john.

computation of HOD in $D(M, \lambda)$,

M a lbr hod pair.

we'll show: $HOD^{D(M,\lambda)}$ is an initial segment of an iterate of M.

if λ is a limit of cutpoints

$\longrightarrow$ nondropping iterate.

o.w. let κ_0 be the 1^{st} strong up to λ:

$\longrightarrow$ cut ~~d~~ off the iteration at the image of κ_0.

prove: if g is M-generic on $Col(\omega, \eta)$,

~~and $\delta > \eta$ is woodin in M,~~

then

$M[g] \models$ "UBH for all ~~[illegible]~~ plus 2 (non dropping anywhere) iteration trees of me with with crits $> \eta$"

adapt the $L[E]$ pf.:

take, in $L[E]$, a pt. wise

def. the counterexample, i.e.

$$N \overset{\sigma}{\cong} \mathrm{Hull}^{L[E]}(\emptyset) \prec L[E]$$

N transitive. have $\mathcal{I} \in N$ with

$N \models b, c$ are two diff. branches. get

$$(*) \begin{cases} \mathcal{M}_b^{\mathcal{I}} \longrightarrow L[E] \\ \mathcal{M}_c^{\mathcal{I}} \longrightarrow L[E], \end{cases}$$

as $\sigma(\mathcal{I}^\frown b)$, $\sigma(\mathcal{I}^\frown c)$ when

$\neg$ UBH for $L[E]$

then compare $\Phi(\mathcal{I}^\frown b)$, $\Phi(\mathcal{I}^\frown c)$;

works by $(*)$. get $\lightning$ as

base model N is pt. wise definable.

$\dashv$

- generic interpretability:

  if $\delta < \lambda$ and is a woodin cardinal in M, then there is for each $\beta < \delta$ a term $\tau \in M$ s.t. f.a. g $Col(\omega, \xi)$-gen,

$$\tau^g = \Sigma_{M|\langle\eta,0\rangle} \cap M[g]$$

(same proof as in grigor)

$$\begin{array}{lll} & - \delta & (\mathbb{R}, \Omega) \text{ hmc or } M|\delta \text{ using cubs } > \xi \text{, plus 2 back-pods} \\ g & - \xi & i \\ & - \eta & M|\eta, \Sigma_{M|\eta} \end{array}$$

- for any $\eta < \lambda$, there is a term τ s.t. for all g in $Col(\omega, <\lambda)$,

$$\tau^g = \Sigma_{M|\langle\eta,0\rangle} \cap \mathbb{R}^*_g$$

So each $\Sigma_{M|\langle\eta, 0\rangle} \in \mathrm{Hom}^*_g$

they are $\leq_W$ cofinal in Hom^*_g :

let $T, U \in M[g]$ where a join of A is $\in \mathrm{Hom}^*_g$. then this set A can be completed in $M[g]|\delta$, for Woodin δ ~~>~~ large enough. can complete A by genericity iteration.

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assume λ is a limit of cutpoints in M.

let M_∞ = dir lim. of all iterates of $M|\lambda$ ~~t~~ via trees $\mathcal{T} \in M|\lambda$ (or ~~in~~, coded by a real in $\mathbb{R}^*_g$, gives same limit).

<u>claim</u>. $\mathcal{M}_\infty = \mathrm{HOD}|\theta^{D(M,\lambda)}$

pf.: ~~then~~

"$\subset$": let $\mathcal{F} = \{(P, \Sigma)$:

(P, Σ) is a lbr hod pair,

Σ fullness pres. $\}$

this is a direct lim system under comparisons.

$\forall (P, \Sigma) \in \mathcal{F}$ there is $\eta < \lambda$ for some iterate (R, Φ) of $(M|\eta, \Sigma_{M|\eta})$

so $(P, \Sigma) \leq^{\mathcal{F}} (R, \Phi)$.

$M_\infty^{\mathcal{F}} = M_\infty \subset \mathrm{HOD}/\theta$.

"$\supset$" Let $A \subset o(\underbrace{M_\infty(M|\eta, \Sigma_{M|\eta})}) \trianglelefteq M_\infty$,

$A \in \mathrm{HOD}$. can read off A from general case:

$$\theta^{D(M,\lambda)} \simeq \pi_{M,\infty}(\kappa), \quad \text{where}$$

κ = the least strongly up to λ.