

*Workshop on*

$L^2$ -Invariants, Measure Theory and  
Geometric Group Theory

MÜNSTER, GERMANY

March 8 – March 12, 2004



# Abstracts

BACHIR BEKKA

## **Kazhdan's Property (T)**

Property (T), a property of locally compact groups introduced by Kazhdan 1967, is a powerful tool with applications in several areas: geometry, ergodic theory, graph theory, combinatorial group theory, von Neumann algebras... We will review the basic definitions as well as the first consequences. We will present the (classical) application to the construction of expander graphs as well as the recently discovered link to the PRA algorithm. We will mention the cohomological approach to Property (T) and the spectral criterion (Zuk, Garland, Gromov). Finally, we will discuss Property (T) for von Neumann algebras (Connes, Jones).

DAMIEN GABORIAU

## **Survey on Orbit Equivalence**

Orbit equivalence theory studies partitions into orbits resulting from measure class preserving group actions. We shall concentrate on probability measure preserving free actions of countable groups. I will present some invariants of orbit equivalence and examine connections with  $L^2$ -invariants and geometric group theory. A particular attention will be paid on  $L^2$ -Betti Numbers (invariant under OE) and on Measure Equivalence (= a classification of countable groups, parallel to quasi-isometry, to which OE naturally leads).

ROSTISLAV GRIGORCHUK

## **A spectrum related to fractal groups as an invariant set of multidimensional rational mappings**

We will describe a method which in some cases leads to a computation of spectra of groups generated by finite automata or to a computation of spectra of their Schreier graphs. One important step of this method is the presentation of a spectrum as an invariant set of a rational multidimensional map. This relates the spectral problem to the theory of strange attractors.

PETER LINNELL

### **$L^2$ -Betti Numbers of One Relator Groups**

We shall determine the  $L^2$ -Betti numbers for a one-relator group. We shall use the techniques to show, for example, that for a left ordered two-generated group, either the group is free or the first  $L^2$ -Betti number is 0.

JOHN LOTT

### **Noncommutative Geometry of Limit Sets**

I'll describe the construction of equivariant K-cycles for the cross-product algebra arising from the action of a discrete group of hyperbolic isometries on its limit set. This extends some results of Connes and Sullivan to arbitrary dimension. I'll also describe how the Patterson-Sullivan measure can be seen as a center-valued KMS state.

WOLFGANG LÜCK

### **Survey on $L^2$ -invariants II**

We continue the survey on  $L^2$ -invariants given by Roman Sauer. We focus on the  $L^2$ -torsion which is a kind of secondary  $L^2$ -invariant, i.e. it is defined if the  $L^2$ -Betti-numbers vanish, and gives more refined information. We present its main properties applications and computations for certain cases. We discuss the conjecture that the vanishing of the  $L^2$ -torsion is an invariant of the measure equivalence class of groups  $G$  for which there exists a finite model for  $BG$  and whose  $L^2$ -Betti numbers all vanish.

NICOLAS MONOD

### **Measure Equivalence of Groups**

Whilst geometric group theory has become a very classical topic, its relative, Measure Equivalence, is a recent focus of interest. Geometric group theory proposes to consider abstract groups as geometric objects, and to study algebraic properties from the point of view of large-scale geometry. Measure equivalence, on the other hand, aims at studying abstract groups through ergodic-theoretic techniques. It is therefore unsurprising that  $L^2$  type invariants are important tools for that programme, and that there are connections to questions in von Neumann algebras. We will give an introduction to this programme, and indicate how our joint work with Y. Shalom imports ideas from geometric group theory into measure equivalence by cohomological means.

YANN OLLIVIER

### **A Panorama of Random Groups**

We will present the philosophy and main results about random groups: critical density and phase transitions phenomenon, algebraic and spectral properties, and construction of groups with prescribed Cayley graphs.

ROMAN SAUER

### **Survey on $L^2$ -Invariants I**

We will give an introduction to  $L^2$ -Betti numbers from the point of view of Lück's dimension theory. This algebraic approach makes it possible to use standard tools from homological algebra like spectral sequences for the study of  $L^2$ -invariants. Some examples, where this can be successfully applied, will be presented.

THOMAS SCHICK

### **Homotopy Equivalence of $L^2$ -eta Invariants**

Using the signature operator of a closed oriented Riemannian manifold  $M$  and its lift to the universal covering, one can define the  $L^2$ -eta invariant of  $M$ . These invariants are defined in terms of a complicated integral using the heat kernel of the operators involved. It turns out that for a torsion free fundamental group which satisfies a certain version of the Baum-Connes conjecture, this invariant does only depend on the homotopy type of  $M$ . We present a new proof of this result, which is originally due to Navin Keswani, which we think sheds some new light on this theorem. Finally, we will discuss generalizations and examples.

ALAIN VALETTE

### **A-(T)-menable Groups**

A locally compact group is a-(T)-menable if it admits a proper, affine, isometric action on a Hilbert space. We will give various characterizations of this class of groups. Using proper actions on spaces with (measured) walls, we will provide several examples. We will give the proof of the recent result by Guentner-Higson-Weinberger: if  $K$  is a field, every countable subgroup of  $GL(2, K)$  is a-(T)-menable. Finally, we will briefly discuss the analogue of a-(T)-menability for  $\text{II}_1$ -factors.

# Schedule

## Monday, March 8

- 10:00–11:00 ROMAN SAUER  
Survey on  $L^2$ -Invariants I
- 11:45–12:45 WOLFGANG LÜCK  
Survey on  $L^2$ -Invariants II
- 15:00–16:00 BACHIR BEKKA  
Kazhdan's Property (T)
- 17:00–18:00 DAMIEN GABORIAU  
Survey on Orbit Equivalence

## Tuesday, March 9

- 10:00–11:00 DIMITRI SHLYAKHTENKO  
Connections between  $L^2$ -Invariants and Free Probability Theory
- 11:45–12:45 YEHUDA SHALOM  
Cohomological Invariants in the Large Scale Geometry of Amenable Groups
- 15:00–16:00 ANTONY WASSERMANN  
Loop groups and Connes-Jones invariants
- 17:00–18:00 DIMITRI SHLYAKHTENKO  
 $L^2$ -Betti Numbers of  $\text{II}_1$ -Factors

## Wednesday, March 9

- 9:15–10:15 ALAIN VALETTE  
A-(T)-menable Groups
- 10:30–11:30 SORIN POPA  
*tba*
- 12:00–13:00 SORIN POPA  
*tba*

**Thursday, March 11**

- 10:00–11:00 ROSTISLAV GRIGORCHUK  
A Spectrum related to Fractal Groups as an Invariant Set of  
Multidimensional Rational Mappings
- 11:45–12:45 MARC BURGER  
Bounded Cohomology and Rigidity Theory
- 15:00–16:00 NICOLAS MONOD  
Measure Equivalence of Groups
- 17:00–18:00 YANN OLLIVIER  
A Panorama of Random Groups

**Friday, March 12**

- 9:15–10:15 ANDRZEJ ZUK  
Spectra of Automata
- 10:30–11:30 JOHN LOTT  
Noncommutative Geometry of Limit Sets
- 12:00–13:00 THOMAS SCHICK  
Homotopy Equivalence of  $L^2$ -eta Invariants
- 14:30–15:30 PETER LINNELL  
 $L^2$ -Betti Numbers of One Relator Groups

**ALL TALKS TAKE PLACE IN LECTURE HALL M5**



# Schedule

Monday Mar 8	Tuesday Mar 9	Wednesday Mar 10	Thursday Mar 11	Friday Mar 12
<i>registration</i>				
Sauer 10:00-11:00	Shlyakhtenko 10:00-11:00	Valette 9:15-10:15	Grigorchuk 10:00-11:00	Zuk 9:15-10:15
<i>tea &amp; coffee</i>	<i>tea &amp; coffee</i>	<i>tea &amp; coffee</i>	<i>tea &amp; coffee</i>	<i>tea &amp; coffee</i>
Lück 11:45-12:45	Shalom 11:45-12:45	Popa 10:30-11:30	Burger 11:45-12:45	Lott 10:30-11:30
<i>lunch</i>	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>
Bekka 15:00-16:00	Wassermann 15:00-16:00	Popa 12:00-13:00	Monod 15:00-16:00	Schick 12:00-13:00
<i>tea &amp; coffee</i>	<i>tea &amp; coffee</i>		<i>tea &amp; coffee</i>	<i>tea &amp; coffee</i>
Gaboriau 17:00-18:00	Shlyakhtenko 17:00-18:00		Ollivier 17:00-18:00	Linnell 14:30-15:30

# Computer facilities

On the first floor of the Mathematical Department there are many (Unix, Linux, and Windows) computers at your disposal.

**Username: `gast478a`**

Ask around for the password. . . . If you have a laptop with a wireless card, then you may also use this username and password to log into the university network.

Wireless LAN name: `Funk-Hoerl`

VPN (Virtual Private Network) server IP address: `172.16.32.1`

## Apart from mathematics...

On Wednesday, March 10, at 19:00 a dinner at *Pinkus Müller*, a traditional brewery and restaurant, is planned. Some locals will meet at 18:30 in front of the entrance of the Mathematical Department and walk to the restaurant. You can simply join them. If you want to go there directly, here's the address:

PINKUS MÜLLER  
Kreuzstr. 4-10  
48143 Münster  
Phone (0251) 45151  
[www.pinkus-mueller.de](http://www.pinkus-mueller.de)

## Some restaurants in Münster

Here is a small selection of restaurants that we enjoy.

- **La Cantina**  
Tibusstr. 7-11. Phone: (0251) 58963. Opens at 18:30, closed on Mondays.  
*Recommended Italian restaurant for small groups. Make sure to reserve a table (unless you are Italian).*
- **Kleiner Kiepenkerl**  
Spiekerhof 47. Phone: (0251) 43416. Opens at noon, closed on Tuesdays.  
www.kleiner-kiepenkerl.de  
*Traditional German cooking in a traditional house with traditionally good service.*
- **Schoppenstecher**  
Hörsterstr. 18. Phone (0251) 47114. Opens at 18:00, closed on Mondays.  
*Good wines, friendly atmosphere, small selection of dishes. Only for very small groups.*
- **Phoenicia**  
Steinfurterstr. 37. Phone (0251) 278794. Opens at 17:00, closed on Mondays.  
*Good Lebanese restaurant near the Hotel am Schlosspark.*
- **Prütt Cafe Restaurant**  
Bremer Str. 32. Phone (0251) 665588. Opens at 17:00  
*Nice vegetarian restaurant. Only for very small groups and further away from the math department, near the train station.*
- **Takanoha**  
Königstr. 45. Phone: (0251) 5105329. Opens at 17:30, and Monday–Saturday also 11:30–15:00.  
www.takanoha.de  
*Japanese cuisine and sushi-bar.*
- **Röstbar**  
Nordstr. 2. Phone (0251) 2843700. Monday–Saturday 9:00–20:00  
www.roestbar.de  
*A little coffee bar, not a restaurant. Freshly roast coffee of different sorts. Delicious cakes and sweets. Wines in the evening. Something for wednesday after the talks.*
- **Schlossgarten**  
Schlossgarten 4. Phone: (0251) 9879696. Opens at 11:00, Sundays brunch.  
*Nice location in the Schlossgarten, good beer. Close to the Mathematical Department.*
- **Habibi Snack**  
Neutor. Almost always open.  
*Kebabs and falafels to go.*

Other recommended spots that offer a variety of restaurants, cafés and *Kneipen* are:

- **Frauenstraße**
- **Kuhstraße**
- **Neubrückenstraße**

and, if you are willing to go a little further away from the Mathematical Department, the **harbor** (Stadthafen I, Hafenweg).

# Participants

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# Organizers

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# Notes

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