# VALUE OF PICTURES IN MODELLING PROBLEMS FROM THE STUDENTS' PERSPECTIVE

Matthias Böckmann and Stanislaw Schukajlow

University of Münster

*Pictures are an important part of human life, and they often accompany modelling problems. In the present study, we investigated whether the extent to which students believe pictures are valuable for understanding modelling problems differs for decorative, representational, and essential pictures. 217 ninth and tenth graders from nine German middle-track classes were randomly assigned to three groups. One group reported the picture-specific utility value of decorative pictures, whereas two other groups reported the utility value of representational pictures and essential pictures, respectively. Students' picture-specific utility value ratings were higher for essential pictures and representational pictures than for decorative pictures, and their utility value ratings were higher for essential pictures than for representational pictures.*

## INTRODUCTION

One important goal of mathematics education is to ensure that students are able to solve problems in the real world using mathematics; thus, modelling competence is part of curricula all over the world (Niss et al., 2007). Similar to the problems in the real world, modelling problems that are presented in the classroom should (and often do) include text and pictures. In order to solve modelling problems, students have to use information that can be found in both the text and the pictures.

The Cognitive Theory of Multimedia Learning explains how humans deal with information that is presented in text and pictures. There are two channels in working memory, one that processes information from words and one from pictures (Mayer, 2005). For an efficient processing of information, the usefulness of the pictures has to be recognized. However, recent studies showed inconsistent results with respect to whether students saw pictures as useful while they solved mathematical problems (Dewolf et al., 2015; Elia & Philippou, 2004). More precisely, it is not clear whether students recognize that pictures can be useful for solving problems with a connection to reality. Moreover, although information in the real world is often presented pictorially, we could not find any studies that investigated students' perceptions of the usefulness of pictures for solving modelling problems in the classroom. The current study focuses on students' perceptions of the extent to which pictures that accompany modelling problems can be useful (or the perceived utility value of pictures) for solving these problems.



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## THEORETICAL BACKGROUND

### Utility Value

Values are an important part of affect and have been investigated from social, psychological, and sociological perspectives (Bishop et al., 2003). The expectancy-value theory links expectancies and personal values and proposes that expectancies and values influence performance, task choice, and motivation (Eccles, 1983). A positive relation between values and students' performance was recently confirmed for problems with and without a connection to reality (Schukajlow, 2017).

In line with the expectancy-value theory researchers underline the importance of the utility (or extrinsic) value for motivation and achievement. A task's utility value describes the learner's perceived usefulness of a task (Eccles & Wigfield, 2002) and refers to the importance of a task or its parts (e.g. text or pictures) for career, grades, an accurate solution, or other indicators of success. In this study, we analyzed the utility value of pictures for understanding modelling problems and thus for the solution process. As the pictures that accompany modelling problems can have different functions in the solution process, we were interested in determining whether students would assign different utility value ratings to different types of pictures.

### Pictures and their functions in problem solving

The term "pictures" describes static visual illustrations such as photos, paintings, or vector graphics. In this study, we used photos as pictures because photos are closely connected to reality and reflect reality more precisely than other types of pictures.

In combination with text, pictures can serve different functions. In the present study, we adapted a taxonomy of pictures for modelling problems that was developed by Elia and Philippou (2004), who specified different functions of pictures in mathematical problem solving. *Decorative pictures* "do not give any actual information concerning the solution of the problem" (Elia & Philippou, 2004, p. 328). *Representational pictures* show the parts of the situation described in the text. *Essential pictures* (called informational pictures by Elia & Philippou, 2004) present information that is essential for solving the modelling problems.

### Utility value and pictures in modelling

A cognitively demanding transfer between real-world problems and mathematics is at the core of mathematical modelling. Activities that are needed to solve modelling problems are understanding the situation and constructing a situation model (Blum & Leiß, 2007). As understanding the problem is the first activity in the modelling process, it can be expected to influence other modelling activities such as constructing a mathematical model and is thus very important for solving a problem. This expectation was confirmed empirically, as the quality of the situation model was found to be closely related to modelling competence (Krawitz et al., 2017; Leiß et al., 2010). Because it is important for students to understand modelling problems, we decided that our study would focus on this modelling activity.

One example of a modelling problem used in our study is the Kite problem (Figure 1). The process of understanding while solving the Kite problem results in the construction of a model of the situation that includes two people, a piece of string, a kite, and the positions of the people and the kite. The height of the kite is unknown, and it can be calculated, for example, by using Pythagoras' theorem and adding an estimate of Lucas' height.

Lucas got a new kite as a birthday present. The kite is 1 m in length and 50 cm in width. Lucas flies the kite with his friend Susan (see picture). They are standing at a distance of 80 m from each other. The kite's string has a length of 100 m. Susan is right under the kite.

How high is the kite flying at this moment?

Figure 1: The Kite modelling problem with a decorative picture

Pictures can support the modelling process by facilitating students' understanding. The decorative picture in Figure 1 does not facilitate an understanding of the problem because it presents only the kite but not the positions of the people flying the kite. Thus, this picture does not enhance a deeper understanding of the problem, which is crucial for constructing a deep situation model and thus also for solving a modelling problem (Krawitz et al., 2017). However, a modelling problem can also include a representational or an essential picture instead of a decorative picture. A representational picture of the Kite problem might show where Susan, Lucas, and the kite are positioned and can help the problem solver structure the information and construct a model of the situation (Figure 2). An essential picture should include important information such as the distance of 80 m between Susan and Lucas in the Kite problem. Without taking this information into account, it is not possible to solve the problem (Figure 3).

A picture's utility value determines whether the picture will be used in the solution process. If students perceive the picture as useful, they might integrate the information from the picture into the solution. In this case, the picture can support the modelling process. This is why we asked students to rate the utility value of different pictures.

Prior findings on the utility value of pictures for solving problems have been contradictory to some extent. Whereas most students recognized that decorative pictures did not help them solve mathematical problems (Elia & Philippou, 2004), students did not always identify the high utility value of pictures with other functions. For example, in the study by Elia and Philippou (2004), students realized the supporting role of representational pictures for solving arithmetic word problems. In the study on realistic problems, however, students did not include information from the picture in their solution, and thus, they did not identify the usefulness of representational pictures for a solution (Dewolf et al., 2015). Further, only a few students recognized the importance of essential pictures for solving arithmetic word problems (Elia & Philippou, 2004).

One explanation for these results is that students could not identify what kinds of information in representational or essential pictures might be helpful for solving problems. The ability to recognize picture-specific utility value might depend, among other things, on the type of problem and might be different for real-world problems.

## HYPOTHESIS

Pictures can help people understand real-world problems. Representational and essential pictures enhance the construction of a situation model because these pictures can act as structural aids and can facilitate the step of understanding in the modelling process. These pictures are supposed to have high utility value in helping people understand the problems, whereas decorative pictures do not support the modelling process and therefore have a low perceived utility value in helping people understand the given problems. In addition, essential pictures are necessary for the solution process and might have a higher utility value than representational pictures. Our considerations led to the following hypotheses: (1) Students will assign higher utility value to representational and essential pictures than to decorative pictures in the extent to which these pictures help them understand modelling problems. (2) Students will assign higher utility value to essential pictures than to representational pictures.

## METHOD

### Sample and design

217 students from nine middle-track classes (lower secondary schools) in grades 9 and 10 (mean age=15.06 years, SD=.79; 49.9% female) participated in the study. The students in each class were randomly assigned to one of the three experimental groups. Each group read modelling problems accompanied by pictures and then rated each picture's specific utility value for understanding the problem. Students in group 1 documented their perceived utility value of pictures with a decorative function. Group 2 reported on the utility value of representational pictures, and group 3 on the utility value of essential pictures. The instructions were: "Read each problem carefully and then answer some questions. **You do not have to solve the problems!**" (cf. Schukajlow, 2017). The participants did not solve the problems because it was not necessary to solve the problems in order to rate a picture's utility value for *understanding* the problem. After reading these instructions, students read each problem and answered the question about utility value.

### Sample problems

In the present study, we used six modelling problems on the topic Pythagoras' theorem, that were developed and tested in prior studies (e.g. Schukajlow, 2017; Blum, 2011). In the present study, we reworked the pictures that accompanied the problems and offered students problems with a decorative, representational, or essential picture. A sample problem (i.e. the Kite problem) with a decorative picture was presented above (Figure 1). Figure 2 presents the same modelling problem with a representa-

tional picture. This picture shows the situation described in the task and can support the problem solver's understanding of the problem.

Lucas got a new kite as a birthday present. The kite is 1 m in length and 50 cm in width. Lucas flies the kite with his friend Susan. They are standing at a distance of 80 m from each other (see picture). The kite's string has a length of 100 m. Susan is right under the kite.

How high is the kite flying at this moment?

Figure 2: The Kite modelling problem with a representational picture

In the experimental condition with an essential picture, numerical information about the distance of 80 m between Lucas and Susan is missing from the text but is presented in the picture (Figure 3). This information is important for solving the problem.

Lucas got a new kite as a birthday present. The kite is 1 m in length and 50 cm in width. Lucas flies the kite with his friend Susan. They are standing far away from each other (see picture). The kite's string has a length of 100 m. Susan is right under the kite.

How high is the kite flying at this moment?


Figure 3: The Kite modelling problem with an essential picture

**Utility value**

A statement about the picture-specific utility value followed each modelling problem. The item that was used to measure the utility value of decorative, representational, and essential pictures for understanding the problem was "The picture helps me understand the problem." A 5-point Likert scale was used (1=not at all true; 5=completely true) to record the students' answers. The picture-specific utility value was measured by calculating mean values of the answers for all six problems for each type of picture. The Cronbach's alpha reliabilities for the utility value scale for decorative, representational, and essential pictures were .73, .82, and .84, respectively.

In order to analyze group differences in the perceived utility values, a one-way ANOVA was computed. For a post hoc analysis, we used Bonferroni comparisons.

## RESULTS

### Perceived utility value

To test Hypotheses 1 and 2 and compare the utility values of pictures for understanding modelling problems, we compared the utility value means for the students who rated the pictures with decorative, representational, and essential functions. Table 1 shows the group means for perceived utility value for the three different types of pictures.

| function of picture | | |
|---|---|---|
| decorative | representational | essential |
| 2.14 (.71) | 3.26 (.89) | 3.74 (.84) |

Table 1: Means (SDs) for perceived utility value

Students assigned the lowest utility value to the decorative and the highest to the essential pictures. As expected, there were significant differences in perceived utility value between the three types of pictures ($F(2, 214)=74.40$, $p<0.01$, $\text{eta}^2=.41$).

| (I) Type | (J) type | Mean Difference (I-J) | Std. Error (SE) | *p* | Cohen's d |
|---|---|---|---|---|---|
| representational | decorative | 1.12 | .13 | <.01 | 1.39 |
| Essential | decorative | 1.60 | .14 | <.01 | 2.06 |
| Essential | representational | .48 | .14 | <.01 | 0.55 |

Table 2: Values from the Bonferroni post hoc analysis of differences in utility value

A post hoc analysis employing t-tests with a Bonferroni correction revealed significant differences between representational and decorative pictures ($t(145)=8.46$) and between essential and decorative pictures ($t(135)=12.36$) (Table 2). Thus, this result confirmed our first hypothesis: Students gave higher utility value ratings to representational and essential pictures than to decorative pictures. Furthermore, we found a significant difference between the utility value ratings of the essential and representtational pictures ($t(140)=3.33$) and confirmed our second hypothesis: Students gave higher utility value ratings to essential pictures than to representational pictures.

## DISCUSSION

In the present study, we analyzed students' utility value ratings of pictures used in modelling problems. The results showed that students' picture-specific utility value ratings differed according to the pictures' functions (Elia & Philippou, 2004). Decorative pictures did not support the modelling process, as they do not include infor-

mation that can help solve the problem and do not facilitate the construction of a situation model. Representational and essential pictures facilitated the construction of a situation model due to their supporting role in the modelling process. As expected on the basis of theoretical considerations and prior findings, students reported a significantly lower utility value for understanding the problems with decorative pictures than for representational or essential pictures. This result is in agreement with Elia and Philippou's (2004) study in which all students recognized that decorative pictures did not help solve a problem and gave decorative pictures lower utility value ratings than representational pictures. However, it is not in line with the results of Dewolf et al. (2015) who did not find indications of the perceived importance of representational pictures for solving realistic problems. One explanation of this finding is a difference in the process of solving realistic problems and the modelling problems used in our study. Whereas students neglect reality when solving realistic problems, they consider reality when solving modelling problems (Galbraith & Stillman, 2006).

The comparison of the utility value ratings of essential and representational pictures confirmed that students gave higher utility value ratings to essential pictures. This finding indicates that students identified information in the picture that was essential for solving the problem. This result was different from Elia and Philippou's (2004) findings. One explanation for the clear differences between the utility value ratings of essential and representational pictures in our study is that the pictures and text were presented simultaneously. Thus, it was easier for the students in our study to identify the usefulness of essential pictures than in Elia and Philippou's (2004) study, in which the students first worked with the text and were given the picture later.

One limitation of our study involves the design of the essential pictures. The numerical information in the essential pictures can attract readers' attention and can thus foster utility value. We tried to counter this limitation by adding "*see picture"* to the text in all conditions. Moreover, students might estimate pictures' utility value superficially, as students might assign higher utility value to such pictures without understanding whether this information is important for solving the problem. This open question should be clarified in future studies. Another important future question is how the utility value of different pictures affects students' modelling performance.

## CONCLUSION

Modelling problems often include pictures. However, to the best of our knowledge, the importance of pictures in modelling problems had not been investigated until now. In the present study, we expanded prior findings to include perceptions of the role of pictures in modelling problems. In our study, students recognized that pictures with different functions had different levels of usefulness, and we encourage teachers and researchers to pay attention to the pictures they use in the classroom.

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