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what is a mouse ?

$\boxed{\text{mouse = iterable premouse}}$

(a) premouse.

(a) pure extender premice :
$L[\vec{E}]$ for $\vec{E}$ a coherent sequence of extenders.

(b) hod premice, $L[\vec{E}, \Sigma]$ for $\vec{E}$ coherent, Σ info on how to iterate.

also : hybrids, relativizations.
premice has a hierarchy :

fine structure : core, projecta

$M|\langle \nu, k\rangle$: the ν^{th} level of M, considered at Σ_k level.

every M is $k(M)$ – sound

$M \longrightarrow N$ $k(M)$ – elementary
really elementary

copied elementary

Σ^*

$\mathrm{ult}(M; E) = \mathrm{ult}_{k(M)}(M; E)$

works below long extenders.

(b) <u>iterability.</u>

(1) M is <u>linearly iterable</u> iff all lin. iterations are well-founded (below $0^\P$)

(2) comparison theorem (below $0^\P$)

if M, N are linearly iterable, then they have a common iterate R s.t. M-to-R or N-to-R does not drop

<u>dodd-jensen</u> if M is lin. iterable and below $0^\P$, and R is an iterate of M, and $\pi : M \xrightarrow{\text{elem.}} R$, then

(1) M-to-R does not drop

(2) $i(\eta) \leq \pi(\eta)$, for $i =$ the iteration map $\eta \in M$.

<u>mouse order</u>: $M \leq^* N$ iff

$\exists R, \pi$ (R is an initial segment of N

and $\pi : M \longrightarrow R$ elem.)

<u>Cor.</u> $\leq^*$ is a prewellorder of mice.

beyond $0^{\P}$:

iteration game :

$G(M, \theta)$: output is a normal tree on M of lh θ.

$G(M, \eta, \theta)$: output is a stack of normal trees.

$G^+(M, \eta, \theta)$: I can drop.

iteration strategy = winning strategy for II.

<u>thm.</u> let P, Q be ctble. pure extender premice.

let Σ, Ψ be ω_1+1 strategies (w.s. for II in $G(P, \omega_1+1)$, in $G(Q, \omega_1+1)$, resp.) then there is an R, a Σ-iterate, and a Ψ iterate s.t. P-to-R does not drop or Q-to-R does not drop.

<u>context</u>. assume AD^+.

Σ, Ψ are ω_1-strategies (sets of reals).

(so extend to ω_1+1 strategies).

<u>problem</u>. we compared (P, Σ) with (Q, Ψ) (partially!), but not P, Q.

<u>what is $\leq^*$?</u>

<u>def</u>. (a) (P, Σ) is a <u>pure extender pair</u> iff (1) P is a pure extender premouse, Σ is an (ω_1, ω_1)-strategy for P,

(2) Σ normalizes well and has strong hull condensation.

<u>normalizes well:</u>

$(\mathcal{T}, \mathcal{U})$ by Σ, both normal

$$\begin{tikzcd} P \arrow[r, dash, "\mathcal{T}"] \arrow[drr, dash] & \cdot \arrow[r, dash, "\mathcal{U}"] & R \arrow[d, "\pi"] \\ & & S \end{tikzcd}$$

iff W is by Σ, and $(\Sigma_W)^\pi = \Sigma_{\mathcal{T},\mathcal{U}}$

$W(\mathcal{T}, \mathcal{U}) = W$

<u>strong hull condensation:</u>

if $\pi : \mathcal{T} \longrightarrow \mathcal{U}$ is suff. elementary and $\mathcal{U}$ is by Σ, then $\mathcal{T}$ is by Σ.

(b) (P, Σ) is a <u>least branch hod pair</u> iff (1) P is a least branch premouse, and

(2) Σ normalizes well, has strong hull condensation

(3) if $P \xrightarrow{s} Q$ via Σ, then

$$\dot{\Sigma}^{Q} \subset \Sigma_s .$$

here, $\Sigma_s(t) = \Sigma(s^\frown t)$ for

$s = \vec{\mathcal{I}}$, $t = \vec{\mathcal{U}}$ stacks

(c) (P, Σ) is a <u>mouse pair</u> iff (P, Σ) is a pure ext. pair or a lbr hod pair

$$\left(\begin{array}{c} \text{mouse} \\ + \\ \text{unique strategy} \end{array} \right) \xrightarrow[\text{appropriate generalization}]{} \text{mouse pair}$$

<u>def.</u> (Q, Ψ) is an iterate of (P, Σ) iff there is a stack s by Σ with last model Q s.t. $\Psi = \Sigma_s$.

<u>comparison</u> (AD^+) let (P, Σ), (Q, Ψ) be countable mouse pairs of the same type. then they

have a common iterate (R, Ω) s.t. either P-to-R or Q-to-R above does not drop.

<u>def.</u> $\pi : (P, \Sigma) \longrightarrow (Q, \Psi)$ is <u>elementary</u> iff $\pi : P \longrightarrow Q$ is elementary and $\Sigma = \Psi^{\pi} =$ π-pullback strategy.

$$\Psi^{\pi}(\mathcal{I}) = \Psi(\pi \mathcal{I}).$$

<u>prop.</u> if $i : P \longrightarrow Q$ is an iteration map by Σ, then $\pi : (P, \Sigma) \longrightarrow (Q, \Sigma_s)$

<u>prop.</u> if $\pi : (P, \Sigma) \longrightarrow (Q, \Psi)$ and (Q, Ψ) is a mouse pair, then so is (P, Σ).

<u>dodd–jensen.</u> if (Q, Ψ) is a iterate of (P, Σ) and $\pi : (P, \Sigma) \longrightarrow (Q, \Psi)$,

is elementary, then

(1) P-to-Q does not drop

(2) $i(\eta) \leq \pi(\eta)$, where i is the iteration map.

<u>defin</u> $(P, \Sigma) \leq^* (Q, \Psi)$ iff $\exists \pi\ (R, \Omega)$ s.t. is an iterate of (Q, Ψ) and $\pi : (P, \Sigma) \longrightarrow$ ~~(Q, Ψ)~~ (R, Ω).

<u>cor.</u> $\leq^*$ is a prewellorder of mouse pairs of fixed type.

<u>other properties.</u> let (P, Σ) be a mouse pair, then

(1) Σ has very strong hull condensation.

(2) Σ fully normalizes well.

(3) Σ is positional

(4) Σ is $OD(\pi)$, where

$$\pi : P \longrightarrow M_\infty(P, \Sigma)$$

dir. lim of all ctbl. iterations.

(5) Σ is Suslin – co-Suslin.