“WHY DON’T YOU MAKE A DRAWING?” MOTIVATION AND STRATEGY USE IN MATHEMATICAL MODELLING

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Motivation is important for students’ learning and strategy use. However, we do not know much about the relations between motivation and the use of strategies such as the drawing strategy. In this study, we assessed the mathematical and strategy-based motivation of 194 ninth- and tenth-grade students using expectancy-value questionnaires. Further, we measured the spontaneous use of drawings for solving geometric modelling problems. We found a positive relation between mathematical and strategy-based expectations of success as well as between mathematical and strategy-based attainment value. Furthermore, mathematical and strategy-based motivation differed in their relation to the use of drawings. These results indicate the importance of both mathematical and strategy-based motivation for strategy use and modelling.

INTRODUCTION

Mathematics as an applied science is part of many other disciplines, such as the natural sciences, computer science, and the social sciences. An application-based view of mathematics is reflected in mathematical modelling. Mathematical modelling involves the use of mathematics to solve real-world problems (Niss, Blum, & Galbraith, 2007). Because of the importance of applications for life and work, countries around the world recommend that mathematical modelling be promoted in mathematics education, and it is included in the mathematics curriculum of different countries. However, prior research has repeatedly demonstrated that students have trouble solving modelling problems (Niss et al., 2007). The use of strategies such as self-generated drawing is considered to have a beneficial effect in overcoming the difficulties involved in solving modelling problems (Galbraith & Stillman, 2006; Hembree, 1992). Positive effects of drawings have been shown for students who made drawings spontaneously. However, why do learners rarely make drawings spontaneously? One possible factor that influences the spontaneous use of drawings is motivation. In the present research, we targeted mathematics and the drawing strategy as the objects of motivation because mathematical and strategy-based motivation might both be important for the spontaneous use of drawings. In this paper, we aimed to examine the relation between mathematical and strategy-related motivation and their importance for the spontaneous use of drawings in mathematical modelling.

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THEORETICAL BACKGROUND

Self-generated Drawings in Mathematical Modelling

By making a self-generated drawing for a mathematical modelling task, the learner visualizes a problem described in the task by representing the objects and their relations to each other in an iconic way. By applying the strategy of making a drawing, we understand both the drawing process and the drawing as a product (Rellensmann, Schukajlow, & Leopold, 2017). As a strategy for learning and problem-solving, making drawings can support various activities in mathematical modelling such as constructing a mental model of the text, discovering errors in the mental model, structuring and simplifying the given situation and constructing a real model, mathematizing the real model, or validating the mathematical result.

Spontaneously making a drawing for a given mathematical word problem has already been shown to be a potentially performance-enhancing strategy for learners (Hembree, 1992; Uesaka et al., 2007). This strategy was found to be more helpful than improving mathematical vocabulary, verbalizing important concepts, or applying other strategies (Hembree, 1992). Thus, making a drawing might also be helpful for solving geometrical modelling problems. Despite the expected positive effects of generating a drawing in mathematical modelling derived from the analysis of modelling activities such as mathematizing, students rarely use this strategy spontaneously. One reason for this result might be students’ motivation. For example, in Pressley's (1986) model of a Good Strategy User, motivational beliefs are suggested to predict the spontaneous use of strategies. Pressley further suggested that if students are motivated to use a strategy, they will use it more often.

Expectancy-value Theory of Motivation

In a broader definition, Middleton and Photini (1999) specified motivation as a reason for human behavior in a specific manner and in each situation. At the core of many theories of motivation are expectancy-value models such as the one by Eccles and Wigfield (1995). These models propose that performance-related decisions (e.g., using a specific strategy) are essentially influenced by two subjective beliefs: expectations of success (ES) and the value attached to the different options that are available. In research, expectations of success have often been estimated via self-concept or via general self-efficacy, which have repeatedly been found to be closely connected to each other (see the overview by Marsh et al., 2019). The value component includes three sub-components: the interest and enjoyment gained from the task (Intrinsic Value, IV), the personal importance of being able to do it well (Attainment Value, AV), and the perceived utility from solving it (Utility Value, UV). Similar to other affective constructs, motivation can target different objects (Schukajlow, Rakoczy, & Pekrun, 2017). The objects of motivation can be learning in general, a specific topic, or even a specific problem. The present research involves mathematical motivation because the object of motivation is mathematics. Motivation that targets a specific strategy or its characteristics as its objects can be called strategy-based motivation. In the present research, we assessed strategy-based motivation by using the drawing strategy because of the importance of this strategy for problem-solving (Hembree, 1992).

Prior research hypothesized a positive relation between expectations of success that targeted different objects in one domain such as mathematics. The reason for this positive relation is that problem-solving activities within mathematics require related abilities and skills. Furthermore, students acquire different abilities and skills in mathematics in parallel in their mathematics lessons or in mathematical activities that they participate outside of school. These considerations were confirmed empirically by Marsh et al. (2019), who demonstrated a positive relation between mathematical expectations of success (that were asked about by referring to mathematics in general) and to specific mathematical problems as objects of motivation. Likewise, a positive relation can be expected between values within the same domain such as mathematics. The expectation that values for different objects in mathematics can be related has been supported by empirical results. For example, the utility value of modelling problems was found to be positively related to the utility value of intra-mathematical problems (Krawitz & Schukajlow, 2018). However, prior empirical results should be interpreted with caution because the differences in the objects of motivation are essential for the relations between the constructs. The relation between mathematical and strategy-based motivation is still an open question.

Motivation and Strategy Use

Many studies have demonstrated the positive effects of expectations of success and value on the use of cognitive and meta-cognitive learning strategies. For example, Virtanen, Nevgi, and Niemi (2013) showed that university students who reported high expectations of success and high intrinsic value were also more likely to report that they organize the learning content in their discipline. Focusing on the relation between mathematical motivation and self-reported learning strategies in mathematics, Berger and Karabenick (2011) found that both expectations of success and value predicted elaboration and metacognitive strategies. However, in these studies, researchers used self-reports to assess the strategies, and the validity of assessing strategies via self-reports has often been criticized in the past. Because of research on the relation between mathematical motivation and self-reported strategies, we suggest a positive relation between mathematical motivation and the use of the drawing strategy.

Moreover, we found only a few studies that analyzed the relation between motivation and the spontaneous use of the drawing strategy. A case study of an eighth-grade girl who did not use a drawing strategy spontaneously at first but used it successfully after being instructed to do so suggests that spontaneous strategy use depends on the perceived efficiency of the strategy and thus also on motivation (Ichikawa, 1993; Uesaka, Manalo, & Ichikawa, 2007). Furthermore, Uesaka et al. (2007) demonstrated that the benefits attributed to learner-generated drawings reported by students were significantly related to the use of drawings. These findings indicate that strategy-based motivation might be important for the spontaneous use of drawings.

RESEARCH QUESTIONS AND HYPOTHESIS

Based on theoretical considerations, we conclude that the spontaneous use of a drawing strategy is related to motivational factors. However, there is a research gap regarding the relation between mathematical and strategy-based motivation as well as to the relation between motivational factors and the use of the drawing strategy. Moreover, we did not find any research that investigated the relation between motivation and making a drawing to solve modelling problems. Therefore, we addressed the following questions in this study:

(1) How are the mathematical motivational constructs (ES, IV, AV, UV MATH) related to the corresponding strategy-related constructs (ES, IV, AV, UV DRAW)?

We expected a positive relation between mathematical and strategy-based expectations of success because the development of the strategic skills involved in making drawings takes place within mathematical learning. We also expected positive relations between the different values of the mathematical and strategy-based constructs. However, as the relations between motivational constructs strongly depend on how close the objects of motivation are to each other, and only a little research has been conducted on strategy-based motivation, these expectations were based mostly on theoretical considerations.

(2) How are mathematical and strategy-based motivational constructs (ES, IV, AV, UV) related to the spontaneous use of the drawing strategy while students solve modelling problems?

Based on the expectancy-value theory, we expected both mathematical and strategy-based motivation to be important for the spontaneous use of drawings. An empirical indication for the positive relation between mathematical motivation and the use of the drawing strategy comes from research on self-reported strategies. One case study and one cross-sectional study carried out with school students on the use of the drawing strategy supported the expectation that students’ strategy-based motivation might be related to spontaneous strategy use.

METHOD

Participants and Research Design

Two hundred twenty German ninth- and tenth graders (49.5% female, M = 14.93 years) of 10 comprehensive classes participated in the study. At the first occasion, the students answered a questionnaire about motivational constructs. After two weeks, they were asked to solve eight geometric modelling tasks. The analysis of students' solutions allowed us to assess their spontaneous use of the drawing strategy. Some students could not participate on both occasions for various reasons. In sum, 194 students participated on both occasions and were included in our analysis.

Measures

The 22-item survey was applied to assess mathematical motivation (MATH, 10 items) and strategy-based motivation with respect to the use of drawings (DRAW, 12 items). Students rated each statement on a 5-point scale (1 = "not true at all" to 5 = "completely true").

Mathematical motivation scale. The mathematical motivational items were adapted in accordance with Eccles and Wigfield (1995). *Expectations of success* (ES MATH) were assessed with three items (e.g., “I am very good at mathematics”). The three components of *value* are *intrinsic value* (IV MATH; 2 items; e.g., "In general, I find working on mathematics assignments very interesting"), *attainment value* (AV MATH; 3 items; e.g., "It is very important to me to be able to solve mathematical problems very well"), and *utility value* (UV MATH; 3 items; e.g., "Mathematics in school is very useful for my professional future after graduation"). The reliabilities of the subscales were mostly good to satisfactory ($.55 < \alpha < .89$). The confirmatory factor analysis revealed that the model with four factors fit the data adequately ($\chi^2/df = 1.72$, SRMR = .04, RMSEA = .06, CFI = .97).

Strategy-based motivation scale. The strategy-based motivation scale with respect to the use of the drawing strategy was assessed with four subscales: *expectations of success* (ES DRAW; 3 items; e.g., “I believe I can make very good drawings for any word problem”), *intrinsic value* (IV DRAW; 3 items; e.g., "I like to make a drawing for a difficult word problem"), *attainment value* (AV DRAW; 2 items; e.g., "It is important to me to be able to make a drawing for a difficult word problem"), and *utility value* (UV DRAW; 4 items; e.g., "Making drawings is important to me because it helps me solve difficult word problems"). The reliabilities of the subscales were mostly good to satisfactory ($.58 < \alpha < .86$). Confirmatory factor analyses showed acceptable values for the model ($\chi^2/df = 3.27$, SRMR = .04, RMSEA = .07, CFI = .95).

Use of drawings. The use of drawings was measured dichotomously for each of eight modelling tasks that could be solved by applying the Pythagorean

Theorem. A code of 0 was assigned to solutions without a drawing and a code of 1 to solutions with a drawing. The measurement showed good reliability (Cronbach's α = .866).

RESULTS

Relations of mathematical and strategy-based motivation. As expected, the analysis of the correlations between mathematical and strategy-based motivation (Table 1) showed moderate positive correlations between ES MATH and ES DRAW as well as between AV MATH and AV DRAW. These results indicate that students who have high expectations of success and ascribe a high attainment value to mathematics are confident that they can use a drawing strategy to solve problems and feel that this strategy is personally important to them. However, we did not find a positive relation between intrinsic value or utility value for mathematical and strategy-based motivation. For example, students who ascribed a higher utility value to mathematics did not differ in their estimation of the utility value of the drawing strategy.

		MATH			
		ES	IV	AV	UV
DRAW	ES	.289**	.255**	.377**	.234**
	IV	-.041	.010	.233**	.087
	AV	.007	.104	.351**	.117
	UV	-.018	.010	.278**	.038

Note. ** $p < .01$, p two-tailed. MATH: mathematical motivation, DRAW: strategy-based motivation, ES: expectancy of success, IV: intrinsic value, AV: attainment value, UV: utility value. Correlations between the same constructs in different domains are presented in grey.

Table 1: Correlations between mathematical and strategy-based motivational constructs

Motivation and the use of drawings. Our analysis of the relation between mathematical motivation and the use of drawings confirmed our expectation for IV MATH (Table 2). Students who attributed high intrinsic value to mathematics used the drawing strategy to solve modelling problems more often. Mathematical expectations of success, attainment value, or utility value in mathematics were not related to the use of drawings. The analysis of the relation between strategy-based motivation and the use of drawings while modelling revealed a more consistent picture and confirmed our expectations. We found positive correlations for all strategy-based sub-constructs IV DRAW, AV DRAW, UV DRAW, and ES DRAW with the use of drawings. These results indicate the importance of strategy-based motivation for the spontaneous use of the drawing strategy. Students who have high expectations of success for the use of drawings and who ascribe high intrinsic, attainment, and utility value to the drawing strategy more often used this strategy spontaneously.

		MATH				DRAW			
		EX	IV	AV	UV	EX	IV	AV	UV
USE	*r*	.047	.180*	.088	.098	.164*	.212**	$.138^{a}$	.172*

Note. ^{a} $p < .10$, * $p < .05$, ** $p < .01$, ** $p < .001$. p: two-tailed. MATH: mathematical motivation, DRAW: strategy-based motivation, EX: expectancy, IV: intrinsic value, AV: attainment value, UV: utility value, USE: spontaneous use of drawings.

Table 2: Correlations between mathematical and strategy-based motivational constructs and the spontaneous use of drawings

DISCUSSION

Based on expectancy value theory (Wigfield & Eccles, 2000), we investigated the relation between mathematical and strategy-based motivation and the importance of motivation for the use of drawings while solving modelling problems. As expected, the analysis of the relation between mathematical motivation and the strategy-based motivation to make drawings showed that mathematical and strategy-based expectations of success were positively related. However, the relation was weak. One reason for this result may be the cognitive structure of the activities: Although the making of drawings as a visual strategy is part of the mathematical curriculum, formal symbolic procedures usually predominate in students' learning in mathematics. Another reason may be the different categories of focused objects (the domain of mathematics vs. the strategy of drawing). As mathematics is a more general object and the drawing strategy is a more specific object, this difference might have an impact on the strength of the relation between the constructs (Marsh et al., 2019). The relation between the personal importance of being good at mathematics (AV MATH) and the personal importance of making good drawings (AV DRAW) was moderate in size. This result revealed that the personal importance of mathematics is closely related to the personal importance of making a drawing to solve mathematical problems. By contrast, the intrinsic and utility values of one object were not related to the values of other. The perceived utility of drawings for solving problems did not depend on whether mathematics was considered useful or not.

The strategy- and mathematics-based motivational constructs differed in their relations with the spontaneous use of drawings during mathematical modelling.

Whereas only the intrinsic value of mathematical motivation was correlated with the use of drawings, all four strategy-based motivational constructs were positively related to the use of the drawing strategy. We suggest that future studies conduct deeper investigations of the relation between mathematical and strategy-based motivation on the one hand and the use of drawings and performance on the other hand. One interesting research question might be whether mathematical motivation has an indirect effect on the use of strategies and performance via strategy-based motivation. In line with results from learning strategy research (Berger & Karabenick, 2011; Virtanen et al., 2013), intrinsic value with respect to mathematics was found to be related to spontaneous strategy use. In addition, as suggested by expectancy-value theory, we found a positive relation between strategy-based expectations of success and the use of drawings in our research. Positive relations between strategy-based values and the use of strategies indicated the importance of values for students' strategy use. Thus, our results confirmed the validity of expectancy-value theory for strategy use.

The results revealed intrapersonal differences when comparing mathematical motivation and strategy-based motivation with respect to making a drawing in mathematical modelling and in problem-solving. Effects of strategy-based motivation on learning outcomes should be addressed more often in future research because it can explain why some students make drawings spontaneously and others do not. Research on strategy-based motivation can be applied not only for the use of the drawing strategy but also to other strategies. Finally, for the practice of teaching, it is important to investigate which teaching interventions improve strategy-based motivation and students' strategic and achievement-related learning outcomes.

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