A NEGATIVE EFFECT OF THE DRAWING STRATEGY ON PROBLEM SOLVING: A QUESTION OF QUALITY?

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Make a drawing is known to be a powerful strategy for solving mathematical problems. But surprisingly, the drawing strategy was found to negatively affect the ability to solve non-linear geometry problems. Our study replicates and extends this finding by addressing the quality of the drawing strategy, which might explain the negative effect. In a randomized controlled trial with 180 students (ninth- to eleventh-graders), we enhanced drawing quality by prompting the students to highlight important elements in their drawings. Our results replicated the negative effect of the drawing strategy on performance and confirmed the quality of the drawing strategy as an important factor that affected the number of linear overgeneralizations. The roles of drawing quality and other factors that might influence the ability to solve such problems are discussed.

INTRODUCTION

The drawing strategy is a heuristic method that is claimed to have strong positive effects on problem solving. However, studies that have investigated the effect of applying the drawing strategy have arrived at divergent findings. Some studies showed that the drawing strategy facilitates problem solving (e.g. Hembree, 1992), whereas others did not find any effects (e.g. De Bock, Verschaffel, & Janssens, 1998), and one study even provided surprising evidence for a negative effect of the drawing strategy on problem solving performance (De Bock, Verschaffel, Janssens, Van Dooren, & Claes, 2003). The present study is aimed at replicating the negative effect of applying the drawing strategy and elaborating on potential explanations for why applying the drawing strategy can hinder problem solving.

THEORETICAL BACKROUND AND RESEARCH QUESTIONS

Drawing Strategy

Applying the drawing strategy involves constructing an external visual representation that corresponds to the structure of the mathematical problem. By drawing, the learner externalizes his or her mental model of the problem situation. This involves re-organizing the given information in such a way that important elements and relations become visible and can be processed more easily after the drawing is constructed (Larkin & Simon, 1987). Hence, drawing makes the key information from the problem explicit and facilitates the process of problem solving (Cox, 1999).

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Empirical evidence for the positive effect of drawing strategy was found in a number of studies (Rellensmann, Schukajlow, & Leopold, 2016; Van Essen & Hamaker, 1990; Zahner & Corter, 2010). Teaching the drawing strategy was even identified as the most effective treatment for improving mathematical problem solving in a meta-analysis conducted by Hembree (1992), in which drawing strategy was compared with other strategies such as verbalizing concepts. However, drawing strategy does not help all students solve the problem. Theoretical models of self-generated drawings emphasize that the benefits of applying the drawing strategy are strongly related to the quality of the use of the strategy (Cox, 1999).

The quality of the use of the drawing strategy is reflected in two properties of the drawing as the final product of the drawing process: the correctness and completeness of the drawing. High-quality use of the drawing strategy implies that students construct a correct drawing (correctness) that explicitly represents the key information from the problem (completeness). The first evidence for the importance of the quality of the use of the drawing strategy comes from research on text-based learning. Supporting students' drawing activities positively affected performance on items that required comprehensive elaboration activities (Van Meter, 2001). Moreover, empirical studies in science and mathematics confirmed theoretical considerations and revealed that the quality of the drawing strategy is positively related to demanding problem solving (Rellensmann et al., 2016; Schwamborn, Mayer, Thillmann, Leopold, & Leutner, 2010; Uesaka, Manalo, & Ichikawa, 2007). Students who constructed drawings of higher quality solved geometrical modelling problems better than other students (Rellensmann et al., 2016). The quality of the drawing strategy is expected to be particularly important when students are required to build connections and draw conclusions from the given information (Van Meter, 2001), as is the case for solving non-routine mathematical problems.

The Drawing Strategy for Solving Non-Linear Problems

An important type of non-routine mathematical problems is the non-linear geometry problem, in which the area or volume of similar figures or solids has to be determined by a given scaling factor. For example: "You need approximately 400 grams of flower seed to lay out a circular flower bed with a diameter of 10 m. How many grams of flower seed would you need to lay out a circular flower bed with a diameter of 20 m?" (De Bock et al., 1998, p. 68). This type of problem is important because it addresses students' strong tendency to engage in linear overgeneralizations – the application of linear models to non-linear situations – which is known to be a common error in problem solving (Van Dooren, De Bock, Janssens, & Verschaffel, 2008). A series of studies conducted by De Bock and colleges (De Bock, Van Dooren, Janssens, & Verschaffel, 2002; De Bock et al., 1998; De Bock et al., 2003) showed that this type of problem is very difficult for students, who often seem to use the linear model in an intuitive manner without being aware of the model they chose (De Bock et al., 2002).

The drawing strategy can be helpful for solving non-linear geometry problems because it provides the opportunity to recognize the non-linear property of the area, and thus, it might facilitate the use of appropriate mathematical procedures. A drawing for a non-linear geometry problem should include the original and scaled figure, which enables the use of visual solution strategies aimed at estimating the relation of the areas (e.g. paving strategies). Contrary to these theoretical considerations, De Bock et al. (2003) showed that applying the drawing strategy did not facilitate the solving of non-linear geometry problems and even affected problem solving performance negatively. What can explain this unexpected finding? In the drawing condition, students between the ages of 13 and 16 were given a drawing that referred to the geometrical object from the problem (e.g. a square). They were then instructed to complete the drawing by using the given scaling factor to add a scaled geometrical object. Students in the drawing condition performed worse than students in the control group, who worked on the same problems without receiving any instructions (23% vs. 44%). An in-depth analysis of students' solutions indicated that the drawing strategy did not elicit visual solution strategies for determining and comparing the sizes of the areas. This argument provides a good explanation for why applying the drawing strategy was not beneficial, but it remains unclear why using a drawing negatively affected problem solving in this study. Another explanation might be that students use the drawing strategy inappropriately, which in turn decreases their performance in solving non-linear geometric problems.

Because of the surprising nature of the negative effects of the drawing strategy, we aimed to replicate De Bock et al.'s (2003) study in order to validate its findings. We expected a negative effect of using the drawing strategy on problem solving performance for non-linear geometry problems. Further, we expected the use of the drawing strategy to increase students' tendency to engage in linear overgeneralizations. As geometrical figures are typically depicted by their circumferences, students' attention is guided toward the linear property of the circumference while drawing instead of toward the non-linear property of the area.

Further, we considered the quality of the drawing strategy as a potential reason for the negative effect of using the drawing strategy on problem solving performance. In particular, we expected that key information such as the area and its non-linear relationship would not be made salient in the drawings so that the quality of drawing strategy would be insufficient with respect to the completeness of the drawings. Therefore, we expected that increasing the quality by highlighting the key information would diminish the negative effect of the drawing strategy on performance because it would prevent the linear overgeneralizations that usually result from drawing.

RESEARCH QUESTIONS

These considerations led us to pose the following research questions:

RQ 1: Does the use of the drawing strategy decrease problem solving performance and increase linear overgeneralizations?

RQ 2: Does increasing the quality by highlighting important information in the drawing diminish the negative effects of the drawing strategy on problem solving performance and on the number of linear overgeneralizations?

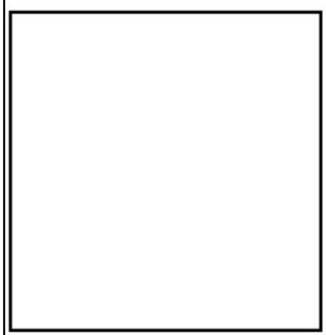
METHOD

Participants and Design

The sample involved 123 students (58.5% female, mean age = 16.19 years) from nine classes, including ninth-graders (11.4%), tenth-graders (48.8%), and eleventh-graders (39.8%). Students came from four high-track schools (German Gymnasium) and one comprehensive school (German Gesamtschule). Students in each class were randomly assigned to one of three groups: Students in the experimental conditions received either drawing (D) or drawing with highlighting (DQ) instructions, aimed at increasing the quality of the drawing strategy. Students of the control group (CG) received no drawing instructions. The instructions were embedded in the tasks given on a paper-and-pencil test. Figure 1 shows the drawing with highlighting instructions (DQ condition) embedded in one of the tasks.

The side of square C is 12 times as large as the side of square D.

a) Draw square D.
b) Hatch the area of square D using a colored pencil.



Square C **Square D**

Figure 1: Sample item with drawing with highlighting instructions. Tasks were adopted from De Bock et al. (2003, p. 449)

To check the implementation of the treatment, we examined whether students in the experimental and control groups followed the instructions by analyzing the numbers of papers with no drawings, drawings (without highlighting), and highlighted drawings in the different conditions (CG: 63% no drawings, 35% drawings, 2% highlighted drawings; D: 7% no drawings, 92% drawings, 1% highlighted drawings; DQ: 8% no drawings, 22% drawings, 70% highlighted drawings). Significantly more drawings and highlighted drawings were made in the respective conditions, indicating that the majority of students followed the instructions as intended for the non-drawing, drawing, and drawing with highlighting groups.

Measures and Data Analysis

Students' performance and the number of linear overgeneralizations were assessed via a problem solving test, which included four experimental items and three additional buffer items. The experimental items were non-linear geometry problems in which the area or volume of a figure (square, circle) or a solid (cube, sphere), respectively, and a scaling factor were given with the question to find the size of the area or the volume of a similar figure. For example: "The side of square C is 12 times as large as the side of square D. If the area of square C is 1440 cm^2, what's the area of square D?" All items were taken from the study by De Bock et al. (2003).

To measure students' performance, we analyzed whether the solutions were correct (coded 1) or incorrect (coded 0). The number of linear overgeneralizations was assessed by analyzing if they were based on a linear model (coded 1) or not (coded 0). Two independent raters rated 20% of the answers to each problem with sufficient inter-rater agreement (Cohen's $\kappa \geq .827$). Scale reliability was satisfactory (Cronbach's $\alpha = .787$ for performance and .715 for linear overgeneralizations). To address the research questions, we compared the mean scores for students' performance and linear overgeneralizations between the CG and D groups (research question 1) and the CG and DQ groups (research question 2) by using *t*-tests. All alpha values we report are one-tailed due to our directional expectations. For reasons of comparability, we followed De Bock et al.'s (2003) procedure and conducted our analysis with only two of the four experimental items. The results remained nearly the same when all items were included in the analysis.

RESULTS

Our first research question was aimed at replicating the negative effect of the drawing strategy on problem solving performance. We found that students in the drawing condition had significantly lower solution scores than their peers in the control group ($M_D = 0.268$, $SD_D = 0.389$; $M_{CG} = 0.476$, $SD_{CG} = 0.460$; $t(80) = 2.203$; $p < .05$; $d_{Cohen} = 0.488$). In line with our expectations, applying the drawing strategy negatively affected students' problem solving performance for non-linear geometric problems.

The first research question further referred to the number of linear overgeneralizations. We found that students in the drawing condition made in tendency significant more linear overgeneralizations than students in the non-drawing condition ($M_D = 0.390$, $SD_D = 0.426$; $M_{CG} = 0.244$, $SD_{CG} = 0.389$; $t(80) = -1.624$; $p = .054$; $d_{Cohen} = -0.358$). As expected, applying the drawing strategy appeared to increase the number of linear overgeneralizations.

The second research question referred to the quality of the use of the drawing strategy and was aimed at investigating whether the negative effect of the drawing strategy could be diminished by increasing the quality. We found that students who used the drawing strategy in a high-quality manner (DQ condition) had significantly lower solution scores than students who did not use this strategy (CG) ($M_{DQ} = 0.220$, $SD_{DQ} = 0.388$; $M_{CG} = 0.476$, $SD_{CG} = 0.460$; $t(77.78) = 2.724$; $p < .01$; $d_{Cohen} = 0.602$). Increasing the quality apparently could not diminish the negative effect of the drawing strategy on performance.

However, a high-quality use of the drawing strategy was found to diminish the negative effect for linear overgeneralizations. Students who used the drawing strategy in a high-quality manner made a similar number of linear overgeneralizations as students in the control group ($M_{DQ} = 0.342$, $SD_{DQ} = 0.425$; $M_{CG} = 0.244$, $SD_{CG} = 0.389$; $t(80) = -1.08$; $p = .141$; $d_{Cohen} = -0.241$). Hence, increasing the quality helped prevent students from making linear overgeneralizations, but it did not help them solve the problems.

DISCUSSION

One of the goals of the present study was to replicate and extend the findings from De Bock et al.'s (2003) study. In line with the previous findings, we found a negative effect of the drawing strategy on students' problem solving performance for non-linear geometry problems. Even the solution scores in our study were very similar to the ones reported by De Bock et al. (2003), indicating that the negative effect is stable across time and different samples. This replication increases the validity of the surprising finding that drawing can hinder students' ability to solve mathematical problems.

Further, our study was aimed at elaborating on potential reasons that might explain the negative effect of applying the drawing strategy. The results confirmed the previous assumption that lower performance is caused by linear overgeneralizations (De Bock et al., 2003). Applying the drawing strategy without supporting students in using it in a high-quality manner increases the number of linear overgeneralizations. The process of drawing seems to guide learners' attention to the linear property of the circumference, which they mistakenly transfer to the area or volume of the figure. Moreover, in both conditions (D and CG), we found that linear overgeneralizations appeared frequently, which, in line with prior research (Van Dooren et al., 2008), highlights the pervasive role of students' tendency to apply linear models.

With the second research question, we investigated the role of the quality of the use of the drawing strategy. We expected that the negative effect of the drawing strategy on performance and on the number of linear overgeneralizations in students' solutions could be diminished by increasing the quality of their strategy use. Quality was increased by addressing the important feature of the drawing strategy to represent key information (completeness of drawings), which was done by instructing students to highlight the area or volume in their drawings. The results partly confirmed the expectations derived from the theoretical considerations.

Contrary to our expectations, improving the quality did not diminish the negative effect of the drawing strategy on students' performance. Hence, even the use of the drawing strategy with an increase in its quality had a negative effect on the ability to solve non-linear geometry problems. A possible explanation is that applying the drawing strategy when drawing the geometrical figures might hinder a covariation view (area as an alterable value that depends on the length of the side), by leading to a static view of a specific figure's lengths and area. Following this consideration, future studies should investigate how the drawing strategy affects different concept images (Vinner, 1997) for linear and non-linear functions. A promising approach for fostering the covariation view might be to construct a drawing by using dynamic geometry software.

Regarding the number of linear overgeneralizations, the results confirmed our expectation that the quality of the use of the drawing strategy is a crucial factor that determines whether the negative effect occurs or not. This finding is in line with previous research demonstrating the important role of the quality with which strategies are applied. A high-quality use of the drawing strategy helped to prevent at least some of the students from falling into the linearity trap, but it did not help the students find the correct mathematical procedure. This result indicates that apart from linear overgeneralizations, students also encounter other difficulties in solving non-linear geometry problems. This highlights the need for qualitative studies to investigate the process of solving non-linear geometry problems with the help of the drawing strategy in order to get a more complete picture of students' difficulties.

Taken together, our findings show that applying the drawing strategy is not a one-size-fits-all solution. Besides increasing the quality of the drawing strategy, teachers should consider that different conceptual images of linear and non-linear functions are essential for problem solving. Reflecting on the advantages and disadvantages of various representations is an important prerequisite for the beneficial use of this strategy. This stresses the need for further investigations on the drawing strategy.

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