EFFECTS OF SHORT-TERM PRACTICING ON REALISTIC RESPONSES TO MISSING DATA PROBLEMS

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In an experimental study with fifth-graders (N=108), we carried out a short-term intervention aimed at improving students' realistic responses to missing data problems. A control group received nonspecific instructions about solving problems with missing data. An experimental group additionally solved a sample problem and discussed possible solutions. At posttest, students from both groups solved missing data problems. In line with our expectations, the experimental group gave more realistic responses to problems with missing data in which the problem statement did not contain any numbers. However, problems that contained data that, at first glance, appeared sufficient to conduct the required calculations were still answered unrealistically by both groups.

INTRODUCTION

Problems encountered in everyday life often do not contain all of the relevant data needed to solve them effectively. Estimations, assumptions, and realistic considerations are necessary to produce solutions to such problems. Therefore, the ability to solve problems with missing data is an important part of mathematical education. Despite the relevance of such problems for everyday life, students seem to separate their real-world knowledge from their mathematical knowledge and tend to fail to consider the context of the problem statement (Verschaffel, Greer, & de Corte, 2000). Moreover, students have been found to have trouble identifying and making assumptions while solving problems that involve missing data (Galbraith & Stillman, 2001). Although different studies have investigated students' responses to realistic word problems (Greer, 1993; Verschaffel, De Corte, & Lasure, 1994; Verschaffel et al., 2000; Yoshida, Verschaffel, & De Corte, 1997), a dearth of research has focused on students' tendency to give unrealistic responses to problems involving missing data. The aim of our study was to investigate the effects of short-term practicing in the solving of problems with missing data on students' realistic responses to these problems in comparison with a control group that received only general instructions about missing data problems.Theoretical backround and research question

Problems with missing data

Word problems with missing data (also called problems with missing information or vague conditions) are problems in which the problem statement does not provide all of the data necessary to solve the problem. These problems are addressed as a part of "ill-defined" problems because they comprise problem elements that are unknown and they possess multiple solutions (for characteristics of ill-defined problems, also called illstructured problems, see Wood (1983), or Jonassen (2000)). Being able to solve such problems may have an impact on students' actual and later lives because most problems encountered in everyday life and professional practice are ill-defined (Jonassen, 2000). The importance of ill-defined problems and hence problems with missing data stands in contrast to their absence in mathematics teaching and in assessments via large-scale studies such as PISA (van den Heuvel-Panhuizen & Becker, 2003).

The effects of prompting students to solve problems with missing data have been investigated in research frameworks for open-ended problems (Silver, 1995; Stacey, 1995) and modelling problems (Galbraith & Stillman, 2001; Schukajlow & Krug, 2014; Schukajlow, Krug, & Rakoczy, 2015). The role of assumptions is thereby mentioned as an underestimated part of successful modelling because it influences the entire modelling process, including the mathematizing of real-world problems (Galbraith & Stillman, 2001). Case studies have shown that students have trouble making assumptions while solving problems with missing data and have suggested that students should practice these problems more often. An experimental study that showed the benefits of problems with missing data was conducted by Schukajlow and Krug (2014). In this study, students in the experimental group were prompted to construct multiple solutions for modelling problems involving missing data. Students in the control group solved modelling problems without missing data. The authors found a positive influence of prompting students to construct multiple solutions for problems with missing data on students' interest in mathematics. In a subsequent study (Schukajlow et al., 2015), an effect on student performance was found via the number of solutions students developed and their experience of competence. Thus, when students practiced solving problems with missing data, their abilities to solve these problems (including the number of realistic responses they gave) improved.

Characteristics of solving problems with missing data

The process of solving problems with missing data includes three central activities. First, students have to recognize that data are missing. Second, students must identify which quantities have to be estimated. Finally, they need to make assumptions about the missing quantities.

Recognizing that data are missing from the task can be demanding for students, and its difficulty depends on the type of problem at hand. If problems with missing data do not contain numbers at all, the need to make assumptions cannot be overlooked. An example of such a problem is: "How many centimeters of toothpaste are used in one month?" In the following, we will call this problem type "problems with no numerical information" (NNI-problems). In other problems, at first glance, there appears to be sufficient data to do the required calculations. However, simply using the standard operations would ignore the realistic nature of the context. An example is: "Mr. Meier wants to have a rope long enough to stretch between two poles that are spaced 12 m apart, but he has only pieces of rope that are 2 m long. How many of these pieces would he need to tie together to stretch between the poles?" (adapted from Greer (1993)).

Realistic considerations are required in order to recognize that data are missing, such as the length of rope that is needed to tie two pieces of rope together and to tie the ropes around the poles. Research on such problems—called problematic problems (P-problems) in the literature—is addressed in the next paragraph.

Problematic problems

Research question

Problems are considered P-problems if the modelling assumptions are problematic, at least if the real-word aspects of the context are taken seriously (Verschaffel et al., 1994). The important issue of P-problems is the strong tendency of students' to neglect their real-world knowledge while solving such problems. This finding was demonstrated by Verschaffel et al. (1994) and Greer (1993) and was replicated in several other studies (Dewolf, Van Dooren, Cimen, & Verschaffel, 2013; Verschaffel et al., 2000; Yoshida et al., 1997). The main reason for the large number of unrealistic responses to P-problems can be understood to come from the inappropriate frequent use of "standard word problems" and how they are applied in teaching-learning situations (Verschaffel et al., 2000). Such so-called standard word problems can be solved through a superficial straightforward application of one or more arithmetic operations to the given numbers, meaning that it is not necessary to make realistic considerations. This leads to restricted conceptions and beliefs about word problems, namely, that every word problem can be solved with a single numerical answer, that all of the relevant data is given, and that each given number is relevant to the solution (Reusser & Stebler, 1997; Verschaffel et al., 2000). Different kinds of arrangements were provided in several studies to challenge these beliefs and to improve the realistic aspects of the responses. The findings of these studies demonstrate that many short-term interventions for P-problems (such as providing a warning or illustrations) do not work (Dewolf et al., 2013; Greer, 1993; Yoshida et al., 1997). However, some studies did find positive effects of increasing the authenticity of the setting or of carrying out long-term interventions with a focus on mathematical modelling (Reusser & Stebler, 1997; Verschaffel & De Corte, 1997).

These considerations led us to pose the following research question:

Will students respond in a more realistic manner if they have been able to practice solving a problem with missing data and are taught to discuss different solutions in a short-term intervention? More precisely: Would a short-term intervention work differently for problems that can obviously be identified as missing data problems because they do not contain any numbers at all (NNI-problems) in contrast to problems for which realistic considerations are necessary to recognize the need to make assumptions (P-problems)?

Taking into consideration the analysis of the processes applied to solve problems with missing data from the previous section, it can be expected that the short-term practicing and discussing of solutions would have different effects on different types of problems. We expected that after practicing and discussing the solutions to a sample problem with missing data, students would give more realistic responses to NNI-problems. For P-problems, however, we expected that greater efforts would be needed to change students' tendency to give unrealistic answers.

METHOD

Sample and design

Our sample involved 108 fifth graders (52 % females, mean age=10.8 years) from four high-track classes (German gymnasium) from two different schools. The classes did not differ in their level of experience in solving realistic word problems. One class from each school was randomly assigned to the experimental condition and another class to the control condition. In order to control for students' mathematical achievement, all students were administered a standardized mathematical computation test (we used the arithmetic section of DEMAT 5+) at the beginning of the study (see Figure 1). After that, both groups received brief general instructions about missing data problems:

> "You will see that the following problems differ from the problems you are usually given because of missing data. These problems are nevertheless solvable. To solve these problems, you have to make assumptions for the missing data."

In addition, the experimental group (EG) engaged in short-term practicing, which included solving a sample problem and discussing different solutions to this problem in the classroom. The sample problem was: "Some friends meet to play. They have 20 small bags of jelly babies. How many bags gets each of them?" Finally, a posttest with four missing data problems was administered to both groups.

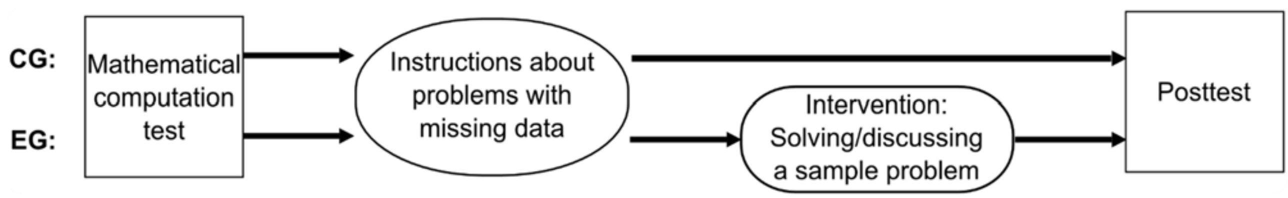

Figure 1: Overview of the study

Missing data problems

In this study, we addressed the two types of missing data problems mentioned above: In NNI-problems, the problem statement does not contain numerical information at all, and thus, making assumptions cannot be overlooked, whereas P-problems provide the opportunity to apply standard operations and therefore require the problem solver to recognize the necessity to make assumptions (see Table 1).

The answers were scored as "realistic" if the solution included realistic assumptions.

NNI-problems	Toothpaste	How many centimeters of toothpaste are used in one month?
	Birthday	Max celebrates his birthday. He wants to eat chocolate marshmallows with his guests. How many packs does he have to buy with his mother?
P-problems	Rope	Mr. Meier wants to have a rope long enough to stretch between two poles that are spaced 12 m apart, but he has only pieces of rope that are 2 m long. How many of these pieces would he need to tie together to stretch between the poles?
	Present	Sina gift-wraps a book. The book measures 5 x 15 x 20 cm. Afterwards, she wants to tie the present with a ribbon. How much ribbon does Sina need?

Table 1: The NNI-problems and P-problems given on the test

These assumptions did not have to be explicitly stated. This means that for the P-problems, that students had to mention that additional rope was required to tie the pieces together and to tie the ropes to the poles ("Rope") and that more ribbon was needed to tie a bow ("Present"). Arithmetical errors were tolerated. In addition, for the "Birthday" item, responses were still coded as realistic even if they did not state that Max also wanted to eat chocolate marshmallows (as long as they mentioned having enough for the guests). This decision was made to eliminate the "problematic" part of the item and ensure that it could be seen as an NNI-problem for which making assumptions was obvious.

RESULTS

As a preliminary result, there were no significant differences between the experimental and control groups on the standardized mathematical computation test (EG: $M = 6.13$, $SD = 3.087$; CG: $M = 6.36$, $SD = 3.006$; $t(106) = -.379$, $p = .71$). The two groups could be seen as having equal levels of mathematical ability.

Descriptive results revealed that there were only a few realistic responses to the P-problems, a finding that is consistent with previous research on P-problems. The NNI-problems were noticeably more often answered in a realistic manner than the P-problems (Table 2).

To answer the research question, we computed a logistic regression with the realistic responses as the dependent variable and the short-term practicing as the independent variable (Table 3). Thereby, the single items from both problem types were considered.

Problem type	Item	Experimental group	Control group
NNI-problems	Tooth	65.38 % (34)	35.71 % (20)
	Birthday	59.62 % (31)	14.29 % (8)
P-problems	Rope	3.85 % (2)	8.93 % (5)
	Present	3.85 % (2)	1.79 % (1)

Table 1: Percentages of realistic responses and total numbers in parentheses

The results revealed a significant effect of practicing for the two NNI-problems: The odds of giving a realistic response were about 3 and 8 times greater for students in the experimental group than for students in the control group for the "Toothpaste" and "Birthday" items, respectively. In contrast to this, we found no effects of practicing on the realistic responses to the P-problems. Thus, as expected, the practicing was successful for the NNI-problems but not for the P-problems.

Problem type	Item	β	*SE*	Wald	*p*	e^{β}	R^2 (Nagelkerke)
NNI-problems	Toothpaste	1.17	0.40	8.56	.003*	3.22	.11
	Birthday	2.14	0.47	20.39	<.001*	8.46	.28
P-problems	Rope	-0.92	0.86	1.14	.287	0.40	.03
	Present	0.77	1.24	0.38	.535	2.16	.02

* Significant at the 5 % level (Bonferroni-adjusted alpha level of .0125 per test (.05/4))

Table 3: Summary of the logistic regression analyses

CONCLUSIONS AND DISCUSSION

The presented study investigated the impact of short-term practicing on students' realistic responses to missing data problems. The effects were analyzed for two kinds of missing data problems. NNI-problems cannot be solved using standard operations because no numbers are presented in the problem statement, and thus, the necessity of making assumptions cannot be overlooked. P-problems, however, provide the opportunity to apply standard operations to the given numbers, and realistic considerations are necessary to recognize the need to make assumptions.

Descriptive findings showed only small numbers of realistic responses to the P-problems. This finding is in line with the results of other studies investigating P-problems, which, for example, have reported 0 % up to 8 % realistic responses to a slightly different version of the "Rope" item (Greer, 1993; Verschaffel et al., 1994; Yoshida et al., 1997). A distinctly higher number of realistic responses were given to the NNI-problems. It can therefore be assumed that recognizing the need to make assumptions in order to provide a numerical answer is a key difficulty in solving P-problems because this feature marks the central difference between P-problems and NNI-problems. It can be expected that the opportunity to apply standard operations confirms students' restricted conceptions and beliefs about word problems and leads to unrealistic responses. This is consistent with the findings of other studies that consider students' conceptions and beliefs as a main reason for their neglect of real-world issues (Reusser & Stebler, 1997; Verschaffel et al., 2000).

The present study addressed these restricted conceptions and beliefs: Students in the control group received general instructions about problems with missing data. In the experimental group, the students additionally solved a sample problem and discussed its solutions in the classroom. The rationale behind the experimental manipulation came from the results of previous studies that showed the necessity of practicing realistic word problems in the classroom (Galbraith & Stillman, 2001). Indeed, constructing and discussing solutions in the classroom had positive effects on students' realistic responses. However, these effects were restricted to the NNI-problems, which do not provide the opportunity to apply standard arithmetic operations at all. No differences between the control and experimental groups were found for the P-problems. These results are in line with previous findings on P-problems, which showed no effects of illustrations, warnings, and other short-term interventions on students' responses (Dewolf et al., 2013; Verschaffel & De Corte, 1997; Yoshida et al., 1997). The open question is whether interventions, which have been shown to have an impact on other problems (e.g., the request to construct two solutions to a problem), can help to stimulate the construction of a situation model and thus increase the number of realistic responses. Schukajlow and Krug (2013) provided some evidence for the impact of prompting students to construct multiple solutions on planning and monitoring. Therefore, additional research is required to investigate the question of whether it is possible to change students' strong tendency to neglect the real world while solving P-problems by improving their monitoring strategies. Further, it seems to be a promising approach to investigate how working with missing data problems can be used to change students' strong tendency to give unrealistic responses to P-problems.

We acknowledge that our study used a limited number of items and item types. Future studies should increase both of these aspects in order to improve the reliability and generalizability of our results.

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