problem session.

① what is $\mathrm{HOD}^{L(\mathbb{R})}$ in the lbr hierarchy?

same for $L[x, g]$.

the same under $AD^+ + \neg AD_{\mathbb{R}}$ + HPC (hod pair capturing).

(steel)

② let $M = M_{sw \cdot \omega^*}$ be the least model with a λ which is the limit of sthgs. what is the mantle?

does it have a bedrock?

Is it the HOD of the AD-model constr. from M by larson-sargsyan-wilson, = the least model containing $\mathbb{R}^*$, Hom^* in which every set of reals is universally baire.

(schindler)

(3) assume κ is singular, strong limit, in fact $\overline{\overline{V_\kappa}} = \kappa$, and $\neg \Box_\kappa$.
is there a proper class model containing all reals satisfying $AD^+ + LSA$?

(adolf)

<u>known</u>: • non-tame mouse (sargsyan)
• very likely: using the methods for trang's proof of $PFA \rightarrow Con(AD_{\mathbb{R}} + \Theta \text{ reg})$ one can obtain $AD_{\mathbb{R}} + \Theta$ regular.

(4) let $M_{1\infty}$ = dir lim of ctbl. iterates of M_1. let $A \in M_{1,\infty}$ must there ex. an $n < \omega$ s.t. either $\{u_m : n \leq m < \omega\} \subset A$ or
$\{u_m : n \leq m < \omega\} \cap A = \emptyset$?

(zhu)

background. we have a characterization of $M_{1,\infty}$ in terms of descr. set thy.

$$\mathcal{M}_{1,\infty} = L_{u_\omega}[O_{\Sigma^1_3}], \text{ where}$$

$$O_{\Sigma^1_3} = \{(\varphi, \alpha) \in \text{Fla} \times u_\omega :$$
$$\exists w \; \varphi(w) \wedge w \text{ codes } \alpha \text{ in the sharp coding}\}.$$

best knowledge so far: for each $n \in \omega$,

$$u_n \in \operatorname{ran}(\pi_{M_1, \mathcal{M}_{1,\infty}}).$$

background: hjorth: "a boundedness lemma for iterations"

zhu: "iterates of M_1"

(5) assume AD. Let κ be a inacc. Suslin cardinal. assume $f : \kappa \longrightarrow \kappa$, and μ is a normal measure on κ. (e.g., μ can be any δ-cof. measure on κ.) Let $\alpha = [f]_\mu$. Is there a characterization of the property

$$W(\alpha) \equiv \text{“} \alpha = w(\Delta) \text{”},$$

where Δ is a pt. class closed under $\exists^{\mathbb{R}}$, $\forall^{\mathbb{R}}$

in part., μ = the ω-cof. measure on κ, can one find an $f : \kappa \longrightarrow \kappa$ s.t.
$W(f(\beta))$ for all β? (Jackson)
and $W([f])$, $[f] = \alpha$.

motivation: if 'yes' the Steel pointclass conjecture fails.

$f > \mathrm{id}$.