall R \in G_q, v \in L^2(\Gamma_p), \xi \in \Gamma_p$$
  2. Intertwine $\widetilde{U}$ with $\underline{U}$ by an isometric map V:
  $$\underline{U}(R)V = V\widetilde{U}(R)$$
  3. Construct a solution $\widetilde{u}^\alpha(p,e)(\xi) := F(\xi A_p^{-1} e)$ of the interwiner property on $L^2(\Gamma_q)$, where F is a numerical function (choice: $F(w) = w^\alpha$).
  4. Use the isometry to get the $\mathcal{H}_q$-valued intertwiner
  $$u^\alpha(p,e) := V\widetilde{u}^\alpha(p,e)$$
- the construction gives a covariantly transforming quantum field: ($\circ =$ contraction over basis of $\mathcal{H}_p$)

$$\Phi^\alpha(x,e) = \int_O \widetilde{\mathrm{d}p}\, \mathrm{e}^{\mathrm{i}px} u^\alpha(p,e) \circ a^\dagger(p) + \text{c.c.}$$

- string-localization

$$[\Phi(x,e),\Phi(x',e')] = 0 \;\forall x + \mathbb{R}^+ e >< x' + \mathbb{R}^+ e' \text{ (*)}$$



- proof
  - separate strings by suitable wedge W and its causal complement W'
  - use covariance and PCT symmetry of the fields $\Phi(x,e)$
  - analytic continuation to desired equation (*)

the continuous spin case

- For $G_{(1/2,1/2\vec{e}_z)}$ the set $\Gamma_{(1/2,1/2\vec{e}_z)}$ is isometric to $\mathbb{R}^2$ via the parametrization

$$\xi(z) = \left(\frac{1}{2}(z^2+1), z_1, z_2, \frac{1}{2}(z^2-1)\right)$$

- Applying the above recipe one obtains the intertwiner

$$u^\alpha(p,e)(k) = \mathrm{e}^{-\mathrm{i}\alpha\pi/2}\int_{\mathbb{R}^2}\mathrm{d}^2z\,\mathrm{e}^{\mathrm{i}kz}\,\underbrace{(B_p\xi(z)e)^\alpha}_{=v^\alpha(p,\xi(z))}$$

pointlike localized currents

- Let $\Phi(x)$ a pointlike localized field with

$$[\Phi(x), \Phi(y)]_\pm = 0 \text{ if } (x-y)^2 < 0$$

due to CCR/CAR for the creation/annihilation operators

$$[a_i(x), a_j^\dagger(y)]_\pm = \delta(x-y)\delta_{ij} \text{ etc.}$$

- Normal ordered products fulfill local commutation relation

$$[{:}\Phi^2(x){:}, {:}\Phi^2(y){:}] = 0 \text{ if } (x-y)^2 < 0$$

because in contracted parts of ${:}({:}\Phi^2(x){:}{:}\Phi^2(y){:}){:}$ use

$$\overbracket{\Phi(x)\Phi}(y) = [\Phi(x), \Phi(y)]_\pm$$

- Similar construction for stringlike fields?

- Form of a pointlike scalar current proposed by Schroer:

$$B(x) := \int_O \widetilde{\mathrm{d}p} \int_O \widetilde{\mathrm{d}p'}\, \mathrm{e}^{\mathrm{i}(p+p')x} a^\dagger(p) \circ u_2(p,p') \circ a^\dagger(p') + \dots$$

  with the double intertwiner function

$$u_2(p,p')(k,k') = \int_{\mathbb{R}^2} \mathrm{d}^2 z \int_{\mathbb{R}^2} \mathrm{d}^2 z'\, \mathrm{e}^{\mathrm{i}(kz+k'z')} F(B_p \xi(z) \cdot B_{p'} \xi(z'))$$

- *Question:* Can B be constructed in a way s.th.
  - relative locality between current and field

$$\langle p,k| \, [B(x), \psi(y,e)] \, |0\rangle = 0 \ \forall x >< y + \mathbb{R}^+ e \ ?$$

    (Schroer)
  - causal commutativity for the currents

$$[B(x,\tilde{x}), B(x',\tilde{x}')] = 0 \ \forall x, \tilde{x} >< x', \tilde{x}'$$

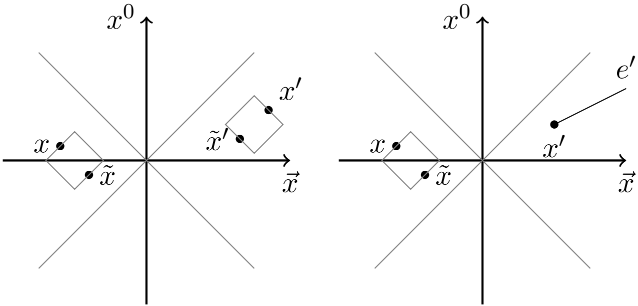
    (Mund/Schroer/Yngvason, proven for the $\langle 0| \cdot |0\rangle$ part)

- Using the definition

$$
\begin{aligned}
& B(x,\tilde{x}) \\
= \; & \int_{\partial V^+} \widetilde{\mathrm{d}p} \int_{\partial V^+} \widetilde{\mathrm{d}\tilde{p}} \int \mathrm{d}\nu(k) \int \mathrm{d}\nu(\tilde{k}) \int \mathrm{d}^2 z \int \mathrm{d}^2 \tilde{z} \\
& F(+B_p\xi(z)\cdot B_{\tilde{p}}\xi(\tilde{z})) \\
& \cdot(\mathrm{e}^{\mathrm{i}px}\mathrm{e}^{\mathrm{i}\tilde{p}\tilde{x}}\mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{\mathrm{i}\tilde{k}\tilde{z}}a^\dagger(p,k)a^\dagger(\tilde{p},\tilde{k}) \\
& \qquad\qquad +\mathrm{e}^{-\mathrm{i}px}\mathrm{e}^{-\mathrm{i}\tilde{p}\tilde{x}}\mathrm{e}^{-\mathrm{i}kz}\mathrm{e}^{-\mathrm{i}\tilde{k}\tilde{z}}a(p,k)a(\tilde{p},\tilde{k})) \\
+ \; & F(-B_p\xi(z)\cdot B_{\tilde{p}}\xi(\tilde{z})) \\
& \cdot{:}(\mathrm{e}^{\mathrm{i}px}\mathrm{e}^{-\mathrm{i}\tilde{p}\tilde{x}}\mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{-\mathrm{i}\tilde{k}\tilde{z}}a^\dagger(p,k)a(\tilde{p},\tilde{k}) \\
& \qquad\qquad +\mathrm{e}^{-\mathrm{i}px}\mathrm{e}^{\mathrm{i}\tilde{p}\tilde{x}}\mathrm{e}^{-\mathrm{i}kz}\mathrm{e}^{\mathrm{i}\tilde{k}\tilde{z}}a(p,k)a^\dagger(\tilde{p},\tilde{k})){:}
\end{aligned}
$$

- the following properties can be shown

$$[B(x,\tilde{x}), B(x',\tilde{x}')] = 0 \ \forall x,\tilde{x} >< x',\tilde{x}'$$
$$[B(x,\tilde{x}), \Phi(x',e')] = 0 \ \forall x,\tilde{x} >< y + \mathbb{R}^+ e$$



- proof similar to string-localization of fields
- involves multiple matrix elements

Summary

- Wigner-Analysis allows for classification of the irreducible repn's of $\mathcal{P}^c$.
- massive case: Particles classified by mass $m > 0$ and spin $s \in \frac{1}{2}\mathbb{N}_0$
- massless case: continuum of inner degrees of freedom("continuous spin")
- Yngvason's Theorem: No pointlike localized Wightman fields in this case
- Construction of intertwiners $\partial V_+ \times H \to \mathcal{H}_p$ gives String-localized fields
- compact localized currents can be defined
- these are local w.r.t. each other and relatively local to the strings

More warped convolutions (?)

- idea: Instead of momenta P (4-dim) in the formula

$$A_{Q_W} = \int \mathrm{d}E_p \, \mathrm{e}^{\mathrm{i}PQ_W p} A \mathrm{e}^{-\mathrm{i}PQ_W p}$$

for the deformation of field operators A (W=wedge region) one *might* use shift operators G (2-dim) discussed before in an expression like

$$A_{Q_W} = \int \mathrm{d}E_a \, \mathrm{e}^{\mathrm{i}GQ_W a} A \mathrm{e}^{-\mathrm{i}GQ_W a}$$

with $\operatorname{supp} \mathrm{d}E_a \subset O_\kappa$...?

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Thank you!