ACTIVATION AND MONITORING OF PRIOR MATHEMATICAL KNOWLEDGE IN MODELLING PROCESSES

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In a qualitative study with eighth to tenth graders (N=18), we investigated whether the activation of prior mathematical knowledge would promote or interfere with solution processes as students solved modelling problems. In addition, we analyzed the role of metacognitive monitoring of knowledge activation. Participants with different prior mathematical knowledge solved modelling problems in which multiple solution approaches were possible. We found that the activation of inappropriate prior mathematical knowledge negatively impacted modelling. Negative effects of prior knowledge also occurred if a second solution for a problem was required because learners stuck to the prior knowledge of their first approach. Monitoring of knowledge activation was rarely found, even when it would have been helpful.

INTRODUCTION

Building a mental model of a real-world situation is particularly important for solving modelling problems (Leiss, Schukajlow, Blum, Messner, & Pekrun, 2010). To build a mental model, students have to structure and simplify the information presented in the problem statement. To decide what information is important, they need to have at least a rough idea of a corresponding mathematical model in mind. Thus, students have to activate prior mathematical knowledge at the very beginning of the solution process. However, an initial strong focus on mathematical issues might occur at the expense of the development of a situational understanding and could lead to solutions that are not adequate from a realistic perspective. Metacognitive monitoring of the activated prior knowledge is considered to play an important role in the decision to either use or ignore the activated prior knowledge. The present article investigates the interplay between prior mathematical knowledge, modelling activities, and monitoring of knowledge activation, with the aim to better understand under what circumstances the activation of mathematical knowledge promotes or interferes with modelling processes.

THEORETICAL BACKROUND AND RESEARCH QUESTIONS

Effects of Prior Knowledge and Monitoring of Prior Knowledge on Performance

Prior knowledge is considered to be an important predictor of performance (Dochy, Segers, & Buehl, 1999). But under certain circumstances, the activation of prior knowledge can have negative effects, as the activation of inappropriate knowledge while solving mathematical problems can lead to a search in the wrong part of the problem space (Kaplan & Simon, 1990). Certain mathematical contents seem to trigger

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inappropriate activation of prior mathematical knowledge. Students were previously found to activate knowledge of proportional relations even when this knowledge was not suitable for the problem at hand. Reasons are seen in the dominant role linearity plays in classrooms and everyday contexts (Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005). Further, it can be hypothesized that knowledge about the topic that was taught most recently is often activated regardless of its appropriateness because, in most classroom situations, this knowledge is typically needed to solve exercises and to succeed on tests. Metacognitive monitoring of knowledge activation was found to be helpful to avoid negative effects of prior knowledge on performance (Stillman, 2011; Stillman & Galbraith, 1998; Van Dooren & Matthew, 2015).

Role of Prior Knowledge for Mathematical Modelling

The translation of a real-world situation into a mathematical model is at the core of mathematical modelling. The translation process requires initial modelling activities such as understanding, structuring, and simplifying the real-world situation in order to transfer it into an adequate mental model of the situation that can be further mathematized (Blum, 2015). Modelling problems often contain superfluous information, and identifying the important information becomes part of the activities of structuring and simplifying. Prior mathematical knowledge can be considered necessary to identify the information that is required to develop a mathematical model. Hence, anticipations of mathematical knowledge might be needed to successfully carry out initial modelling activities. On the other hand, impulsively activated mathematical knowledge has been suggested to promote superficial solutions in which situational constraints are neglected, especially if no metacognitive activities to monitor the activation of knowledge are conducted (Stillman & Galbraith, 1998). Cue salience and its interaction with prior knowledge is thereby seen as particularly important because it can trigger the activation of inappropriate knowledge. Activation of inappropriate prior mathematical knowledge and a lack of metacognitive activities devoted to monitoring knowledge activation might account for why students have trouble solving modelling problems, but little is known about the interplay between these factors and students' solution processes.

Research Questions

These considerations led us to pose the following research questions:

1. To what extent does the activation of prior mathematical knowledge promote or interfere with modelling processes?
2. Is metacognitive monitoring used to determine the appropriateness of the activated mathematical knowledge?

METHOD

Participants and Data Collection

The sample involved 18 eighth to tenth graders (9 girls and 9 boys between the ages of 14 and 16) from four middle-track classes (German Realschule) from two different schools. We selected participants by following the principle of maximum variation sampling (Patton, 2015, p. 283). As selection criteria, we focused on the background variables mathematical ability, reading comprehension, and prior mathematical knowledge. Mathematical ability was estimated with math grades and reading comprehension via a general standardized test (Leiss et al., 2010). Mathematical knowledge about circles could help or inhibit problem solving. Thus, we chose eight students who had not yet covered this topic in their mathematics classes and ten students who had studied this topic before participating in the investigation. The interviews were conducted individually. First, each participant worked on the problems "Wind turbine" and "Ferris wheel" using the think-aloud method to verbalize his or her approach (Figures 1 and 2). Second, a stimulated recall interview was conducted in which the participant watched the problem solving videos along with the interviewer and commented on his or her own (i.e. the student's) actions spontaneously or when requested to do so by the interviewer. At the end of the stimulated recall interview, students were asked to find a second solution for the "Wind turbine" problem.

"Wind turbine" problem

Wind energy is the fourth largest type of energy in Germany and is therefore an important part of energy production. Because wind turbines are very large, they are also called wind giants. Overall, a wind turbine is about 150 meters high. The radius of the windmill is 45 meters. This is exactly the length of one of the blades. The three blades are mounted at a height of about 95 meters on a so-called nacelle. The nacelle is rotatable so that the blades of the wind turbine can align themselves with the wind direction. The speed at which the blade tip rotates is about 40 meters per second at an average wind speed. If the wind blows too hard, the system switches off. At a medium wind speed, a blade will return to its initial position after 6 seconds.

How many meters will the blade tip cover in one turn of the wind turbine?

Figure 1: "Wind turbine" problem

"Ferris wheel" problem

The London Eye is the third largest Ferris wheel in the world. It stands directly on the banks of the Thames. Overall, the Ferris wheel is 140 meters high and has a huge diameter of 125 meters. From the highest point of the Ferris wheel, you can see for 40 km. For passengers to board and exit, the wheel does not have to stop because it turns very slowly. The speed is only 10 meters per minute. A ride on the Ferris wheel is expensive. It costs 25 euros but also takes 40 minutes.

At what altitude above the water level will a person be 10 minutes after boarding?

Figure 2: "Ferris wheel" problem

To stimulate the activation of different prior knowledge, we decided to use problems to which different solution approaches could be applied. The first problem "Wind turbine" can be solved by either calculating the circumference of the circle ($C=2\cdot\pi\cdot45m\approx283m$) or using the proportional relation of time and travel distance ($d=40m*6=240m$). For the "Ferris wheel" problem, constructing an adequate mental model of the situation and recognizing that the position of the gondola after 10 minutes is a quarter of one rotation are crucial to applying an appropriate solution method. The result can be calculated by adding up the length of the radius and the height of the base ($125:2+140-125=77.5$ m). The problem can also be solved with other approaches (e.g. trigonometric functions), but other approaches did not come up in the interviews.

Data Analysis

The problem solving and stimulated recall interviews were transcribed and sequenced. Sequences of the stimulated recall interviews were assigned to the related problem solving sequences in order to collect more indications of whether prior knowledge was activated. The transcripts were analyzed using qualitative content analysis (Mayring, 2014). A category scheme was used to code the sequences with regard to modelling activities, prior mathematical knowledge, metacognitive monitoring of knowledge activation, and the appropriateness of the solution. More specifically, the modelling activities were divided into initial modelling activities (understanding/structuring the problem) and later modelling activities. Prior mathematical knowledge was categorized into subcategories referring to different mathematical contents (e.g. circle calculation or proportional relations). The occurrence of metacognitive monitoring of knowledge activation was recorded. Different solution qualities (correct, partial, incorrect and processing canceled) and different qualities of the mental model of the situation (adequate, not adequate) were distinguished. Content-analytical quality criteria such as the stability and reproducibility of the analysis were tested by calculating intra- and inter-coder reliability for more than a quarter of the material with satisfactory agreement (Cohen's kappa calculated for each dimension ranged between $.691 \leq \kappa \leq .878$). Disagreements about the coding were discussed and validated consensually.

RESULTS

Because of space limitations, we present only the most important results and exemplarily sketch two examples of solutions to the "Ferris wheel" problem in which aspects of prior mathematical knowledge were found to promote or interfere with problem solving.

For the first research question, we analyzed what kind of prior mathematical knowledge was activated and how this knowledge interacted with the modelling processes. Learners who had prior knowledge of circle calculation often activated this knowledge ("Wind turbine" problem: 6 of 10 students; "Ferris wheel": 6 of 10 students). For the "Wind turbine" problem, they activated this knowledge even more often than knowledge that referred to proportional relations, although the approach of calculating the circumference of the circle is more difficult and prone to errors (circle calculation: 6 of 10 students; proportional relations: 2 of 10 students). This was found despite the fact that these learners also had prior knowledge of proportional relations, which was verified in the interviews. Learners without prior knowledge of circle calculation usually used prior knowledge of proportional relations (“Wind turbine” problem: 6 of 8 students; “Ferris wheel”: 5 of 8 students). Regarding the supporting or interfering effect of the activated knowledge, we found a big difference between the problems. For the “Wind turbine” problem, in one third of the solution processes, knowledge of circle calculation or proportional relations was already activated in the initial modelling activities of understanding and structuring. In the largest number of cases (12 of 18 students), the activation of knowledge of circle calculation or proportional relations led to appropriate approaches and correct solutions. But after applying one approach, most learners had trouble applying a second approach. They tried to apply their prior knowledge of their first approach again, but they did not step back and activate their prior knowledge of other mathematical contents. The transcript below illustrates this difficulty as described by one of the learners.

29:25 158 Ella: So this problem, the first one [“Wind turbine”], I thought was relatively easy because, as I said before, you only had to calculate the circumference here. But the first solutions are always easy, but then to come up with the second … because then you are so fixated on one calculation and then you also think that it is now the only one. It’s just difficult then to still be open to another way.

In the “Ferris wheel” problem, activation of knowledge about circle calculation or proportional relations in initial modelling activities was often found to be accompanied by inadequate mental models of the situation (15 of 18). For example, students who activated prior knowledge of circle calculations in initial modelling activities (5 of 10) figured out that this was not fruitful and either applied a second approach (3 of 10) or canceled their processing (2 of 10). On the other hand, the activation of prior knowledge of proportional relations (10 of 18) typically led to a single attempt in which the learners used this knowledge to calculate the distance traveled instead of the height above the water level as requested and reported the distance traveled as a result (10 min·10 m/min=100 m). Hence, in almost all cases, the activation of prior knowledge of proportional relations resulted in incorrect solutions.

The second research question was about the use of monitoring activities to control the activation of prior mathematical knowledge. Monitoring of knowledge activation was found only very rarely (“Wind turbine” problem: 2 of 18; “Ferris wheel” problem: 1 of 18). In particular, for initial modelling activities, no metacognitive monitoring was found at all. Moreover, there were no differences between the “Wind turbine” and “Ferris wheel” problems, even though for the “Ferris wheel” problem, it was essential to monitor one’s knowledge activation in order to recognize the inappropriateness of certain prior knowledge. Moreover, we found indications that even if students identified contradictions in the solution, they did not change their solution. The case of Pia presented below exemplifies this issue.

In the following, two solutions to the "Ferris wheel" problem are sketched. In the first case, Tabea is a learner with high reading comprehension skills and high mathematical performance. Her solution process is characterized by a long period in which she engages in the initial modelling activities of understanding and structuring. Although Tabea has no prior knowledge of circle calculation, she activates such knowledge and mentions that "hopefully this has nothing to do with π." She later explains that she knows about π because of a poster in her classroom. Her first idea is to calculate the circumference of the circle and divide the result, but she does not know how to do it. She mentions that "there must be something else that I have overlooked" and starts to read the problem statement again and transfers important information into a sketch (Figure 3). The sketch and her prior knowledge of fractions help her to recognize that 10 minutes corresponds to a quarter rotation. She calculates the length of the radius and interprets it as equal to the height she was searching for. However, her solution fails to take into account the base of the Ferris wheel.

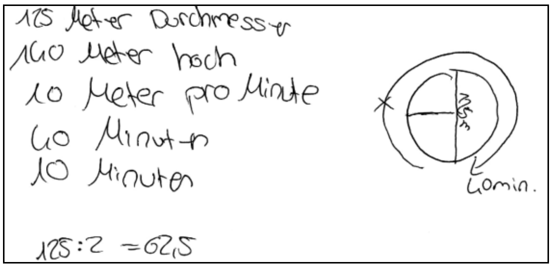

Figure 3: Tabea's solution to the "Ferris wheel" problem

The second case is Pia, a student with rather weak reading comprehension skills and weak mathematical performance. Like Tabea, her process of solving the "Ferris wheel" problem begins with a long period in which she engages in initial modelling activities. She reads the problem statement several times and also sketches the situation. Pia uses prior knowledge of proportional relations to interpret the speed of 10 m per minute as "in one minute, I am ten meters high" and to create a table to calculate the distance traveled after ten minutes (Figure 4, left). In the sequences presented below, she writes down and comments on her solution.

18:06	37	[*pause*] So, I am not one hundred percent sure, but um.
18:10	38	[*writes*] After ten minutes, it is located at a height of 100 meters. Okay, I'm done.

I: Are you at least satisfied with your solution?

P: No, not really, actually this is not right.

I: Okay, what is wrong?

P: That, if you are 100 meters high, you have actually only gone this far [draws a sketch (Figure 4, right) to explain the difference between the distance traveled and the altitude].

In the stimulated recall interview, Pia is able to explain that she is aware of the discrepancy between her solution, which presents the distance traveled, and the height she was searching for (Figure 4, right).

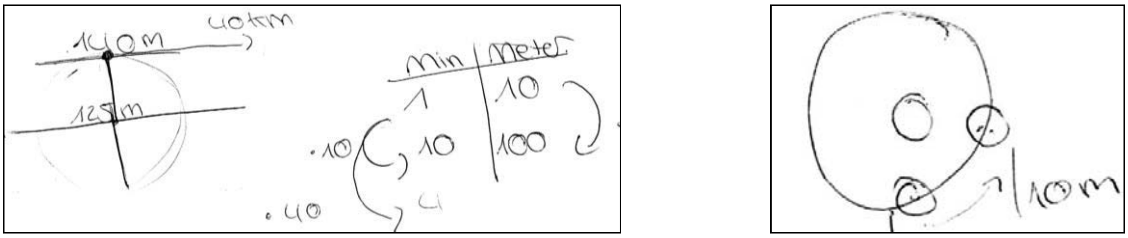

Figure 4: Pia's solution and the sketch in which Pia explains the discrepancy between her solution and the question

In summary, Tabea is one of the rare examples where the activation of inappropriate prior knowledge did not lead to an incorrect solution to the "Ferris wheel" problem (Tabea's solution was categorized as partially correct). On the other hand, in Pia's case, her prior knowledge of proportional relations was used to come up with a superficial solution, and even her recognition of discrepancies did not lead her to search for appropriate prior knowledge.

SUMMARY AND DISCUSSION

In the present study, we investigated whether the activation of prior mathematical knowledge would promote or interfere with solution processes in solving modelling problems. The positive or negative impact of the activated prior mathematical knowledge depended on the appropriateness of the knowledge. Students tended to activate inappropriate knowledge if some information in the problem statement looked promising at first glance but did not match the problem's demands. In these cases, especially the activation of prior mathematical knowledge in initial modelling activities was accompanied by inadequate mental models of the situation and incorrect solutions. This can be considered an indication that supports the hypothesis that impulsively activated mathematical knowledge can promote superficial solutions (Stillman & Galbraith, 1998). Prior knowledge of proportional relations and circle calculation were both activated frequently, even if these types of knowledge were not appropriate for solving the problem at hand. The inappropriate activation of knowledge of proportional relations is in line with previous research that demonstrated that students tend to overgeneralize proportional relations (Van Dooren et al., 2005). Students' frequent activation of prior knowledge of circle calculation indicates that the most recently learned subject is an important although unexplored factor that should be addressed in future studies. Further, it was found that learners had trouble finding a second solution because they stuck to the prior knowledge they had activated for the first solution. This indicates that a first solution impedes the search for a second solution, and this should be considered an aggravating factor when multiple solutions are required. A low occurrence of metacognitive monitoring was found, although in some of the solution processes, metacognitive activities could have helped students recognize the inappropriateness of the activated knowledge and might have stimulated a search for prior knowledge that was more appropriate. Therefore, a lack of metacognitive monitoring can also be considered as one reason for students' low success in solving the modelling problems (Stillman & Galbraith, 1998). Teaching methods that were found to stimulate monitoring activities such as prompting each student from the very beginning to find two solutions (Schukajlow & Krug, 2013) might help students recognize the inappropriateness of prior knowledge.

Despite methodological limitations such as the limited number of participants, our findings can contribute to a better understanding of the role that prior mathematical knowledge plays in modelling processes and might inspire further studies.

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