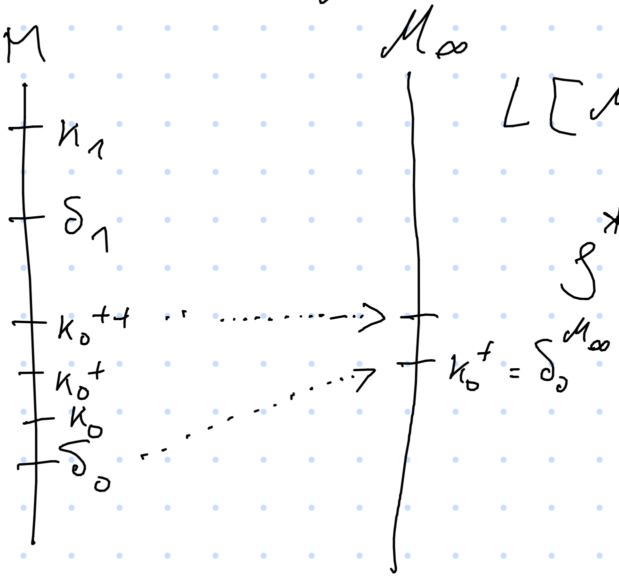
Recall: $M = M_{swsw}$ 2018-07-26



$L[\mathcal{M}_\infty, \xi \mapsto \xi^*] = \mathcal{V}_0$

$\xi^* = \min\{\pi_{P,\infty}(\xi) \mid P \in \text{system}\}$

$\mathcal{V}_0$, $\mathcal{M}_\infty$ have the same $H_{\delta_0^{\mathcal{M}_\infty}}$

In M, $\mathcal{M}_\infty$ is fully iterable wrt trees living on $\mathcal{M}_\infty | \delta_0^{\mathcal{M}_\infty}$.

In $\mathcal{V}_0$, $\delta_0^{\mathcal{M}_\infty}$ is Woodin.

$$\tilde{E} = \{ (\nu, a, X) \mid \nu > \kappa_0, a \in [\nu]^{<\omega}, E_\nu^M \neq \emptyset, X \in (E_\nu^M)_a \cap \mathcal{V} \}$$

"$\in$" $\mathcal{V}$

In $\mathcal{V}$, $\mathcal{M}_\infty$ is fully iterable wrt trees living on $\mathcal{M}_\infty | \delta_0^{\mathcal{M}_\infty} = \mathcal{V} | \delta_0^{\mathcal{M}_\infty}$.

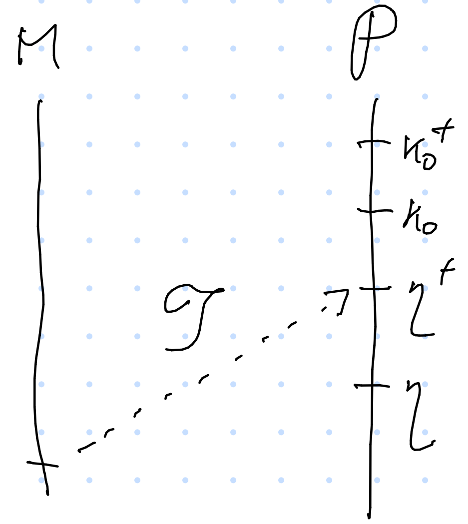
<u>Today</u> <u>Lemma</u> $\exists \mathbb{P} \in \mathcal{V}_0$, $\mathbb{P} \subseteq \mathcal{V}_0 | \delta_0^{\mathcal{V}_0}$, $\mathbb{P}$ has the $\delta_0^{\mathcal{V}_0}$-c.c., $M | \kappa_0^+$ is $\mathbb{P}$-generic over $\mathcal{V}_0$ and $\mathcal{V}_0[M | \kappa_0^+] = M$.

"$HOD^{L(\mathbb{R})}$ Vopěnka and Woodin's extender algebra" website

Proof of the lemma

In V_0 define $\mathcal{L}^*$ by
atomic formulae: "$\check{\xi} \in \dot{a}$" $\forall \xi < \delta_0^V = \kappa^{+M}$
close under $\neg, \bigwedge, \bigvee$ of length $< \delta_0^V$.

$\mathcal{P}[M|\eta^+] = M$



There is one canonical name $\tau \in M^{\mathbb{B}^M_{\delta_0}}$ for $M|\kappa_0^+$ in a way that

$\tau_{\mathcal{P}} := \pi_{M,\mathcal{P}}(\tau)$ $\qquad$ $\tau_{M_\infty} = \pi_{M,M_\infty}(\tau)$

is a canonical name for $M|\kappa_0^+$ in $\mathcal{P}^{\mathbb{B}^{\mathcal{P}}_{\delta_0^{\mathcal{P}}}}$.

Say $\varphi \in \mathcal{L}$ iff $\varphi \in \mathcal{L}^*$ and

$\exists p \in \mathbb{B}^{M_\infty}_{\delta_0^{M_\infty}}$ $\quad p \Vdash_V^{\mathbb{B}_{\delta_0^V}} \tau_{M_\infty} \models \varphi^*$

$\shortparallel$

$\mathbb{B}^V_{\delta_0^V}$

$\varphi \leq_{\mathcal{L}} \psi \iff \forall p \in \mathbb{B}^V_{\delta_0^V}$

$p \Vdash \tau_{\mathcal{M}_\infty} \models \varphi^*$ then $p \Vdash \tau_{\mathcal{M}_\infty} \models \psi^*$.

$\leq_{\mathcal{L}}$ is a partial order in $\mathcal{V}$.

<u>Claim</u> $M | \kappa_0^{+M}$ is $(\mathcal{L}; \leq_{\mathcal{L}})$-generic over $\mathcal{V}_0$.

<u>Proof of claim.</u>

- $(\mathcal{L}; \leq_{\mathcal{L}})$ has the $\delta_0^{\mathcal{V}_0}$-c.c.:

Suppose $(\varphi_i \mid i < \delta_0^{\mathcal{V}_0}) \in \mathcal{V}_0$ in an antichain in $\mathcal{L}$.

$$p_i \Vdash \tau_{\mathcal{M}_\infty} \models \varphi_i^*$$

$$\cap$$

$$\mathbb{B}_{\delta_0^{\mathcal{V}_0}}^{\mathcal{V}_0}$$

If $p \leq p_i, p_j$ in $\mathbb{B}_{\delta_0^{\mathcal{V}_0}}^{\mathcal{V}_0}$ then

$$p \Vdash \tau_{\mathcal{M}_\infty} \models \varphi_i^* \wedge \varphi_j^* = (\varphi_i \wedge \varphi_j)^*. \quad ⚡$$

$M | \kappa_0^{+M}$ is $(\mathcal{L}; \leq_{\mathcal{L}})$-generic $/ \mathcal{V}_0$.

Define $g = \{\varphi \in \mathcal{L} \mid M | \kappa_0^+ \models \varphi\}$.

g is $(\mathcal{L}; \leq_{\mathcal{L}})$-generic.

Let $A \in \mathcal{V}_0$ be a maximal antichain in $(\mathcal{L}; \leq_{\mathcal{L}})$.

If $g \cap A = \emptyset$, then $M|\kappa_0^+ \models \underbrace{\bigwedge_{\varphi\in A} \neg\varphi}_{\in \mathcal{L}^*}$.

Why is $\bigwedge_{\varphi\in A} \neg\varphi$ in $\mathcal{L}$?

For any $P \in$ system, $P[M|\underbrace{\eta^+}_{\delta_0^{\prime\prime P}}] = M$. So

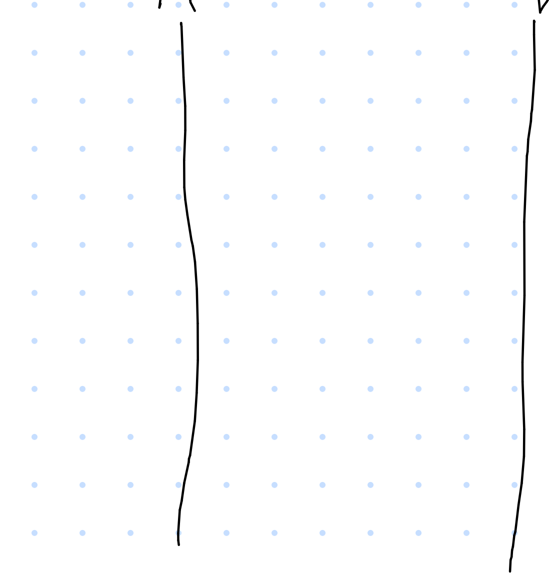
$\tau_P^{M|\eta^+} \models \bigwedge_{\varphi\in A} \neg\varphi$, so $\exists p \in M|\eta^+$ $p \Vdash \tau_P \models \bigwedge_{\varphi\in A} \neg\varphi$.

If P is sufficiently far out in the system
$\exists p \in \mathbb{B}^{\nu_0}_{\delta_0^{\nu_0}}$ $p \Vdash^{\mathbb{B}^{\nu_0}_{\delta_0^{\nu_0}}}_{\nu_0} \tau_{M_\infty} \models \left(\bigwedge_{\varphi\in A} \neg\varphi\right)^*$, so
$\bigwedge_{\varphi\in A} \neg\varphi \in \mathcal{L}^*$.

$M|\kappa_0^+ \in V[g]$ as

$\xi \in M|\kappa_0^+ \iff$ "$\check{\xi} \in \dot{a}$" $\in g$.

M $\qquad$ $V \subseteq V[M|\kappa_0^+] = M$

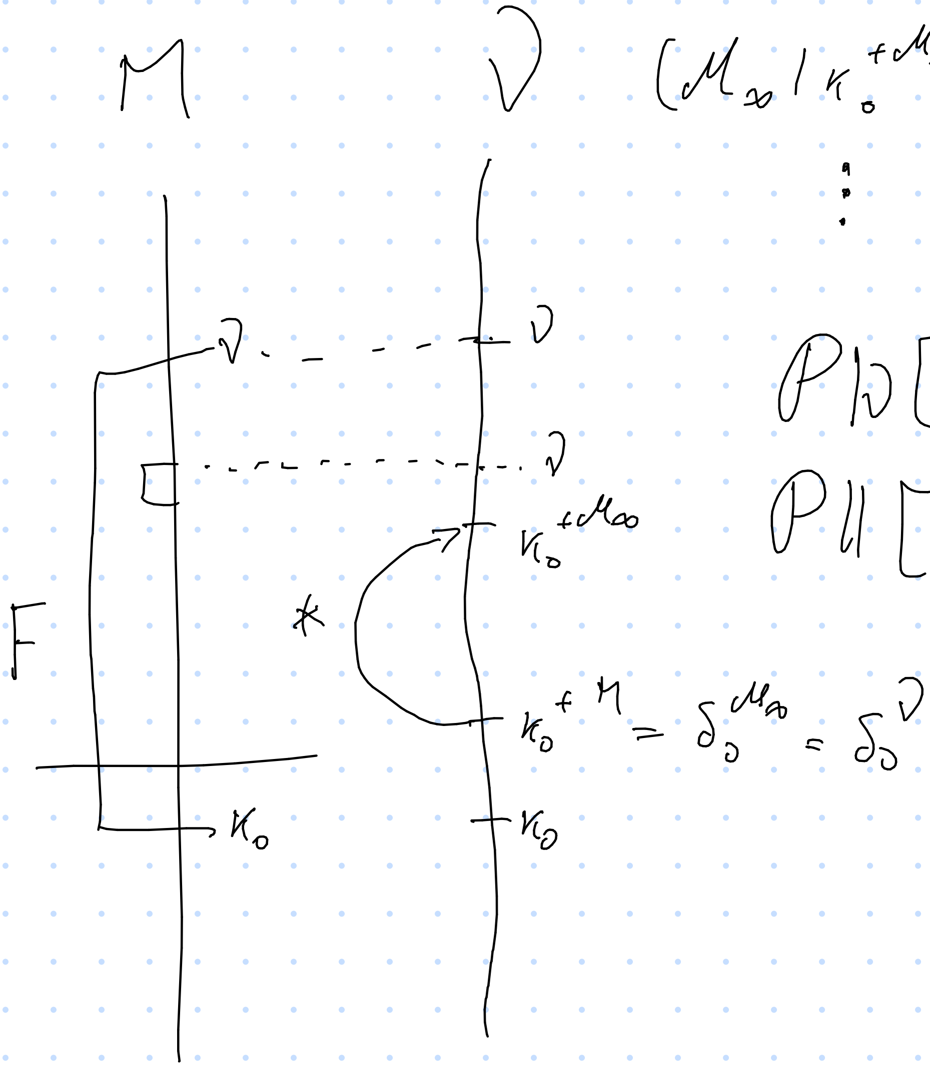


"$\ddot{e}$" V

$\tilde{E}$ = sequence of extenders with index $> \kappa_0$ $\restriction \nu_0$

The earlier proof of $\tilde{E}$ "$\in$" $\mathcal{V}$ actually showed

$$(E^M_\nu \restriction \operatorname{crit}(E^M_\nu)^+ \mid \nu > \kappa_0) \text{ "}\in\text{" } \mathcal{V}.$$



$(\mathcal{M}_\infty | \kappa_0^{+\mathcal{M}_\infty}, *)$

$\vdots$

$$\mathcal{P} | \nu [M | \kappa_0^+] = M | \nu$$

$$\mathcal{P} \| [M | \kappa_0^+] = M \| \nu$$