Martin Zeman.

assume thruout: all mice are tame

work with Grigor Sargsyan.

start with a hod pair (P, Σ)

P is a suitable pm, full

δ

ω cardinals on top of δ, δ woodin

if κ is a cardinal cutpoint of P,

then $N \trianglelefteq P$ where $N | \kappa =$ $P | \kappa$ is sound, projects to κ and is weakly it.ble (i.e., all ctbl. $\bar{N} \longrightarrow N$ are $\omega_1 + 1$ it.ble)

Σ is a fullness-preserving iteration strategy for P: if $\mathcal{T}$ is a stack of normal trees acc. to Σ and

$i^{\mathcal{T}} : P \longrightarrow M^{\mathcal{T}}_\infty$ is total, then

$M^{\mathcal{T}}_\infty$ is full.

assume Ω is measurable in V, do the K^c construction following CMIP with the following modification.

- background condition:
  the certificates (N, G) need to be closed under Σ, i.e., if $J \in N$, then $\Sigma(J) \in N$.

- in the constr. put an extender F with $\mathrm{crit}(F) = \kappa$ on the sequence iff F is κ^+-certified, i.e., for each $\mathcal{A} \subset \mathcal{P}(\kappa)$ s.t. $\overline{\overline{\mathcal{A}}} \leq \kappa$ there is a certificate which agrees with F on $\mathcal{A}$.

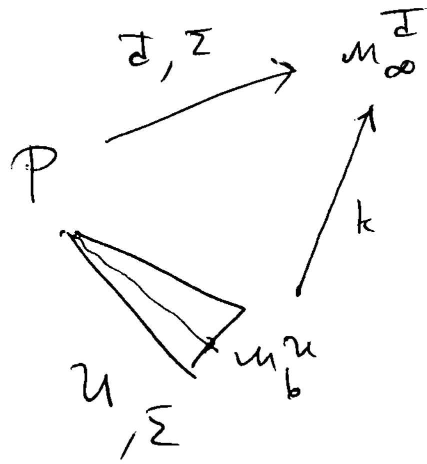
<u>with this:</u>

(1) the proof of Cheap Covering goes thru.

(2) if $\mathcal{N}_\alpha$ is a model of the K^c-construction, $\sigma : \bar{\mathcal{N}} \longrightarrow \mathcal{N}_\alpha$ is countable, then $\bar{\mathcal{N}}$ is ω_1+1 it. the and if $\mathcal{T}$ is a countable normal tree on $\bar{\mathcal{N}}$, then $\mathcal{T}$ has a max. branch.

<u>We assume:</u> Σ has branch condensation,

i.e., $\forall$ $\mathcal{T}, \mathcal{U}$ stacks of normal trees $\mathcal{T}$ acc. to Σ incl. the last branch, $\mathcal{U}$ acc. to Σ, no last branch.



if b cof. branch thr $\mathcal{U}$ and $\exists k$ making the diagram commute, then $b = \Sigma(\mathcal{U})$.

point is: prove that K^c is iterable in V_Ω and has a woodin cardinal.

(1) P ~~into~~ iterates into an initial segment of ~~the~~ K^c- ~~[illegible]~~ (using Σ), and ~~[illegible]~~

if $P \xrightarrow{\mathcal{T}, \Sigma} P' \trianglelefteq K^c$,

then the cardinals of of P' are cardinals of K^c.

in part., $\delta^{P'}$ is woodin in K^c.

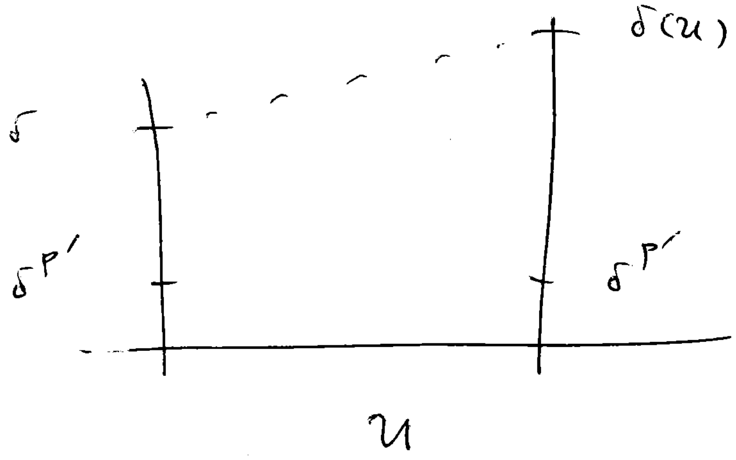
(2) iterability of K^c below $\delta^{P'}$.

show that $\Sigma_{P', \mathcal{T}}$ is an iteration strategy for K^c below $\delta^{P'}$.

(3) what if there are woodin cardinals in $K^c > \delta^{P'}$.

let δ be the least such. we describe an iteration strategy for K^c in the interval $(\delta^{P'}, \delta)$.

sample case:



there is no Q-structure for $M(\mathcal{U})$.

in $K^c \| \delta$ let $(N_\xi : \xi < \delta)$ be a fully backgrounded construction with crit. pts $> \delta^{P'}$.

compare P' against the construction. this gives a tree $\overline{T}$. using P-constructions can show the are $\mathcal{Q}$-structures in K^c at every this stage.

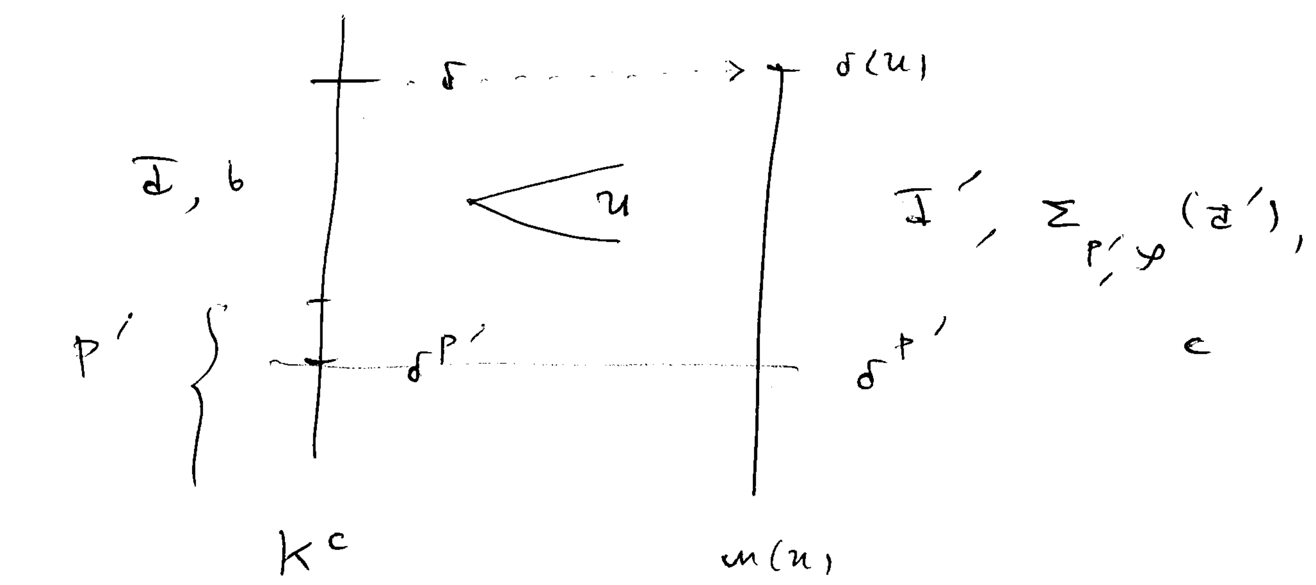
(use the fact that there are no Woodin cardinals in K^c between $\delta^{P'}$, δ.) also, the models $\mathcal{N}_\xi$ don't move in the comparison.

<u>so</u>: $\mathcal{T} \in K^c$.

<u>observe</u>: K^c is full at its cardinal cutpoints.

(by comparing against the K^c-construct.)

this gives: $\mathcal{T}$ is maximal.



if c is a cofinal well-founded branch thru $\mathcal{U}$, then

$i_c^{\mathcal{U}}$ maps δ cofinally onto $\delta(\mathcal{U})$.

Let $b = \Sigma_{P', \varphi}(\mathcal{T})$.

note: $b \notin K^c$, as o.w. ~~cf(δ(T))~~ $cf(\delta) = \omega$ in K^c, but is is woodin in K^c.

Consider a similar situation in $M(\mathcal{U})$. Let $\mathcal{T}'$ be the comparison tree of P' against the L[E] constr. ~~of~~ of $M(\mathcal{U})$ with crits $> \delta^{P'}$.

~~[illegible]~~

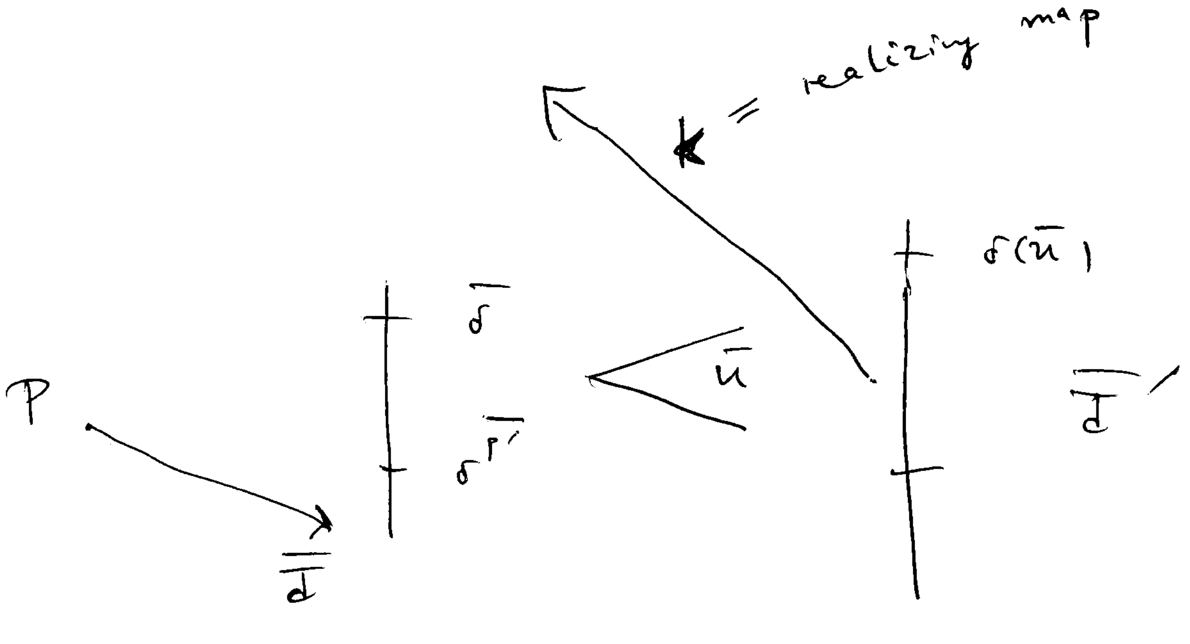
now we define the value of the iteration strategy Γ for K^c.

(*) $\Gamma(\mathcal{U})$ = the unique cofinal branch c thru $\mathcal{U}$ s.t. $i^{c}_{\mathcal{U}}(\mathrm{ran}(i^{\mathcal{I}}_{b})) \subset \mathrm{ran}(i^{\mathcal{I}'}_{b'})$.

where $b' = \Sigma_{P;\varphi}(\mathcal{I}')$.

<u>woodin.</u> there can be at most one such branch c because $i^{\mathcal{U}}_{c}$ " b is a cofinal subset of $\delta(\mathcal{U})$.

<u>existence</u>: let the following be a corr. substr. of the picture on p. 6.



all coll. trees are acc. to Σ, using Σ has branch condensation.

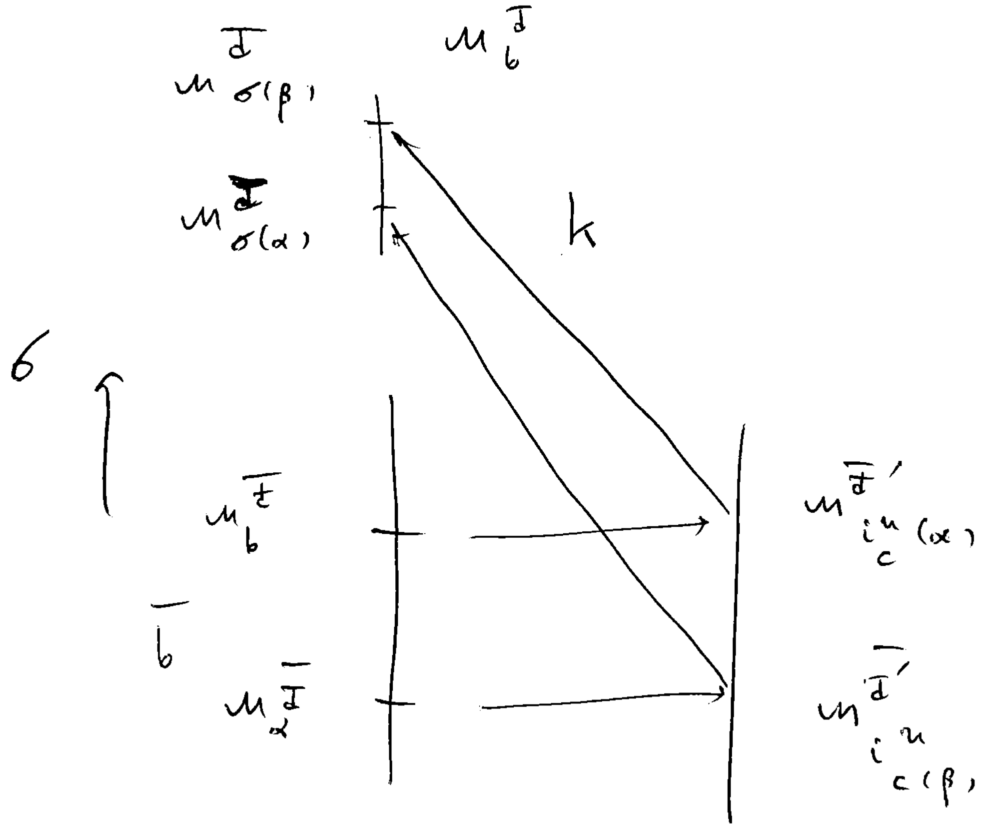
easy : $\bar{I}' \in H$.

$$\sigma : M_\infty^{\bar{I}'} \longrightarrow M_\infty^{\bar{I}} .$$

So by branch condensation :

$$\bar{\Sigma}_{\bar{P}', \bar{\varphi}} (\bar{I}') = \Sigma_{\dots} (\bar{I}') .$$

we look at the direct lim model along the branch determined by "i^u_c" $\bar{b}$.



so we have

$$\begin{tikzcd}
P \arrow[rr, "\Sigma\ j"] \arrow[dr, "\Sigma{,}\ \bar{j}"'] & & M_b^{J} \\
& M_{b^*}^{\bar{J}'} \arrow[ur, "k"'] &
\end{tikzcd}$$

so $k \circ \bar{j} = j$.

by branch condensation:

$$b^* = \Sigma_{P,\varphi}(\bar{J}').$$

so $b^* = \bar{\Sigma}_{\dots}(\bar{J}')$ satisfies $(*)$ on p.8 for the collapsed situation.

but there can be only one such, so $b^* \in$ collapse, so get existence of a branch as desired.