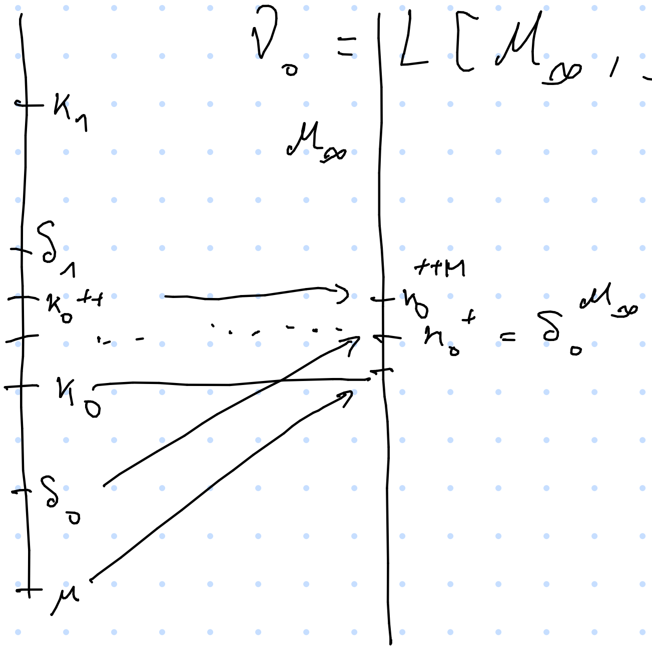
$M = M_{swsw}$ $\qquad V_0 = L[\mathcal{M}_\infty, g \mapsto g^*]$

2018-07-25




$$g^* = \min \{ \pi_{\mathcal{P},\infty}(g) \mid \mathcal{P} \in \text{system} \}$$

$$g^* = \pi_{\mathcal{P},\infty}(g) = \pi_{\mathcal{P},\infty}\Big(\underbrace{\pi_{\mathcal{P},\infty}(\bar{g})}_{= \pi^s_{\mathcal{P},\infty}(\bar{g})}\Big) \qquad \bar{g} = \pi_{\mathcal{P},\infty}(g)$$

$$= \pi_{\mathcal{P},\infty}(\pi^s_{\mathcal{P},\infty}(\bar{g}))$$

$$= \underbrace{\pi_{\mathcal{P},\infty}(\pi^s_{\mathcal{P},\infty})}_{\shortparallel}(\pi_{\mathcal{P},\infty}(\bar{g}))$$

$$= \pi_{\mathcal{M}_\infty, \mathcal{M}_\infty^{\mathcal{M}_\infty}}(g)$$

<u>Lemma</u> $V_0 = \mathrm{HOD}_{\mathcal{E}}^{M^{Coll(\omega, <\kappa_0)}}$, where $\mathcal{E}$ is defined as follows:

Let $M = L[E]$.

Say $E' \sim E$ iff $L[E'], M = L[E]$ have a common strong cutpoint $\eta < \kappa_0$ and if $g$ is $Coll(\omega, \eta)$-

generic / $L[E], L[E']$, then $L[E'][g] = L[E][g]$ and they are the same when construed as $g$-premice.

$\mathcal{E} = \{E' \mid E' \sim E\}$.

<u>Proof</u>. $\mathcal{V} \subseteq HOD_{\mathcal{E}}^{M^{Coll(\omega, <\kappa_0)}}$:

From any point in $\mathcal{E}$, may define the very same $\mathcal{V} = L[\mathcal{M}_\infty, \xi \mapsto \xi^*]$ in a uniform way, working inside $M^{Coll(\omega,<\kappa_0)}$.

$HOD_{\mathcal{E}}^{M^{Coll(\omega,<\kappa_0)}} \subseteq \mathcal{V}$:

Say $\xi \in A \iff M^{Coll(\omega,<\kappa_0)} \models \varphi(\xi, \vec{\alpha}, \mathcal{E})$

$\boxed{\text{For all } P \in \text{system} \quad P^{Coll(\omega,<\kappa_0)} = M}$

$$\iff P^{Coll(\omega,<\kappa_0)} \models \varphi(\xi, \vec{\alpha}, \underset{\{E' \mid E' \sim E^P,\ E' \text{"}\in\text{"} P^{Coll}\}}{\underset{\shortparallel}{\mathcal{E}}})$$

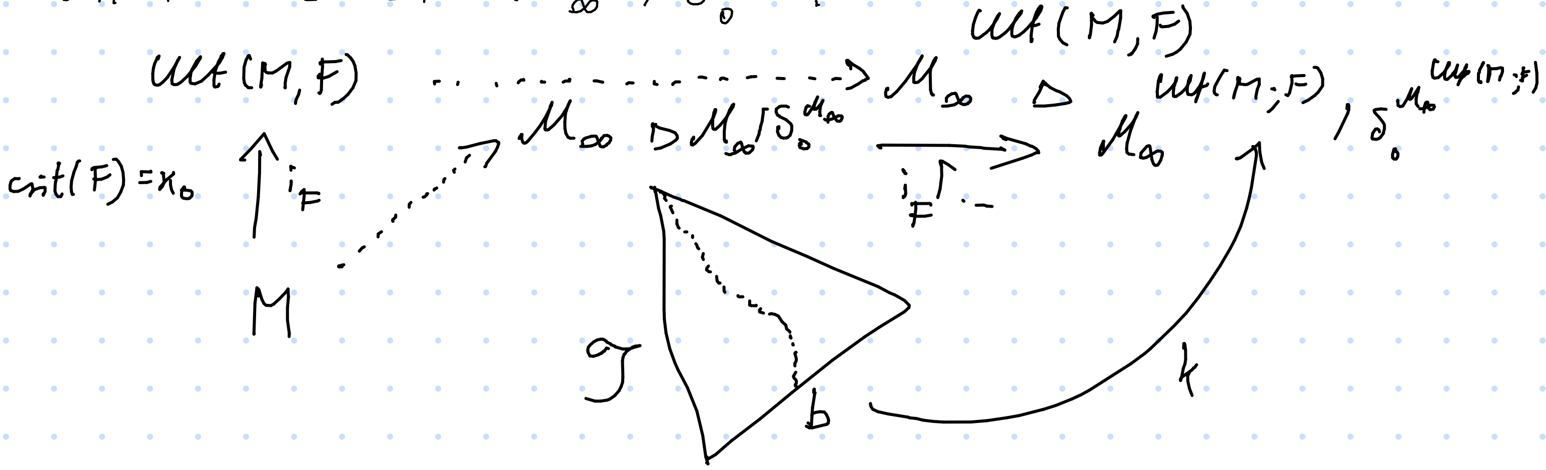
$$\iff \mathcal{M}_\infty^{Coll(\omega,<\kappa_0^{\mathcal{M}_\infty})} \models \varphi(\xi^*, \vec{\alpha}^*, \mathcal{E})$$

$\downarrow$ as defined in $\mathcal{M}_\infty^{Coll(\omega,<\kappa_0^{\mathcal{M}_\infty})}$ from $E^{\mathcal{M}_\infty}$

$\dashv$

**Corollary** $H^{\mathcal{V}}_{\delta_0^{\mathcal{M}_\infty}} = H^{\mathcal{M}_\infty}_{\delta_0^{\mathcal{M}_\infty}}$.

**Lemma** $M$ knows $\Sigma_{\mathcal{M}_\infty}$, restricted to trees in $M$ which live on $\mathcal{M}_\infty | \delta_0^{\mathcal{M}_\infty}$.




⊣

**Lemma.** $\delta_0^{\mathcal{M}_\infty}$ is Woodin in $\mathcal{V}$.

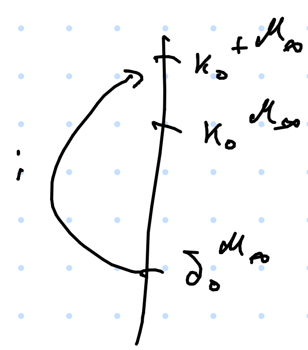
**Lemma** $\cdot$ $\mathcal{M}_\infty | \delta_0^{\mathcal{M}_\infty}$ iterable in $M$

- $H^{\mathcal{V}}_{\delta_0^{\mathcal{M}_\infty}} = H^{\mathcal{M}_\infty}_{\delta_0^{\mathcal{M}_\infty}}$
- $\delta_0^{\mathcal{M}_\infty}$ remains Woodin in $\mathcal{V}_0$.

**Lemma.** Let $i \in M$ be the map which arises from iterating $\mathcal{M}_\infty$ below $\delta_0^{\mathcal{M}_\infty}$ to make $\mathcal{M}_\infty | \kappa_0^{+\mathcal{M}_\infty}$ generic in a canonical fashion.

Let $Q = (\mathcal{M}_\infty | \kappa_0^{+\mathcal{M}_\infty}; i)$.

Then $Q = Hull^Q(\delta_0^{\mathcal{M}_\infty} \cup ran(i))$.



$(\Longrightarrow \mathrm{card}^M(\kappa_0^{+\mathcal{M}_\infty}) = \delta_0^{\mathcal{M}_\infty} = \kappa_0^{+M}$ in $M$.)

Let $\tilde{E} = \{ (\nu, a, X) \mid \nu > \kappa_0,\ a \in [\nu]^{<\omega},$
$E_\nu^M \neq \emptyset,$
$X \in (E_\nu^M)_a \cap \nu \}$

<u>Lemma</u>. $\tilde{E}$ "$\in$" $\mathcal{V}$.

<u>Proof</u>. $\mathcal{V}$ is uniformly definable in any $P \in \mathcal{M}_\infty$-system.

$(\nu, a, X) \in \tilde{E}$ iff $\ldots \wedge X \in (E_\nu^M)_a \cap \mathcal{V}$

iff $\ldots \wedge X \in (E_\nu^P)_a \cap \mathcal{V}\ \forall P$

iff $\ldots \wedge X^* \in (E_{\nu^*}^{\mathcal{M}_\infty})_{a^*} \cap \mathcal{V}$.

$\dashv$

<u>Corollary</u>. $\mathcal{V}$ knows $\Sigma_{\mathcal{M}_\infty}$ restricted to trees in $\mathcal{V}$ which live on $\mathcal{M}_\infty | \delta_0^{\mathcal{M}_\infty} = \mathcal{V} | \delta_0^{\mathcal{V}}$.

<u>Corollary</u> $\mathrm{card}(\kappa_0^{+\mathcal{M}_\infty}) = \delta_0^{\mathcal{V}}$ in $\mathcal{V}$.

<u>Lemma</u> $\mathcal{V} = L[\mathcal{M}_\infty, \xi \mapsto \xi^*]$ is a ground of $M$ witnessed by some forcing $\mathbb{P} \in \mathcal{V}$ of size $\delta_0^{\mathcal{V}}$ which has the $\delta_0^{\mathcal{V}}$-c.c. in $\mathcal{V}$.

($\mathbb{P} \neq$ extender algebra).