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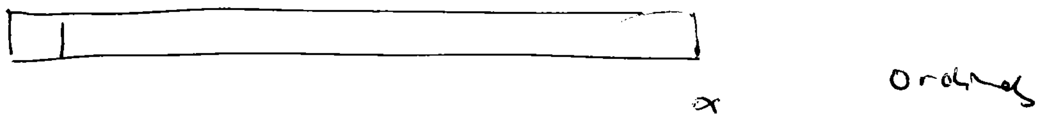
recognizable sets of ordinals.

(joint with merlin carl)

ordinal turing machines OTM



lim time λ lim state

1. p states $\longmapsto$ states
2. content of the cells.
3. head position : lim inf

$x \subset \alpha$ (OTM-) computable from α

$\iff$ x is an output of a computation which has α as an additional input.

x computable$^{\alpha}$ $\iff$ $x \in L$.

x is recognizable iff there is a
$\cap$
α program P and some β

$$\forall y \subset \alpha$$

$$(\; P(y, \beta) = 1 \iff x = y \;)$$

output of computation
with input y and
β as add. input.

without additional ordinal parameter:

$x \subset \omega$ computable iff
x is ~~reg~~ recognizable $\iff$

$x \in L_\sigma$, where
σ is least with $L_\sigma \prec_{\Sigma_1} L$
[illegible]

assume that $M_1^\#$ exists.

<u>question.</u> is every recognizable set
of ordinals in L?

def. $x \subset \alpha$ is in the recognizable closure if there are $x = x_0, x_1, \ldots, x_n$, x_n recognizable, x_i recognizable from x_{i+1}.

lem. Suppose that $x \subset \alpha$. TFAE.

x is constructible from $y \subset \beta$

1. y is recognizable
2. there is some φ, some γ s.t. y is the unique subset Z of β s.t. $L[Z] \models \varphi(Z, \gamma)$.
3. y implicitly definable over L, i.e. there is some φ ~~s.t.~~ and some γ s.t. y is the unique $z \subset \beta$ s.t. $(L, \in, z) \models \varphi(z, \gamma)$

M^α = the α^{th} iterate of M_1 by the least measurable cardinal + its images let $M^\infty = \bigcap_{\alpha \in OR} M^\alpha$

lem. if $x \subset \alpha$ is recognizable,
then $x \in M^\infty$, iff $\alpha < \omega_1$.

question. is every recognizable
subset ω_1 in M_1 ?

lem. if $\mathbb{P}$ is homogeneous, G is
$\mathbb{P}$-generic over V.
Supp. that $y \subset OR$ is recognizable,
$y \in V[G]$. then $y \in V$.

lem. Supp. $M_1^\#$ ex., ZF + DC holds,
every $X \subset \omega_1$ is in some $L[y]$,
some $y \in \omega$.
let $\mathbb{P}$ be homogeneous, $\mathbb{P}$ doesn't
coll. ω_1, $\mathbb{P}$ forces choice.
then every recognizable subset of ω_1 is in M_1.

<u>question.</u> supp. that H_{ω_2} is closed under $M_1^\#$?
then is every rec. subset of of ω_1 in M_1 ?

<u>questi.</u> can we add recognizable sets by forcing assuming large cardinals ?

<u>lem.</u> (steel) supp. $\alpha < \omega_1^M$.
say $\rho_\omega(N) = \omega$, N has height α, N sound, N is Π^1_2 it. the . then $N = M_1 \| \alpha$.

<u>lem.</u> supp. that $\alpha < \omega_1^{M_1}$, $\rho_\omega(M_1 \| \alpha) = \omega$.
let $y \subset \omega$ be a can. code for $M_1 \| \alpha$. then y is recognizable.

then if $x \in M_1 \| \alpha$,

x is constructible from y.

$\Rightarrow$ $M_1 \| \omega_1^M$ is recognizable to

ω_1^M .