EFFECTS OF PROMPTING STUDENTS TO USE MULTIPLE SOLUTION METHODS WHILE SOLVING REAL-WORLD PROBLEMS ON STUDENTS' SELF-REGULATION

Kay Achmetli, Stanislaw Schukajlow, André Krug

University of Münster, Germany

In the project MultiMa (Multiple solutions for mathematics teaching oriented toward students' self-regulation), we investigated the effects of prompting students to use multiple solution methods while solving real-world problems on their learning. In this quasi-experimental study, we compared three treatment conditions. In one condition, students solved real-world problems by using multiple solution methods. These solution methods consisted of a solution using a table and a solution using differences. In the other two conditions, the same real-world problems were solved using only one of the methods. About 307 ninth graders from twelve middle track classes took part in this study during four lessons. Before and after a teaching unit, students' self-regulation was tested.

INTRODUCTION

The development of multiple solutions is an important component of school curricula in different countries. Encouraging students to use multiple solution methods improves students' mathematical knowledge. However, we do not know much about the influence of the use of multiple solution methods on students' self-regulation, which is crucial for lifelong learning. As solving real-world problems is an important part of mathematics education, we chose this type of task to investigate the effects of prompting students to use multiple solution methods on students' self-regulation while solving real-world problems.

THEORETICAL BACKGROUND

Self-regulation

Boekaerts (2002) defines self-regulation as "students' attempts to attain personal goals by systematically generating thoughts, actions, and feelings at the point of use, taking account of the local conditions." Thus, self-regulation is divided into three main parts: (1) students' orientation toward the attainment of their own goals, (2) the thoughts, feelings, and actions that can help them to attain these goals, and (3) working toward the attainment of their goals. It is further set within the framework of local conditions.

Self-regulatory processes can be acquired from and are sustained by social as well as self-sources of influence. Zimmerman (2000) describes four developmental levels of self-regulatory skills. The development of self-regulation begins on the first level, which is called an observational one. On this level, learners vicariously observe and imitate skills from a proficient model. On the level of emulation, learners imitate these

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skills with social assistance before they can work independently under structured conditions on the next level (the level of *self-control*). A *self-regulated* level is achieved when learners can flexibly and systematically adapt their performance to changing conditions.

Multiple solutions and self-regulation

Heinze, Star, and Verschaffel (2009) claim that the ability to use multiple representations (or multiple solution methods) and to flexibly switch between a range of representations is a critical component of the skills needed to solve mathematical problems. Recently, some experimental studies were carried out to identify the influence of prompting students to construct multiple solutions on students' learning in mathematics (Rittle-Johnson & Star, 2007). Students who developed two solution methods for the same task outperformed students who developed one solution at a time. Comparing two solution methods for the same problem or presenting two solution methods using different problems improved students' procedural flexibility. Students who developed two solution methods were more flexible in their choice of the appropriate solution method. In addition, Große and Renkl (2006) state that reflecting on various solution methods helps learners to apply methods more flexibly and effectively. Furthermore, Tabachneck, Koedinger, and Nathan (1994) found that it was more effective to employ a combination of strategies than to rely on a single strategy for solving algebra problems. Flexibility and adaptivity are important parts of self-regulatory skills. Prompting students to construct multiple solutions can improve their flexibility and adaptivity and thereby also improve their self-regulation.

The influence of prompting students to construct multiple solutions while solving real-world problems with missing information on students' self-regulation was investigated in the study by Schukajlow and Krug (2012). The results showed that, while controlling for self-regulation on a pre-test, students in the condition in which multiple solutions were prompted reported significantly higher self-regulation on the post-test than students in the condition in which they were instructed to develop one solution only.

Multiple solutions, modelling, and self-regulation

We distinguish between three types of multiple solutions that can be constructed in solving real-world problems (cf. a similar approach by Tsamir, Tirosh, Tabach, & Levenson, 2010). First, multiple solutions may result from variability in mathematical solution methods. The second type of multiple solutions can be developed if students have to make assumptions about missing data and thus arrive at different outcomes/results. The third one includes variability in mathematical solution methods as well as in different outcomes/results. The effects of prompting the second kind of multiple solutions on students' self-regulation were examined by Schukajlow and Krug (2012). In the current paper, we explored the effects of prompting the first type of multiple solutions on students' self-regulation.

The important activities that need to be implemented while modelling consist of simplifying a complex situation that is presented in the task, mathematizing, and working mathematically to reach a mathematical result. While solving a real-world problem, there are several ways in which the learner can simplify the problem, mathematize, or work mathematically. Solution methods can be pre-formal or formal ones while the outcome/results stay the same. Whereas formal solution methods are the final stage in a genetic development, pre-formal solution methods refer to a certain basis of formal argumentation, but are codified in a non-formal way (Blum, 1998).

To illustrate a solution process and to exemplify two pre-formal solution methods, we will analyze the solution of the task "BahnCard," which was developed in the framework of the project MultiMa. First, the problem solver has to understand the problem "BahnCard" and construct a model of the situation. Then the model of the situation needs to be simplified and structured and the important values need to be identified. These values are the costs per year for each card and the amounts for the outward and return journeys that would be paid using each card.

BahnCard

Mr. Besser lives in Hamburg. His parents live in Bremen. For the outward and return journeys with the "Deutsche Bahn" (German Rail), Mr. Besser has to pay 100 €. There are two special offers, the so-called "BahnCard 25" and the "BahnCard 50." The prices for each year and the prices for the outward and return journeys from Hamburg to Bremen for owners of the "BahnCards" are listed below.

BahnCard 25	BahnCard 50
Price per year: 59 € Price for the outward and return journeys: 75 € Number of customers: 3.1 million	Price per year: 240 € Price for the outward and return journeys: 50 € Number of customers: 1.6 million

Mr. Besser is going to buy a "BahnCard." When is it worth buying the "BahnCard 25" and when the "BahnCard 50"? Write down your solution.

Figure 1: Modelling task "BahnCard"

Next, the simplified situation needs to be mathematized, and different mathematical solution methods can be applied to solve the problem. One solution method that can be applied is a "pre-formal solution method using differences." In order to solve a real-world problem using differences, one has to understand the meaning of the important values and to transfer information between reality and mathematics several times. Whereas the "BahnCard 50" is 181 € (= 240 € - 59 €) more expensive than the "BahnCard 25," each outward and return journey with the "BahnCard 25" is 25 € (= 50 € - 25 €) more expensive than with the "BahnCard 50." Obviously, one has to calculate a difference for the costs per year and a difference for the cost per journey as well as to interpret the mathematical results. The question is how often one has to take a trip with the more expensive "BahnCard 50" until the cheaper prices for the journeys pay off. This is exactly after 7.24 (= 181 € ÷ 25 €) journeys per year. This result has to be interpreted—for example, "For up to 7 journeys per year, the "BahnCard 25" is cheaper."—and validated. Another way to solve this problem is to apply a "pre-formal solution method using a table." Students can make assumptions about a possible number of journeys per year (e.g. 1, 3, 6…), calculate the total cost for owners of the "BahnCard 25" and the "BahnCard 50," compare the costs, identify up to what number of journeys owners should take the "BahnCard 25", and write a recommendation about which offer is preferable for a certain number of journeys.

This analysis of solving the task "BahnCard" shows two ways to solve a real-world problem. Specifically, using different solution methods leads to the same result.

Being able to choose between different solution methods grants problem solvers the ability to solve tasks more flexibly and monitor their own solution process. Therefore, we assumed that similar effects as by Schukajlow and Krug (2012) could be found in our present study in which we prompted another way to provide multiple solutions to real-world problems: multiple solution methods (MSM). In addition, we assumed that the effects on self-regulation would not differ between our one solution method (OSM) conditions.

RESEARCH QUESTIONS

1. How many solutions will students develop in the MSM-condition and will there be differences in the number of solutions developed between the MSM-condition and the OSM-conditions?
2. Will students' self-regulation differ according to the opportunity to develop multiple solution methods? In particular, will students in the MSM-condition report more self-regulation than students in the OSM-conditions?
3. Will students' self-regulation differ between the different types (i.e., table vs. differences) of prompted solution methods? More precisely, will there be differences in the reported self-perceptions of students in the OSM-conditions?

METHOD

Design and sample

307 German ninth graders (48.26% female; mean age=14.6 years) were asked about their self-regulation before and after a teaching unit (see Figure 2). The teaching unit consisted of two sessions: the first and second lessons as well as the third and fourth lessons. Four schools with three middle track classes each took part in this study. Each of the twelve classes was divided into two parts with the same number of students in each part. The average achievements in the two parts did not differ, and there was approximately the same ratio of males and females in each part. There were three different treatment conditions "multiple solution methods" (MSM), "one solution method (table)" (OSM1), and "one solution method (differences)" (OSM2). At each school, there were six different groups, which were evenly assigned to the three treatment conditions. Furthermore, each part of a class was assigned to a different treatment condition. In total, there were 24 groups: eight groups in the MSM-condition, eight in the OSM1-condition, and eight in OSM2-condition. The students in MSM, OSM1, and OSM2 were taught in different classrooms.

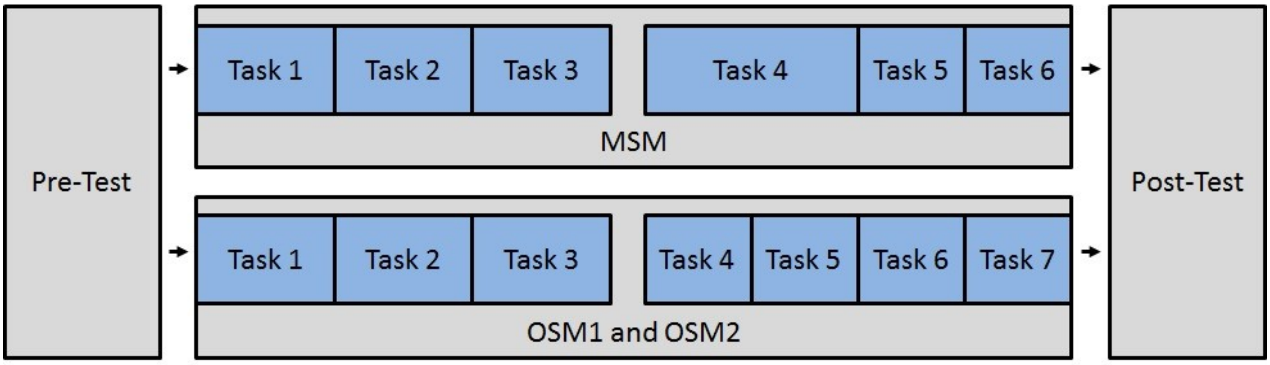

Figure 2: Overview of the study design

To implement the treatment, which consisted of solving real world problems using different solution methods, three teaching scripts were developed. Six teachers who participated in this study received these scripts with all tasks to be administered and a detailed plan for each teaching unit. Each teacher taught the same number of student groups in each treatment condition, so the influence of a teacher on students' learning did not differ between conditions. In each lesson, at least one member of the research group was present to videotape and to observe the implementation of the treatment.

Treatment

In the recent study, we used the student-centered learning environment from the DISUM project, which was complemented by direct instructions for the teaching unit. In all treatment conditions, the same methodological order was implemented for the first session. In the first session, a teacher first demonstrated how real-world problems can be solved using one specific method (in the OSM-conditions) or using multiple solution methods (in the MSM-condition). Then students solved tasks using the demonstrated solution methods according to a special kind of group work (alone, together, and alone) and discussed their solutions with the whole group in the classroom. The teacher summarized the key points of each treatment condition. Furthermore, in the MSM-condition, the teacher emphasized the development of two different methods.

In the second session, four problems were solved in the OSM-conditions and three tasks were addressed in the MSM-condition by applying the same kind of group work. After the fourth task in the MSM-condition, the teacher highlighted and summarized the link between the two methods and fostered discussions about students' preferences for one or the other solution method, whereas in the OSM-conditions, an additional task was given. Finally, in the MSM-condition, students had the opportunity to choose their preferred solution method to solve the last two tasks and discussed their choice in the classroom.

Four out of six tasks given in the MSM-condition required the development of the two solution methods: "Use two different solution methods to solve this problem. Write down both solutions." In the OSM-conditions, students solved a standard version of this task (see e.g. Fig. 1) using the demonstrated solution method.

Measures

After the second and third lessons, students were asked to report their self-regulation using a 5-point Likert scale (1=not at all true, 5=completely true) before and after a teaching unit (see Figure 2). The sample item is "While learning mathematics, I set my own goals that I would like to achieve." This scale consists of 6 items and was adapted from the longitudinal PALMA study (Pekrun et al., 2007). Reliability values (Cronbach's Alpha) for self-regulation were .66 and .75 on the pre-test and post-test respectively. The number of solutions developed (0=no solution; 1=one solution; 2=two solutions; 3=more than two solutions) was estimated by two independent raters for 20% of the tasks. The values for inter-rater agreement (Cohen's Kappa) were between .89 and .94.

RESULTS

For statistical analysis, we used t-tests, and examined that our data met the statistical assumptions for applying these tests. Levene's tests showed that there was heterogeneity of variance for some measures. For these tests, we used the adjusted values for degrees of freedom and t-values.

Number of solutions developed

First, we investigated how many solutions were developed across all problems in the MSM-condition. The analysis of students' answers showed that 1% of the students did not solve any of the posed problems, 5% of the students used one solution method, and 94% used two or even more than two solution methods. Thus, nearly all of the students in the MSM-condition used two or more solution methods (mean=1.92, standard deviation SD=0.25) as intended in our study. In the OSM-conditions, students did not or rarely used two or more solution methods (mean=1.01, SD=0.08 and mean=1.04, SD=0.24). The t-tests (MSM-OSM1: $t(116)=34.0$; $p<0.001$; effect size Cohen's $d=4.97$ and MSM-OSM2: $t(194)=25.2$; $p<0.001$; $d=3.61$) indicated that there were highly significant differences between the numbers of solution methods that were used in the respective conditions. These results revealed that nearly all students will use multiple solution methods while solving real-world problems if one prompts them to do so.

Multiple solutions and self-regulation

To examine the influence of prompting students to use multiple solution methods while solving real-world problems on students' self-regulation, we compared self-regulation on post-tests while taking into account the respective pre-test measures. The t-tests indicated that there were no significant differences between the MSM-condition and the OSM-conditions (MSM-OSM1: $t(185)=0.33$; $p=0.78$ and MSM-OSM2: $t(169)=0.36$; $p=0.72$). Thus, students in the MSM-condition did not report more self-regulation on the post-test than students in the OSM-conditions when controlling for self-regulation on the pre-test.

Self-regulation	Pre mean (SD)	Post mean (SD)	Adjusted post[a] mean (SD)
MSM	3.64 (.59)	3.47 (.63)	3.47 (.57)
OSM1	3.60 (.67)	3.48 (.73)	3.50 (.58)
OSM2	3.62 (.57)	3.54 (.70)	3.50 (.56)

a Adjusted by the pretest.

Table 1: Students' self-regulation on the pre-test, post-test, and adjusted post-test.

Different solution methods and self-regulation

To investigate the potential impact of prompting students to use different types of solution methods (i.e., table vs. differences) on students' self-regulation, we compared self-regulation in the one-solution conditions on the post-tests, taking into account the pre-test measures. The adjusted post-test means for self-regulation in the two OSM-conditions were identical with just a minor difference in the SD. A t-test showed that there were no significant differences between self-regulation in the OSM-conditions ($t(170)=0.36$; $p=0.97$). Thus, in the present study, we were able to confirm our assumption that students' self-regulation does not differ according to the type of solution method applied.

DISCUSSION

The results indicated that there were significant differences in the number of solutions developed between the MSM-condition and the OSM-conditions, as intended in our recent study. Furthermore, there was no difference in the impact of prompting different solution methods on the self-perceptions of students' self-regulation. However, we did not find any effects of prompting students to use multiple solutions on students' self-regulation. Although prompting the use of multiple solutions has previously been shown to increase flexibility (Rittle-Johnson & Star, 2007) and also self-regulation (Schukajlow & Krug, 2012), we could not find any effects of prompting students to use multiple solution methods on self-regulation in the recent study. One explanation for this result may be that students in the MSM-condition were not instructed to use certain solution methods according to the specific task but were rather instructed to use their preferred method to solve all tasks of this type. The highest level of self-regulation in Zimmerman's hierarchal order – flexibly and systematically adapting one's performance to changing conditions – was not achieved in the MSM-condition. The ability to choose a solution method based on individual-, task-, and context-specific criteria is an important part of being flexible and adaptive (Heinze et al., 2009). These criteria should be taken into account in future studies.

Compared to the results by Schukajlow and Krug (2012), where significant differences in students' self-regulation were found, students did not have the opportunity to make assumptions about missing information and to apply their assumptions to the task. This lack of autonomy could be a reason for the failure to find an increase in students' self-regulation in the current study.

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