Experiences of competence and autonomy during a teaching intervention on mathematical modelling

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Experiences of competence and autonomy are essential for intrinsic motivation, social development, and well-being. However, little is known about how these constructs are related to previous achievement in mathematics. In the present study, we investigated whether ninth-graders (N = 83) with different levels of achievement differ in their experiences of competence and autonomy and whether their experiences change during a teaching intervention focused on modelling. We found that high- and low-achieving learners did not differ in competence or autonomy. However, competence and autonomy developed differently across the time period of the teaching intervention. The experience of competence increased for high achievers, and autonomy decreased for low achievers. The results contribute to a better understanding of how intra- and interindividual differences are related to competence and autonomy and provide insights for effective interventions.

Keywords: competence, autonomy, individual differences, need satisfaction

Introduction

An important prerequisite for students' motivation is the satisfaction of their basic needs for competence, autonomy, and relatedness (Deci & Ryan, 2000). Students' motivation, in turn, is related to their academic choices, engagement, persistence when facing difficulties, and performance (Wigfield et al., 2015). Teaching interventions can facilitate need satisfaction by providing students with opportunities to experience competence, autonomy, and relatedness. One possibility is to teach modelling problems. Typical characteristics of modelling problems are that they contain superfluous or missing information and can be solved in different ways. Hence, learners can develop their own solutions, which might facilitate experiences of competence and autonomy. However, the effectiveness of a teaching intervention also depends on how the opportunity is perceived and used by an individual learner, along with the person's previous experiences and individual characteristics (Brühwiler & Blatchford, 2011). Further, there is a lack of research on situational components of motivation (Schukajlow et al., 2023). The experience of autonomy was found to vary noticeably across several measurement points (Rakoczy et al., 2022), highlighting the importance of investigating state (situational) motivation. In the present study, we analyzed the differences in experiences of competence and autonomy in high- and low-achieving students at five measurement points during a teaching intervention on mathematical modelling. The intervention provided opportunities for need satisfaction by using modelling problems and self-regulative teaching methods. The research questions of the study were:

RQ1: Do high- and low-achieving learners differ in general in their experiences of competence and autonomy while solving modelling problems?

RQ2: To what extent do changes in students' experiences of competence and autonomy during a teaching intervention on modelling depend on students' achievement levels?

Theoretical framework

Experiences of competence and autonomy in solving modelling problems

Self-determination theory (Deci & Ryan, 2000) proposes that the satisfaction of three basic needs–the needs for competence, autonomy, and relatedness–is the basis for intrinsic motivation, social development, and well-being. In the present paper, we focus on competence and autonomy because of their importance for the learning of mathematics (Schukajlow et al., 2023). The need for competence refers to the need to feel effective and to be able to attain desired outcomes. The need for autonomy refers to volition and the desire to act in accordance with one's integrated sense of self (Deci & Ryan, 2000). According to self-determination theory, supporting learners' experiences of competence and autonomy promotes motivation and engagement and also affects performance.

Consequently, interventions should enable the satisfaction of basic needs, including experiences of competence and autonomy. One possible approach is to present problems that can be solved in different ways so that learners can develop their own solutions in ways that are adaptive to their individual learning prerequisites, thus potentially supporting their experiences of competence and autonomy (Schukajlow & Krug, 2014). Take for example the "Tree track" problem (Figure 1).

Tree Track Adventure Park

Mr. Meier is the owner of a tree track adventure park. He opened this attraction for young and old guests 12 years ago. Now he has to replace the steel ropes that the safety lines clip onto. These steel ropes are very expensive. To replace them, Mr. Meier needed more of the steel rope than originally planned. He has only 22 m of steel rope left for the last zip line between two trees. The two trees are 17 m apart. The steel rope needs to be attached to the starting tree at a height of 10 m and to the ending tree at a height of 8.5 m so that visitors can then slide from one tree to the other. [An additional 12 m of steel rope is required to attach the rope to the trees.]

Does Mr. Meier have to buy more steel rope, or is the existing steel rope sufficient?

Figure 1: The "Tree track" modelling problem

This problem can be solved in different ways by using the Pythagorean Theorem, by drawing to scale, or by using triangle inequality. In addition, different assumptions can be made for the additional length of rope needed to tie around the trees, ranging from rough estimations to sophisticated considerations that include computations of the circumference of the tree. Modelling problems, such as the "Tree track" are often thought to promote learners' experiences of competence and autonomy. However, there are also studies that do not support this assumption (Krawitz & Schukajlow, 2018).

Differences between high and low achievers in their experiences of competence and autonomy

Empirical studies have provided support for the positive relationship between the experience of competence, the experience of autonomy, and intrinsic motivation in mathematics (e.g., Schukajlow & Krug, 2014). However, according to supply-use models (e.g., Brühwiler & Blatchford, 2011), teaching outcomes depend not only on the learning opportunities provided in class but also on individual learners' perceptions. Individual learning processes (e.g., learning strategies, attention, and

effort) and individual preconditions (e.g., cognitive and motivational conditions) are of central importance, as they shape how learners perceive and use their learning opportunities. Thus, solving modelling problems such as the "Tree track" in class can be expected to have different impacts on motivation, depending on individual differences, such as different achievement levels. A student with high mathematical achievement in the past will probably have more confidence in their ability to solve the problem, probably needs less support, and might therefore feel more autonomous in solving the problem. This autonomy, in turn, might lead to higher engagement, perseverance, and better outcomes.

Research has focused primarily on achievement as the dependent variable of need satisfaction, but there is some evidence that achievement is reciprocally related to need satisfaction. In a longitudinal study, Wang et al. (2019) demonstrated that prior academic achievement of third- and fourth-graders in the subjects Mathematics, English, and Chinese predicted later need satisfaction, including experiences of competence and autonomy. This finding supports the assumption that learners with higher previous achievement are more likely to experience higher competence and autonomy than their peers with lower achievement.

Change in experiences of competence and autonomy over the course of interventions

To better understand the situational nature of constructs, minimize retrospective distortion, and analyze dynamics and motivational trajectories across lessons, state-based measures are needed (Hannula, 2012). Recent studies (Rakoczy et al., 2022) have shown that learners' experience of autonomy can vary noticeably across different measurement points. For example, Rakoczy et al. (2022) investigated students' experience of autonomy at 10 measurement points across two consecutive art lessons. They found that the largest amount of variance (62%) occurred between measurement points, with less variance (35%) observed between students. Similar results are reported for science class with substantial variabilities between measurement points (39 to 56%) and between students (37 to 57%), depending on which facet of autonomy was considered.

The teaching intervention in the present study and hypothesis

The teaching intervention in the present study focused on mathematical modelling and specifically on dealing with superfluous or missing information. The intervention was conducted in the context of a larger project involving two different experimental conditions: one focusing on superfluous information and the other on missing information. Here, we decided to combine the conditions, as there was no difference between them. A self-regulative teaching method that was shown to be beneficial for teaching modelling in a previous study (Durandt et al., 2022) was used. Essential elements of this teaching method are individual solving phases, cooperative work in small groups, and reflection on the content learned in the classroom as a whole. Further, scaffolding principals were used to create an autonomy-supportive setting without overwhelming students with the demands of modelling problems. For example, we provided concrete manipulatives to facilitate understanding (Bruner, 1966) and promote learners' experiences of autonomy and competence (Reyes, 2019). In addition, the modeling problems used in the intervention can be solved through the application of various mathematical procedures and strategies, which was found to enhance students' experience of autonomy and competence in previous research (Schukajlow & Krug, 2014). Consequently, the

teaching intervention provided students with opportunities to experience competence and autonomy. However, an open question is whether there are inter- and intraindividual differences in perceptions of competence and autonomy. Based on these considerations and prior research (Wang et al., 2019), we expected that high-achieving learners would experience higher levels of competence and autonomy than low-achieving learners. High-achieving learners might feel more competent in solving the problems and need less support, facilitating more autonomous actions. Further, we expected variation in competence and autonomy between teaching units, as previous studies have indicated variation between measurement points (Rakoczy et al., 2022). However, we did not have clear expectations of whether there would be different developmental trends (rise or fall) in the development of competence and autonomy for high- and low-achieving learners.

Method

Sample, procedure, and teaching intervention

The sample involved 83 ninth-graders (55.4% female; mean age 14.82 years) from two middle-track schools (German Realschule). A performance test on mathematical modelling was administered before the teaching intervention. The teaching intervention consisted of five units administered across four lessons (180 min total):

- Introductory example of superfluous or missing information in modelling (Unit 1)
- Classifying given modelling problems regarding superfluous or missing information (Unit 2)
- Solving a modelling task using a prestructured worksheet (Unit 3)
- Estimating the accuracy of given solutions to modelling problems (Unit 4)
- Solving two modelling problems holistically and describing differences between problems in challenges regarding superfluous or missing information (Unit 5)

The "Tree track" problem (Figure 1) was one of the modelling problems used in Unit 5.[1] After each unit, students answered a questionnaire about their experiences of competence and autonomy.

Measures

Modelling performance was measured before the teaching intervention by using seven modelling problems. Students' solutions to the modelling problems were coded using a five-step coding scheme ranging from wrong solutions (coded 0) to solutions in which the four modelling activities structuring, mathematizing, working mathematically, and interpreting were correctly applied (1 point for each activity for a total of 4 points; interrater reliability $\kappa > .652$). Students' performance was estimated with weighted likelihood estimator (WLE) parameters obtained with a partial credit IRT model (WLE reliability: .646).

Experiences of competence and autonomy were assessed with 4-point scales ranging from 1 (*not at all true*) to 4 (*completely true*). The scales consisted of three items each and had been validated in

[1] Figure 1 shows the version of the problem presented to students in the missing information condition. Students in the superfluous information condition were given the same problem with the following sentence included in the text at the end of the situational description: "An additional 12 m of steel rope is required to attach the rope to the trees."

previous studies (Rakoczy et al., 2022; Schukajlow & Krug, 2014). Sample items, means, and reliability measures for the scales are presented in Table 1. Internal consistency (Cronbach's α) for the experience of competence was low for the first measurement point (α = .411), but we decided to keep the construct because the α-values were satisfactory for the other measurement points.

Table 1: Constructs and sample items, means (*M*), and standard deviations (*SD*) of items across all five measurement points, range of α as a measure of internal consistency for each scale

Construct	Sample item	*M*	*SD*	α
Competence	During unit [number of unit], I felt able to master what I was doing.	2.61	0.61	.411 - .806
Autonomy	During unit [number of unit], I could decide for myself how to work.	3.00	0.56	.714 - .931

Data analysis

A median split for modelling performance (*Mdn* = -0.69) was used to divide learners into two groups called "high achievers" (n_1 = 39 participants) and "low achievers" (n_2 = 42 participants). To address our research questions, we computed a repeated-measures ANOVA for each construct with the factor "unit" including the five teaching units as a within-subjects factor, the factor "group" as a between-subjects factor comparing high- vs. low-achieving students, and competence or autonomy, respectively, as the dependent variable.

Results

The first research question was about differences between high- and low-achieving learners. Contrary to our expectations, the results of the repeated-measures ANOVA showed no significant main effect of "group." High- and low-achieving learners did not differ in their mean values of competence, $F(1, 79) = 0.01$, $p = .949$, $\eta^2 < .001$ ($M_{high} = 2.62$ $SD_{high} = 0.60$; $M_{low} = 2.60$ $SD_{low} = 0.63$), or autonomy, $F(1, 79) = 2.33$, $p = .131$, $\eta^2 = .029$ ($M_{high} = 3.10$ $SD_{high} = 0.53$; $M_{low} = 2.91$ $SD_{low} = 0.58$) across the five teaching units.

To address the second research question, we analyzed the development of experiences of competence and autonomy in high and low achievers during the teaching intervention. A repeated-measures ANOVA revealed interaction effects between "group" and "unit" for competence, $F(3.61, 284.86) = 2.85$, $p = .029$, $\eta^2 = .035$, and autonomy, $F(4, 79) = 2.83$, $p = .025$, $\eta^2 = .035$, indicating that high- and low-achieving learners differed in the changes in their experiences of competence and autonomy. For high-achieving learners, their experience of competence significantly increased from the first, second, and third units to the fifth unit (Table 2), but for low-achieving learners, no differences were found for the experience of competence between the units despite a significant increase from the first to the second unit. Figure 2 illustrates the development of experience during the teaching intervention for high and low achievers. For the experience of autonomy, high-achieving learners reported similar experiences of autonomy for the different teaching units. For low-achieving learners, the experience of autonomy decreased during the teaching intervention with significantly lower values in the first unit compared with the third, fourth, and fifth units and in the second unit compared with the third and fifth units (Table 2).

Table 2: Means, standard deviations, and mean differences ($Unit_{row}$ - $Unit_{column}$) in experiences of competence and autonomy for high and low achievers

		High achievers						Low achievers					
	Unit	*M*	*SD*	2	3	4	5	*M*	*SD*	2	3	4	5
Competence	1	2.50	0.67	-0.03	-0.08	-0.15	**-0.33***	2.54	0.59	**-0.20***	-0.09	-0.02	-0.02
	2	2.53	0.69		-0.04	-0.12	**-0.29***	2.74	0.73		0.11	0.18	0.18
	3	2.57	0.72			-0.08	**-0.25***	2.63	0.80			0.07	0.06
	4	2.65	0.77				-0.17	2.56	0.88				-0.01
	5	2.82	0.72					2.56	0.88				
Autonomy	1	3.15	0.81	0.18	0.12	0.03	-0.08	3.12	0.67	0.07	**0.32***	**0.28***	**0.37***
	2	2.97	0.78		-0.06	-0.15	-0.26	3.05	0.72		**0.25***	0.21	**0.29***
	3	3.03	0.79			-0.09	-0.20	2.80	0.73			-0.04	0.05
	4	3.13	0.68				-0.10	2.84	0.85				0.09
	5	3.23	0.69					2.75	0.89				

Note. *mean difference is significant at the .05 level (two-tailed *t*-test).

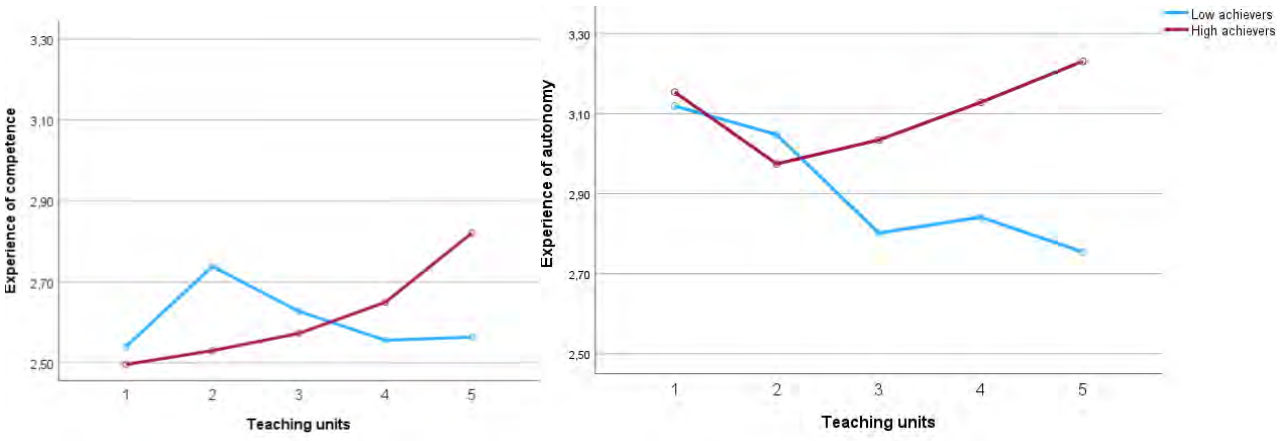

Figure 2: Graphs illustrating the development of experiences of competence and autonomy during the teaching intervention for high and low achievers

Discussion

The present study was designed to examine the relationship between previous achievement and experiences of competence and autonomy over the course of a teaching intervention. Contrary to

previous findings (Wang et al., 2019), high- and low-achieving learners did not differ in their experiences of competence and autonomy when their ratings from the five measurement points were aggregated. However, their experiences of competency and autonomy changed differently over the course of the teaching intervention. This finding is in line with previous studies that have demonstrated large amounts of within-person variability in the experience of autonomy (Rakoczy et al., 2022). In addition, it provides support for the state-like nature of the constructs and the need to examine and discuss differences between measures that focus on state versus trait aspects of the constructs in future research.

Further, our analysis of the development of experiences of competence and autonomy over the course of the teaching intervention indicates that the experience of competence increased in high achievers, and the experience of autonomy decreased in low achievers. A possible explanation for this finding is that fading out of the scaffolding comprised less structured and more difficult tasks at the end of the intervention. For high achievers, solving challenging tasks offered an opportunity to experience competence and autonomy, whereas low achievers needed more support and thus experienced lower levels of competence and autonomy.

An important limitation of the present study is that additional data from observations of lessons or interviews will be necessary to find out more about the reasons for the changes in students' experiences of competence and autonomy. Future studies should additionally use qualitative analyses to better understand the processes involved in these changes.

The results help to shed light on the relationship between prior achievement and experiences of competence and autonomy, indicating different motivational trajectories for high- and low-achieving learners. In addition, the results contribute to self-determination theories, as they highlight the importance of considering inter- and intraindividual differences in need satisfaction. The elements of the intervention (self-regulative teaching, scaffolding, multiple solutions) seem to have varying effects on the experience of competence and autonomy for learners at different levels of proficiency. Future studies need to find out what elements contribute or hinder experience of competence and autonomy and how low-achieving learners' experiences of competence and autonomy can be facilitated to prevent them from suffering from decreases in their need satisfaction. One practical implication is to offer more support to low-achieving learners in teaching interventions.

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