john.

Ω

Hom_∞

iteration strategy for f.s. extender model — next $\mp$

Λ


hypothesis

AD^+ + NLE

↑

no long extenders

scaled pointclasses
suslin cardinals
iteration strategies for
coarse Γ-woodins.

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iteration strategies for fine structural structures.

assume AD^+ + NLE = there is no
iteration strategy for a pure
extender pm with a long
extender.

a <u>pure extender pair</u> is a pair

(P, Σ) s.t.

- P is a pure extender premouse, cm., (λ-indexed — ms would be okay)
  every P has a degree of soundness, $k(P)$, attached.
  $\rho(P) = \rho_{k(P)+1}(P)$
  $p(P) = p_{k(P)+1}(P)$.
- $P | \langle \nu, k \rangle$ = initial seg. Q of P
  s.t. $\hat{o}(Q) = \nu$,
  $k(Q) = k$.
- Σ is an ω_1 strategy for P with scope HC,
  a winning strategy for II in
  $G^+(P, \omega_1, \omega_1)$
  $\uparrow$
  I, II produce stack of weakly normal trees on P.

• Σ normalizes well + has strong hull condensation.

<u>normalizes well:</u>

$(\mathcal{T}, \mathcal{U})$ a stack by Σ.

$$\begin{tikzcd}
P \arrow[r, "\mathcal{T}"] \arrow[drr, bend right, "W"'] & Q \arrow[r, "\mathcal{U}"] & R \arrow[d, "\pi"] \\
& & S
\end{tikzcd}$$

$\leftarrow$ last model

$W = W(\mathcal{T}, \mathcal{U})$ is by Σ

W normal

<u>want:</u>

$$\underbrace{(\Sigma_{W,S})^{\pi}}_{\text{pullback of } \Sigma_{W,S} \text{ via } \pi} = \Sigma_{\mathcal{T},\mathcal{U},R}$$

pullback of $\Sigma_{W,S}$ via π.

that's 2-normalizes well.

Σ normalizes well iff every tail Σ_s 2-normalizes well.

<u>strong hull ~~class~~ condensation:</u>

if $\mathcal{U}$ is a normal tree on P by Σ, and $\Phi : \mathcal{T} \longrightarrow \mathcal{U}$ is a "pseudo hull embedding," then $\mathcal{T}$ is by Σ.

<u>Defn.</u> a least branch hod pair (lbr hod pair) is a (P, Σ) ~~[illegible]~~ s.t.

- P is a least branch premouse (constructed for $\vec{E}$ and strategy info $\dot{\Sigma}^P$).
- Σ normalizes well.
- $\dot{\Sigma}^P \subset \Sigma$, in fact if S is a

stack by Σ with last model R,

then $\dot{\Sigma}^R \subset \Sigma_{S,R}$.

1^{st} part from "normalizing iteration trees and comparing iteration strategies"

2^{nd} part: handwritten notes.
both are on web page.

<u>def.</u> a mouse pair is a (P, Σ) of one of the two variations.

<u>basic theory of mouse pairs:</u>

comparison, dodd-jensen, background construction, lbr hod pairs used to compute HOD in a particular case.

question on complexity of (P, Σ), in part., optimal suslin representations.

Comparison sketch. Σ, Ψ suslin, co-suslin.

assume AD^+, and let (P, Σ), (Q, Ψ) be pure extender pairs.

want to find (R, Φ), a pair, s.t. for some $\mathcal{T}$ by Σ + $\mathcal{U}$ by Ψ,

either (1) P-to-R does not drop
$\quad\quad$ $\parallel$
$\quad$ comm last model; allow final drop.

and $\Phi = \Sigma_{\mathcal{T}, R} = \Psi_{\mathcal{U}, R}$,

or (2) Q-to-R does not drop, and

$\Phi = \Sigma_{\mathcal{T}, R} = \Psi_{\mathcal{U}, R}$.

pf.: let N^* be a coarse Γ-woodin, w/ $P, Q \in HC^{N^*}$, and Σ, Ψ are well in Δ, captured by N^*.

N^* has a $\leq \delta^{N^*}$ u.b. representation for Σ, Ψ.

Let $\mathbb{C} = \left(\left(M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k} \right) : \nu \leq \delta^{N^*}, k < \omega \right),$

a backgrd constr. in N^*.

<u>show</u>: there is some $(M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k})$ which

is an iterate of (P, Σ),

same for (Q, Ψ).

why?: take any $(R, \Phi) = (M^{\mathbb{C}}_{\nu,k}, \Omega^{\mathbb{C}}_{\nu,k})$

compare with (P, Σ).

(a) no strategy disagreements show up.

(b) only the P-side moves.

1980's.

(a): say $P \xrightarrow{\mathcal{I}} S$

$$S | (\alpha, k) = R | (\alpha, k)$$

and $\Sigma_{\mathcal{I}, S(\alpha,k)} \neq \Phi_{R|\langle\alpha,k\rangle}$.

$$\begin{tikzcd}
\mathcal{M}_b^{\mathcal{U}} \arrow[r] & i_b^{\mathcal{U}^*}(\mathcal{R}|(\alpha,h)) & \in & \mathcal{M}_b^{\mathcal{U}^*} \\
S \arrow[u, "\mathcal{U}"] & R|(\alpha,h) \arrow[u] & \in & N^* \arrow[u, "i_b^{\mathcal{U}^*}"'] \\
P \arrow[u, "\mathcal{I}"] \arrow[uu, bend left, "W=W(\mathcal{I}^\frown\mathcal{U}^\frown b)"] \arrow[ur, "\mathcal{I}"'] \arrow[uur, "\mathcal{U}^\frown b", "i_b^{\mathcal{U}^*}(\mathcal{I})"'] &&&
\end{tikzcd}$$

$\mathcal{U}$ on $S(\alpha,k)$ s.t. $\Sigma_{\mathcal{I},\, S(\alpha,h)}(\mathcal{U}) \neq \Phi_{R|(\alpha,h)}$

u.b.-ness of Σ in N^*:

$i_b^{\mathcal{U}^*}(\mathcal{I})$ is by Σ.

[branch condensation would then give $b = \Sigma_{\mathcal{I},\, S(\alpha,h)}(\mathcal{U})$.]

but:

W hull of $i_b^{\mathcal{U}^*}(\mathcal{I})$,

hence W is by Σ, by the strong hull condensation.

so $(\mathcal{I}, \mathcal{U}^\frown b)$ is by Σ. $\dashv$