

-1-

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what is a mouse?

mouse = iterable premouse

(a) premouse.

(a) pure extender premice :

$L[\vec{E}]$  for  $\vec{E}$  a coherent sequence of extenders.

(b) hod premice,  $L[\vec{E}, \Sigma]$  for  $\vec{E}$  coherent,  $\Sigma$  info on how to iterate.

also: hybrids, realizations.

premise has a hierarchy:

fine structure: core, projects

$M| \langle \nu, k \rangle$  : the  $\nu^{\text{th}}$  level of  $M$ ,  
considered at  $\Sigma_k$  level.

every  $M$  is  $k(M)$ -sound

$M \rightarrow N$   $k(M)$ -elementary  
weakly elementary

cofinal elementary

$\Sigma^*$

$$\text{ut}(M; E) = \text{ut}_{k(M)}(M; E)$$

works below long extenders.

⑥ iterability.

(1)  $M$  is linearly iterable iff all  
lin. iterations are well-founded

(below  $O^{\#}$ )

(2) comparison theorem (below  $O^{\#}$ )

if  $M, N$  are linearly iterable, then they  
have a common iterate  $R$  s.t.

$M$ -to- $R$  or  $N$ -to- $R$  does not drop

Dodd-Jensen if  $M$  is lin. iterable and

below  $O^{\#}$ , and  $R$  is an iterate of  $M$ ,

and  $\pi: M \xrightarrow{\text{elem.}} R$ , then

(1)  $M$ -to- $R$  does not drop

(2)  $i(\gamma) \leq \pi(\gamma)$ , for  $i =$  the iterat' map  
 $\gamma \in M$ .

mouse order:  $M \leq^* N$  iff

$\exists R, \pi$  ( $R$  is a stack of  $N$

and  $\pi: M \rightarrow R$  elem.)

corr.  $\leq^*$  is a prewellorder of mice.

beyond  $O^\Pi$ :

iteration game:

$G(M, \theta)$ : output is a normal tree on  $M$  of lh  $\theta$ .

$G(M, \gamma, \theta)$ : output is a stack of normal trees.

$G^+(M, \gamma, \theta)$ : I can drop.

iteration strategy = winning strategy for  $\Pi$ .

thm. In  $P, Q$  be cttle. pure extendible premice.

Let  $\Sigma, \Upsilon$  be  $\omega_1+1$  strategies (w.s.  
 for  $\Pi$  in  $G(P, \omega_1+1)$ , in  $G(Q, \omega_1+1)$ , resp.)  
 then there is a  $R$ , a  $\Sigma$ -chrm, and  
 a  $\Upsilon$  chrm st.  $P$ -to- $R$  does not  
 drop or  $Q$ -to- $R$  does not drop.

context. assume  $AD^+$ .

$\Sigma, \Upsilon$  are  $\omega_1$ -strategies (sets of reals).

(so extend to  $\omega_1+1$  strategies).

problem. we compared  $(P, \Sigma)$  with  $(Q, \Upsilon)$   
 (partially!), but not  $P, Q$ .

what is  $\leq^*$ ?

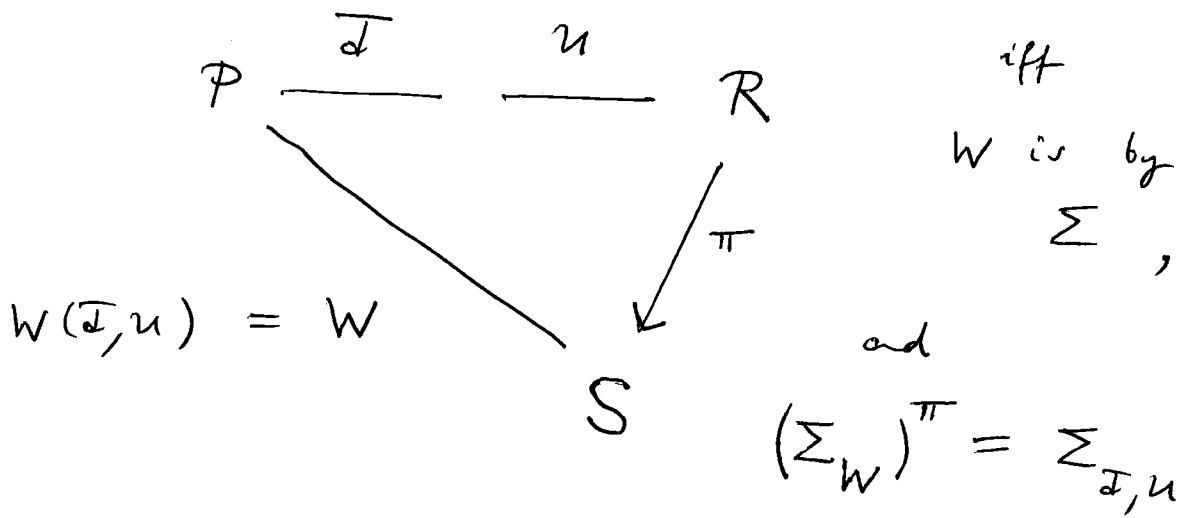
def. (a)  $(P, \Sigma)$  is a pure extend pair

iff (1)  $P$  is a pure extend premouse,  
 $\Sigma$  is a  $(\omega_1, \omega_1)$ -strategy for  $P$ ,

(2)  $\Sigma$  normalizes well and has strong hull condensation.

normalizes well:

$(\mathcal{I}, \mathcal{U})$  by  $\Sigma$ , both normal



$W(\mathcal{I}, \mathcal{U}) = W$

strong hull condensation:

if  $\pi: \mathcal{I} \rightarrow \mathcal{U}$  is suff. elementary and  $\mathcal{U}$  is by  $\Sigma$ , then  $\mathcal{I}$  is by  $\Sigma$ .

(6)  $(P, \Sigma)$  is a least branch hod pair

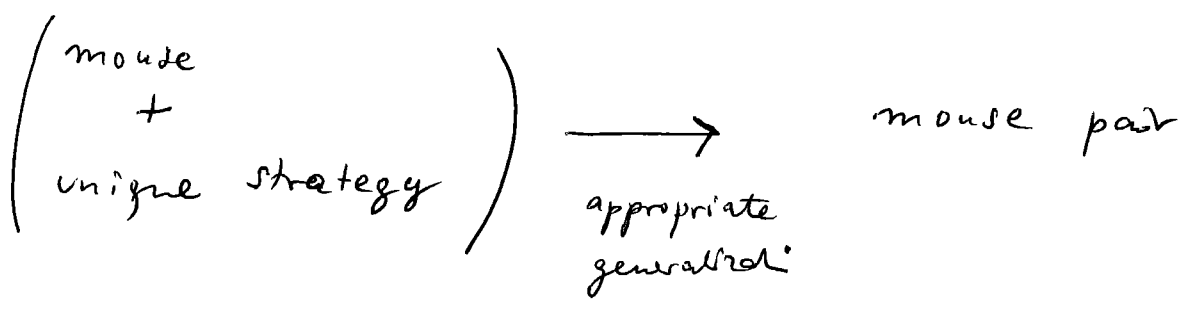
iff (1)  $P$  is a least branch premouse, and

(2)  $\Sigma$  normalizes well, has strong hull condensation

(3) if  $P \xrightarrow{S} Q$  via  $\Sigma$ , then  
 $\dot{\Sigma}^Q \subset \Sigma_S$ .

here,  $\Sigma_S(t) = \Sigma(s \hat{=} t) \sim$   
 $s = \vec{I}, t = \vec{u}$  stacks

(c)  $(P, \Sigma)$  is a mouse pair iff  
 $(P, \Sigma)$  is a pure ext. pair or a  
 lbr hod pair



def.  $(Q, \Upsilon)$  is an iterate of  $(P, \Sigma)$   
 iff there is a stack  $s$  by  $\Sigma$  with  
 last model  $Q$  s.t.  $\Upsilon = \Sigma_s$ .

comparison  $(AD^+)$  let  $(P, \Sigma), (Q, \Upsilon)$  be  
 either mouse pairs of the same type. then they

has a common iterate  $(R, \Omega)$  st.

either  $P$ -to- $R$  or  $Q$ -to- $R$  above  
does not drop.

def.  $\pi: (P, \Sigma) \rightarrow (Q, \Psi)$  is elementary iff

$\pi: P \rightarrow Q$  is elementary and  $\Sigma = \Psi \pi =$

$\pi$ -pullback strategy.

$$\Psi^\pi(I) = \Psi(\pi I).$$

prop. if  $i: P \rightarrow Q$  is an iterate map

by  $\Sigma$ , then  $\pi: (P, \Sigma) \rightarrow (Q, \Sigma_S)$

prop. if  $\pi: (P, \Sigma) \rightarrow (Q, \Psi)$  and

$(Q, \Psi)$  is a mouse pair, then so is

$(P, \Sigma)$ .

dodd-jenw. if  $(Q, \Psi)$  is a iterate

of  $(P, \Sigma)$  and  $\pi: (P, \Sigma) \rightarrow (Q, \Psi)$

is elementary, then

(1)  $P \rightarrow Q$  does not drop

(2)  $i(\gamma) \leq \pi(\gamma)$ , where  $i$  is the critical map.

defn'  $(P, \Sigma) \leq^* (Q, \Upsilon)$  iff

$\exists \pi (R, \Omega)$  s.t. is a iterate of  $(Q, \Upsilon)$  and  $\pi: (P, \Sigma) \rightarrow (R, \Omega)$ .

cor.  $\leq^*$  is a prewellorder of mouse pairs of fixed type.

othr properties. Let  $(P, \Sigma)$  be a mouse

pair, then

(1)  $\Sigma$  has very strong hull condensation.

(2)  $\Sigma$  fully normalizes well.



(3)  $\Sigma$  is positional

(4)  $\Sigma$  is  $OD(\pi)$ , where

$$\pi : P \longrightarrow M_{\infty}(P, \Sigma)$$

↑  
dir. lin of all em.  
choices

(5)  $\Sigma$  is  $sublin - co - sublin$ .