<u>nam</u>

the core model induction.

<u>def.</u> (mouse operator) let Υ be a set.
a Υ- <u>mouse operator</u> $\mathcal{F}$ with domain $\mathbb{D}$
has the following properties.

$\mathcal{F}(x) \lhd Lp^{\Upsilon}(x)$,

sound

where $Lp^{\Upsilon}(x) = \bigcup \{ M \mid M$ is a Υ-mouse on x, $\}$

$\rho_\omega(M) = x$

Υ is an operator, just like $\mathcal{F}$.

examples of $\mathcal{F}$:

① $\mathcal{F}$ can be a 1st order mouse operator.

e.g., $x \longmapsto x^{\#}$

$x \longmapsto M_1^{\#}(x)$

there is a Σ_1 fla φ, some param a s.t.
$\mathcal{F}(x) =$ the 1st level $M \lhd Lp^{\Upsilon}(x)$ s.t.
$M \models \varphi(x, a)$.

(so $\mathcal{I}$ would be undef. for x's which don't see a.)

② $\mathcal{I}$ can be "strategic."

<u>properties of $\mathcal{I}$:</u>

① (condensation) if $\pi : \bar{m} \longrightarrow \mathcal{I}(x)$ suff. elementary, and $\pi(\bar{x}) = x$. then $\bar{m} \trianglelefteq \mathcal{I}(\bar{x}) \trianglelefteq L_p^{\Upsilon}(\bar{x})$. (in most cases, $=$)

② (determines itself on generic extensions) there is a formula $\varphi(x, y, z)$ s.t. $\forall N \models ZFC^-$, N is closed under $\mathcal{I}$, then for any N-generic g, $N[g]$ is closed under $\mathcal{I}$ and $\varphi(-)$ defines $\mathcal{I}$ on $N[g]$, i.e., $\forall x, y \in N[g]$

$$\mathcal{I}(x) = y \iff N[g] \models \varphi(x, y, a).$$

③ (relativizes well) there is a formula $\varphi(x,y,z)$ s.t. $\forall a, b \in \mathrm{dom}(\mathcal{F})$, with $a \in L_1(b)$, then when $N \models ZFC^-$, closed under $\mathcal{F}$, if $\mathcal{F}(b) \in N$, then $\mathcal{F}(a) \in N$, and $\mathcal{F}(a)$ is the unique $x \in N$ s.t. $N \models \varphi(x, a, F(b))$.

---

a mouse operator $\mathcal{F}$ is "nice" if it has the above properties.

<u>def.</u> (maximal model)

let $\Omega = \{ A \subset {}^{\omega}\omega : \exists \mathcal{F}$ $\mathcal{F}$ is a nice mouse operator + $A \leq_W \mathcal{F} \restriction \mathbb{R}$ $[+ \; L(A, \mathbb{R}) \models AD^+]^{*)} \}$

the core model induction analyzes how <u>complicated</u> Ω is.

*) under e.g. PFA, it doesn't matter if you put this here or cross it out.

Sample problems:

(1) Con(PFA) $\longrightarrow$ Con($\exists$ supercompact cardinal)

(2) Con(ZF + DC + ω_1 is supercompact)

$\longrightarrow$ Con($\exists$ a woodin)

Con(ZF + DC + ω_1 is strongly compact)

$\longrightarrow$ Con(lim of woodins)

e.g., PFA + $\exists$ measurable cardinal, say κ.

show that if F is a nice operator, then $M_1^{F,\#}$ is defined.

Succ. steps in the core model induction,

$$\forall x \quad M_1^{F,\#}(x) \lhd Lp^F(x)$$

is the least active F-mouse M s.t. $M \models$ "there is a woodin cardinal"

Φ is defined on ~~ct~~ a cone of H_κ^V

?

defined on $V_\kappa^{V[g]}$ for $g \subset Col(\omega, <\kappa)$

V-generic.

(define Ω in $V[g]$.)

- ~~should~~ extend Φ to $H_{\kappa^+}^{V[g]}$.

can't have $L_{\kappa^+}^{\Phi}[x]$ computes κ^+

correctly, other take a hull of

$(L_\tau^\Phi[x]; \in, \mathcal{U})$, $\tau = \kappa^{+L^\Phi[x]}$,

$\mathcal{U} = L^\Phi[x] \cap$ measure on κ.

gives $\Phi^\#(x)$.

- then show $M_1^{\Phi,\#}(x)$ is defined

for $x \in dom(\Phi)$.

deny. $\exists x \in dom(\Phi)$ s.t.

$M_1^{\Phi,\#}(x)$ is not defined.

So $K^\Phi(x)$ exists.

let $\gamma < \kappa$ be s.t. $(\gamma^+)^{K^{\mathcal{F}}(x)} = \gamma^+$

since $\square_\gamma$ holds in $K^{\mathcal{F}}(x)$.

but PFA $\Rightarrow \neg\square_\gamma$. contradiction.

<u>limit steps:</u>

- put previous operators together.

$cf(\underset{\sim}{\Delta}) = \omega$: easy.

$cf(\underset{\sim}{\Delta}) > \omega$: most of the work is done.

if $\underset{\sim}{\Delta}$'s Solovay sequence has limit length, then need ~~[illegible]~~ hod mice.