gabriel

thm. (gitik - schindler - shelah) if $\omega_1 \le cf(\kappa) < \kappa$

$\Phi_0(\kappa) \equiv$ $S = \{\alpha < \kappa : 2^\alpha = \alpha^+\}$ is stat. + costat, then $\forall x \subset \kappa$ bounded

$\forall n \quad M_n^\#(x)$ ex.

- assuming all mice are tame,
if $\Phi_0(\kappa)$ then $\exists M$ pm s.t.
$M \cap OR = \kappa$ and
$M \models$ "$\forall \alpha \exists \delta > \alpha \ \delta$ is woodin." $\equiv \Phi_1(\kappa)$

$\Phi_0(\kappa) + \forall \alpha \ \alpha < \kappa \Rightarrow \beth_\alpha(\alpha) < \kappa$

$\beth_\alpha(0) = 2^\alpha$

$\beth_\alpha(\beta+1) = 2^{\beth_\alpha(\beta)}$

$\beth_\alpha(\gamma) = \sup_{\beta < \gamma} \beth_\alpha(\beta)$ if γ lim.

<u>thm.</u> (steel) if Ω is measurable, and $\neg\Phi_1(\Omega)$. then $\exists P$ set p.m. $K(P)$ up to Ω exists.

<u>thm.</u> (jensen-steel) no inner model with a woodin cardinal + ZFC proves K exists.

---

defin
$\tilde{K}(\tau, \Omega)$ s.t. for μ strong limit

$$K | cf(\mu) = \tilde{K}(cf(\mu), \mu^+) | cf(\mu).$$

we want that $\forall \tau < \kappa$ $cf(\tau) = \tau$
$\exists \mu < \kappa$ strong limit s.t. $cf(\mu) = \tau$
and $\mu^+ < \kappa$.

we want to make sense of $K(P)|\kappa$.
then we can run the gitik-sch-shelah proof.

<u>τ-p.m.</u> <u>def</u>. a potential τ-p.m. is a structure (M, η) where M is a p.m.,

η is a strg automorphism of M and we define cores + projecta etc. as usual, but $\eta \in$ language.

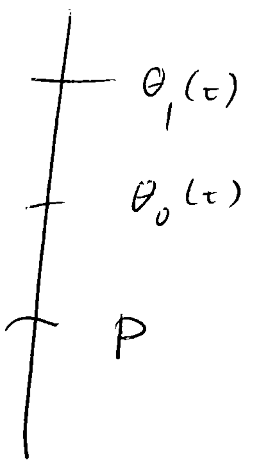
<u>rmk.</u> we are interested in

$$K^c(P, \theta_0(\tau), \theta_1(\tau)) \text{ constructions}$$

with robust extenders, where $\tau < \kappa$ is a reg. cardinal.

$$\theta_1(\tau) = \inf \{ \operatorname{crit}(\dot F^{W_\xi}) : \xi < \gamma \} .$$

$$\theta_0(\tau) < \operatorname{crit}(\dot F^{W_\xi}) \longrightarrow \operatorname{crit}(\dot F^{W_\xi}) \neq \tau .$$



$\operatorname{crit}(F) > \theta_1(\tau)$ + $\operatorname{cf}(\operatorname{crit}(F)) \neq \tau$, then F needs to be added.

<u>def.</u> given M a p.m. s.t. $M \cap OR = \kappa$,
we say M is $\leq (\omega, \omega_1, <\kappa)$ iterable iff
M is iterable f.a. trees $\mathcal{T}$ s.t. $lh(\mathcal{T}) < \kappa$
+ $\mathcal{T}$ lives on an initial segment of M.

<u>th.</u> (steel) there is a full P s.t. f.a. $K^c(P, \theta_0(\tau), \theta_1(\tau))$ constr., and any strong cutpt. η of $K^c(P, \theta_0(\tau), \theta_1(\tau))$
s.t. $\forall \delta > \eta$ $K^c(—) \models$ "δ is not woodin,"
the r-p.m. $(K^c(P, \theta_0(\tau), \theta_1(\tau), \eta)$ is $(\omega, <\kappa)$ iterable.

<u>th.</u> (steel) $\exists P$ s.t. P is full and f.a. $\theta_0(\tau), \theta_1(\tau)$ large enough,

- $K^c(P, \theta_0(\tau), \theta_1(\tau))$ is $(\omega, \omega_1, <\kappa)$ iterable,
- $K^c(P, \theta_0(\tau), \theta_1(\tau)) \models$ "$\neg\exists$ woodin above $o(P)$"

<u>def.</u> (jensen-steel) given τ, Ω s.t. $\omega_3 \leq \tau < \Omega$, $2^{<\tau} < \Omega$, $\forall \alpha < \Omega$ $\alpha^\omega < \Omega$, $\tau^{++} < \Omega$.

Γ is (τ, Ω, W) thick iff $\exists C \subset S(W) \cap OR$ s.t.

- $cf(S(W)) \geq \Omega$, C is a τ-club in $S(W) \cap OR$
- $\forall \eta \in C$ $cf(\eta)$ is not measurable in W

<u>def.</u> (jensen-steel) given (τ, Ω) like in the previous def, if $S(W)$ is W-thick,

$Def^W = \bigcap \{ H^{S(W)}(\Gamma) : \Gamma$ is (τ, Ω, W) thick$\}$

$\tilde{K}(P, \tau, \Omega)$ = common tr. collapse of all

Def^W s.t. $S(W)$ is

(τ, Ω, W) thick, $W \cap OR = \Omega$.

<u>claim.</u> if W is a stable universal weasel, then $L[S(W)] \models \Omega$ is not woodin.

[$W \cap OR = \Omega$

proof:

compare W, $K^c(P, \theta_0(\tau), \theta_1(\tau)) | \Omega$.

note that $\exists Q \trianglelefteq S(K^c(P, \theta_0(\tau), \theta_1(\tau)) | \Omega)$

Q kills the woodin property of Ω.

since W and $K^c(P, \theta_0(\tau), \theta_1(\tau)) | \Omega$ are universal, if $\mathcal{I}_0$ on W, and $\mathcal{I}_1$ on $K^c(P, \theta_0(\tau), \theta_1(\tau))$,

$i_0'' \Omega \subset \Omega$, $i_1'' \Omega \subset \Omega$,

der. f

then $\mathrm{ult}(S(W), E_0) = S(M_\infty^{\mathcal{I}_0})$

$= S(M_\infty^{\mathcal{I}_1}) = \mathrm{ult}(K^c(-); E_1)$.

der. f.

<u>th.</u> (jensen-steel) if $S(W)$ is W-thick, then there are class many $\alpha < \Omega = W \cap OR$ with the hull property.

weak covering for $\tilde{K}(P, \tau, \Omega)$

~~def~~ def. (jensen-steel) W_0 is a very soundness witness for $\tilde{K}(P, \mu^+, \Omega)$ iff W_0 is $S(W_0)$ thick and $\mu^+ \subset \mathrm{Def}^{W_0}$.

thm. (jensen-steel) if $\mu < \kappa$ is a singular strong limit, then f.a. Ω large enough,

$$\mu^{+\tilde{K}(P, \tau, \Omega)} = \mu^{+V}.$$

given some μ_0, sing. strong limit, let $\mu_1 > \mu_0$ be a sing. strong limit s.t. $cf(\mu_1) > \mu_0$ + W_1 is a very soundness witness for $\tilde{K}(\mu_1^+, \Omega)$ for Ω large enough;

$$\mu_1^{+W_1} = \mu_1^{+V}.$$

$W_1 | \mu_1^+ \models \mu_1$ is the largest ~~ca~~ cardinal and

$\mathrm{Def}^{W_1} \supset \mu_1^+$.

(gitik, shelah) if $S = \{\alpha < \kappa : 2^\alpha = \alpha^+\}$

is stat. + costat. $\Longrightarrow$

$\neg (*)_\kappa$, ~~a sequence (κ_j : $j\in i$)~~

f.a. clubs $C \subset \kappa$

$\exists$ $(\kappa_n : n<\omega)$ in C

$\mathrm{cf} \prod \kappa_n^+ > (\sup \kappa_n)^+$.

<u>claim</u> (steel) let $\alpha \geq o(P)$ be

a cardinal of $\tilde{K}(P, \tau, \Omega)$ and

Q is a pm of height $\leq \Omega$ that

agrees with $\tilde{K}(P, \tau, \Omega)$ below α. then

1) Q is α – strong and properly small

iff

2) f.a. $<\alpha$ – strong properly small p.m.
P s.t. $|P| = \Omega$, (P, Q, α) is a $<\kappa$ –
it. th phalanx .

2) $\Longrightarrow$ 1) : compare

$(W, \tilde{K}(P, \tau, \Omega), \alpha)$ with

W, [illegible] W is a very [illegible]

[illegible] etc.