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In what follows, let M be a transitive model of ZFC. An $x \in {}^{\omega}\omega$ is called a *Cohen real* over M iff $g_x = \{x \upharpoonright n \colon n < \omega\}$ is $\mathbb{C}$-generic over M.

Problem 1. Assume M to be countable. Show that the set of $x \in V$ such that x is a Cohen real over M is comeager.

Problem 2. Let g be $\mathbb{C}$-generic over M. Show that in $M[g]$ there is a perfect set of Cohen reals over M.

Problem 3. Let $\mathbb{P} \in M$ be a partial order such that $M \models$ "$\mathbb{P}$ has the κ-c.c.," where κ is an uncountable regular cardinal in M. Let g be $\mathbb{P}$-generic over M. Show that if $f \colon x \to M$ is a function with $x \in M$ and $f \in M[g]$ then there is a function $g \in M$ with the following properties.

(a) $\mathrm{dom}(g) = x$,
(b) $\mathrm{Card}(g(u)) < \kappa$ in M for every $u \in x$, and
(c) $f(u) \in g(u)$ for every $u \in x$.

Problem 4. Let $\mathbb{P} \in M$ be a partial order. Let g be $\mathbb{P}$-generic over M, and let h be $\mathbb{P}$-generic over $M[g]$. Show that $M[g] \cap M[h] = M$.

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