SET THEORY COURSE WINTER TERM 2020-21, EXERCISE SHEET NO. 3, NOV 12, 2020

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For sets x and y write $(x, y) = \{\{x, y\}, \{x\}\}$.

Problem 1. Show that for all sets x, y, x', y' we have that $(x, y) = (x', y')$ if and only if $x = x'$ and $y = y'$. (x, y) is the *ordered pair* of x and y.

$x \times y = \{(u, v) \colon u \in x \wedge v \in y\}$, the *Cartesian product* of x and y. Also, ${}^y x = \{f \colon f$ is a function from y to $x\}$.

Problem 2. Show in ZFC that for all x and y, $x \times y$ exists. Show in ZFC that for all x and y, ${}^y x$ exists. Which axioms from ZFC do you make use of in each case?

Problem 3. Let $\mathsf{ZFC}^{-\mathrm{Pair}}$ be ZFC without the pairing axiom. Show the pairing axiom in the theory $\mathsf{ZFC}^{-\mathrm{Pair}}$. Which axioms do you make use of for that?

Problem 4. Show in ZF that AC (the axiom of choice as defined in the lecture) is equivalent to the following statement:

$$\forall x (\forall y \in x \, y \neq \emptyset \to \exists \text{ a function } f \text{ with } \operatorname{dom}(f) = x \wedge \forall y \in x \, f(y) \in y).$$

Problem 4. Show that there is a set A of pairwise non-isomorphic linear orders on $\mathbb{N}$ such that $A \sim \mathbb{R}$ (i.e., there is a bijection from A onto $\mathbb{R}$).

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