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ORIGINALARBEIT

The Role of the Situation Model in Mathematical Modelling—Task Analyses, Student Competencies, and Teacher Interventions

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Abstract In a study within the DISUM research project, we investigated the role that the construction of situation models plays as an essential prerequisite for understanding a given mathematical modelling task, using a sample of 21 9th grade classes ($N = 416$). Specific task characteristics, general mathematical competence, reading competence, and teacher interventions aiming at understanding the situation model were analyzed as crucial factors influencing students' ability to solve modelling tasks. The results show that: (1) strategies for constructing an adequate situation model have a significant influence on modelling competence, (2) mathematical reading competence and intra-mathematical competence can explain almost one third of the variance of the performance on the modelling test, (3) teacher interventions may encourage students to adopt strategies facilitating the construction of situation models, but an increase of modelling competence requires separate strategy training.

Keywords Modelling competence · Reading comprehension · Situation model · Word problem · Teacher intervention

Mathematics Subject Classification (2000) 97C30 · 97C70 · 97D40 · 97D70 · 97M10

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Zur Rolle des Situationsmodells beim mathematischen Modellieren – Aufgabenanalysen, Schülerkompetenzen und Lehrerinterventionen

Zusammenfassung Im Rahmen einer Studie des Forschungsprojekts DISUM wurde in 21 Realschulklassen der Jahrgangsstufe 9 ($N = 416$) untersucht, welche Rolle das Konstruieren des Situationsmodells als wesentliche Voraussetzung für das Verstehens einer gegebenen Aufgabenstellung bei der Bearbeitung von Modellierungsaufgaben spielt. Als zentrale Einflussfaktoren für die Modellierungskompetenz der Schüler wurden spezifische Aufgabeneigenschaften, die allgemeine mathematische Kompetenz und die Lesekompetenz identifiziert sowie Interventionen, mit denen Lehrpersonen auf das Verstehen einwirken. Dabei zeigen die Ergebnisse der vorliegenden Studie u.a., dass (1) Strategien zur Konstruktion eines adäquaten Situationsmodells einen signifikanten Einfluss auf die Modellierungskompetenz haben, (2) mathematische Lesekompetenz und innermathematische Kompetenzen knapp ein Drittel der Leistungsvarianz des Modellierungstests erklären können, (3) Lehrerinterventionen im Unterricht zwar die Anwendung solcher situationsmodellbezogener Strategien bei den Schülern anregen können, individuelle Leistungssteigerungen aber offenbar eines gesonderten Strategientrainings bedürfen.

1 Mathematical Modelling

1.1 Modelling Competence

Both on the national and on the international level, applications and modelling have been an important topic in mathematics education (the relevant state-of-the-art is documented in ICMI Study 14, Blum et al. 2007). Since the first results of the TIMSS and the PISA study and the resulting introduction of national Education Standards in Germany with modelling as one of the six central competences (cf. Leiss and Blum 2006), mathematical modelling has also been the focus of interest regarding classroom teaching in Germany. As desirable as this development may be—long demanded by mathematics educators—, it must be emphasized that this has to be accompanied by adequate research and developmental activities. So far there exists little empirical knowledge about how this complex competence of modelling can be acquired by students, which cognitive processes go on while modelling and above all how teachers can intervene appropriately and support substantial learning processes (cf. the overview on different studies on modelling competence by Maaß 2006, p. 119). A large empirical field of research is thus opened up for mathematics education with the crucial question, especially for teaching practice: Which (sub)competencies must students acquire in order to be able to solve modelling tasks such as in Fig. 1.

1.2 The Cognitively Oriented Cycle of Mathematical Modelling

A first approach here comes from the studies of the so-called "cognitive modelers" (Kaiser and Sriraman 2006) who deal with the description of individual modelling processes of students. In these studies, the three-step modelling cycle of Pollak (1979) was adjusted towards the requirements of research to represent all essential cognitive

Diapers

Mr and Mrs Brettleimer have a baby. Mr Brettleimer wants to buy cloth diapers that can be washed in the washing machine and used almost forever, for 469 €. The washing costs amount to 0.05 € per diaper.

Mrs Brettleimer thinks that cloth diapers are too expensive. She prefers buying non-recyclable diapers for the three years her baby needs diapers for 0.25 € per diaper.

Which of the two possibilities should family Brettleimer chose?
Give reasons for your answer.

Fig. 1 Modelling task "Diapers"

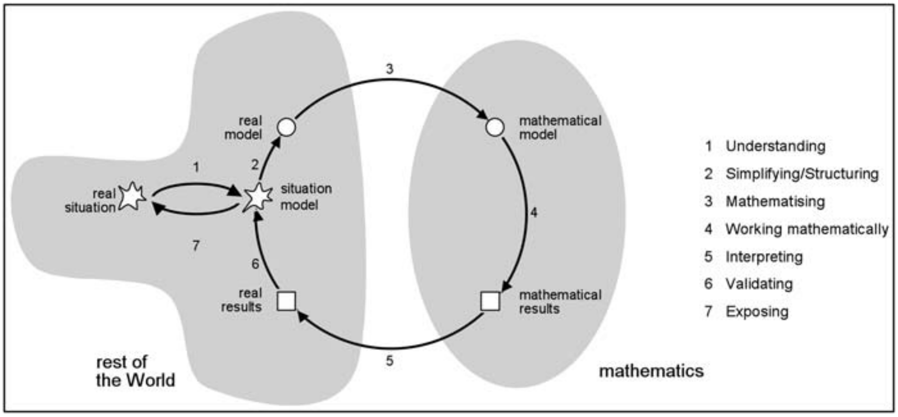


Fig. 2 Modelling cycle

processes within the process of solving modelling tasks (cf. Galbraith and Stillman 2006, and for an overview on various descriptions of modelling processes Borromeo Ferri 2006). In particular, two additional elements proved to be especially important which include essential cognitive processes for solving modelling tasks: the so-called "situation model" on the one hand, and the so-called "real model" on the other. Correspondingly, these components have been used in DISUM for the construction of the following seven-step modelling cycle which also serves as a basis for the present study (Blum and Leiss 2007). See Fig. 2.

It is important to note that this diagram only shows an idealized illustration of theoretical solution processes whereas actual processes are normally not so linear (Leiss 2007; Borromeo Ferri 2007). The value of this model is its usefulness for describing such processes and particularly for providing appropriate names for the necessary sub-competencies (Maaß 2006). Based on this modelling cycle, an exemplary cognitive analysis of the modelling task "diapers" (see Fig. 1) leads to the following ideal-typical seven modelling steps.[1] See Fig. 3.

[1]Of course, one has to assume that the very mastery of those seven sub-competencies does not necessarily result in the successful treatment of a modelling task as the main complexity of such tasks consists in the necessary integration of those sub-competencies.

general description	example: the task "diapers"
1. First, the text and maybe a photo have to be read and the problem situation has to be understood by the problem solver, that is an *individual mental model of the real situation* has to be constructed (see section 2.1).	
2. For example by making assumptions or selecting given data the situation has to be simplified, structured and made more precise, leading to a *real model* of the situation.	- Which diaper sort causes less costs? - 469,-€ package 0,05 € per diaper - 0,25 € per diaper - 3 years (365 days) 5 diapers per day
3. Based on basic ideas (vom Hofe 1998) of different mathematical concepts, mathematisation transforms this real model into a *mathematical model*.	$y_1 = 469 + 0.05 \cdot x$ $y_2 = 0.25 \cdot x$ $x = 3 \cdot 365 \cdot 5$ $y = y_1 - y_2$
4. Then mathematical tools like rearranging a term or the rule of three are used, yielding a *mathematical result.*	$y = -626$
5. By activating basic ideas again, the mathematical result has to be interpreted in the real world as a *real result* for the given problem.	Difference in prices approx. 600 €
6. The next step is a *validation* of the real result: Is it reasonable? Is the accuracy appropriate? Are the assumptions/ simplifications adequate? Accordingly, one might go round the modelling loop several times.	- , , ... ? - additional costs for the washing machine? - additional costs for a bigger garbage can? - 3, 4, 5, 6 diapers per day? - time needed for washing - ecological aspects ...
7. The process ends with an exposition of a final answer to the original problem.	If Mr. and Mrs. Brettleimer need 5 diapers per day for 3 years then Cloth diapers are 600 € cheaper than non-recyclable diapers.

Fig. 3 Seven modelling steps. Even if this *mathematical model* suggest that the mathematical model is only an intra-mathematical construct it is always linked with the given real situation of the task. So Niss (1989) describes the mathematical model as a triple (c, o, f), consisting of a real context c, a composition of mathematical objects o and a function f which assigns the elements of c to the elements of o

The importance of knowing the relevant modelling steps for teachers as well as for researchers is underlined by Treilibs et al. (1980, p. 4):

> *The full range of these high level skills is needed if children are later to be able to use mathematics in tackling problems of concern to them. The weakest link in their modelling chain will set the limits on what they can do.*

2 The Construction of a Situation Model as a Basis for Modelling Activities

2.1 The Construct "Situation Model"

Understanding the content of a task is fundamental for solving it since such tasks cannot be solved by simply superficially combining numbers and operations found within the text of the task (cf. De Corte et al. 2000 for examples of this superficial strategy). Correspondingly, the constructivist assumption that learners can merely create a subjective (re)construction of tasks lead more than 20 years ago to the introduction of an individual mental situation model as an element that can explain the difficulty of mathematical solution processes:

> *The situation model is the personal cognitive structure to which the process of understanding is directed. The situation model is the cognitive correlate to the situation structure either supposed from the author's point of view or understood from the readers' point of view* (translated from Reusser 1989, p. 136f.).

In particular with modelling tasks, an interplay of the characteristics of the real situation—normally presented in written form—(Verschaffel et al. 2000, p. Xf.) of intrapersonal aspects (Artelt et al. 2001, p. 69ff.) and especially in the classroom of social aspects (Reusser 1988, p. 333) determines the mental model of the task in the student's cognition. These two factors that are central for the presented test-based study, can be characterized by the following features:

Intrapersonal aspects:

- reading competence,
- prior knowledge about the context,
- general cognitive competence,
- general mathematical competence,
- attitudes and beliefs towards mathematics, mathematical tasks or the context,

Task characteristics:

- format,
- context,
- semantic structure,
- mathematical structure.

Transferring this aspects to the task "diapers" given above, a student whose parents use diapers for their baby will develop a different mental image of the task not only because of the contextual knowledge but also because of the emotional involvement than a student who is an only child. This individually shaped perception of a given task and the resulting individual situation model are the basic principles of all further solution processes.

Independence-oriented support by the teacher has to take these individual aspects into consideration. This is why Verschaffel et al. (2000, p. 97), among others, provide suggestions—though rather non-systematically—for teacher interventions that could be helpful for constructing the situation model ("Try to describe the task in your own words" or "Try to put yourself in the situation of the task").

2.2 Reading Competence

However, since instruction for tasks is usually given in a written form in mathematics education, both the students' reading competence and corresponding teacher interventions ("Read the text again carefully") will play a central role in addition to prior knowledge (cf. Stillman 2000) as an empirically known factor of influence.

Reading competence is one of the basic prerequisites for participating in many domains of social life and for knowledge acquisition in different domains (cf. Kamil et al. 2000). Traditionally, reading competence is defined as the capability of understanding written texts. Understanding means the construction of a coherent mental representation while reading a text. During the past decades, however, a noticeable extension of the concept of reading competence could be observed. In the course of American Pragmatism more attention was paid to what kind of knowledge, abilities and skills were necessary for a modern person to cope with the job market and everyday life. Since photos, pictures, drawings, charts, diagrams and other kinds of imageries and texts are widespread in modern society, the text repertoire considered within reading research and the conceptualization of reading competence have been extended considerably (cf. e.g. Mosenthal and Kirsch 1991). In this context, Schnotz and Dutke (2004, p. 63) write: "In fact, reading competence has to be regarded as a competence of understanding a written document which contains verbal information in the form of characters as well as pictorial information in the form of graphic symbols". Reading competence is also increasingly regarded as a domain-transcending competence (Kirsch et al. 2002, p. 15) that has to be encouraged in various school subjects. Creating and interpreting graphs, charts, diagrams and pictures as well as operating with mathematical symbols are parts of mathematics in school and represent a part of mathematical competence. Reading such information sources can thus be characterized as "mathematics oriented reading".

Whereas the significance of mathematics oriented reading for mathematical competence is obvious, there are few studies that investigate the correlation between the general reading competence as an understanding of task texts, on the one hand, and the mathematical competence—especially modelling competence—of the students, on the other (for an exception see, e.g., Reikeras 2006). Before discussing our research question concerning this topic, we will first explain the cognitive relationship between reading and the (mathematics oriented) situation model in detail.

2.3 Basic Cognitive Principles of Reading

The reading process is a complex process that takes place at the word, sentence and text levels at the same time. The three levels are not linear additive so that individual letters, words and sentences are read sequentially and the acquired information is fed into the reader's working memory. Instead one speaks of two concurrent processes interacting: The one is controlled by the text and the other by the reader's knowledge structures (cf. a summary about this by Goldmann and Rakestraw 2000). Only when the resulting mental representation of facts described in the text is coherent we can speak of understanding the text. Coherency here means "the linkage of mental units in a coherent unity" (Schnotz 1994, p. 17). Until now there exist only a few descriptions of how a mental representation of a text is constructed in a person's memory. Currently it is assumed that both a propositional representation (text basis) and a situation model are constructed when reading (Kintsch and Greeno 1985). Information is stored in the situation model as a whole. This whole can be presented as a kind of picture where various sensory information (auditory, visual, tactile, etc.) are brought together. The situation model makes it possible to directly read off conclusions about the situation (Johnson-Laird 1983). As conclusions play an important role in solving mathematical tasks, the meaning of a situation model for a successful task solving process can be seen as relevant (cf. e.g. research on text problems by Mayer and Heagarty 1996).

2.4 Research Questions

The above-mentioned findings combined with a strong correlation between reading competence and the competence of solving reality-oriented tasks that had been described in PISA (Artelt and Schlagmüller 2004) leads to three main research questions: One can ask on the one hand whether the difficulty of constructing an adequate situation model is influenced by specific task characteristics (research question 1). On the other hand one can ask whether dealing with reality-oriented tasks actually demands more competencies from a student than only the construction of the situation model, including intra-mathematical competencies (research question 2). And to state it more constructively: To what extent is it possible for the teacher to enhance students' modelling competence by systematically promoting students' (mathematical) reading competence (research question 3)? In the following these three questions will be specified.

1. Tasks

(1a) Is it possible to reliably rate the theoretical difficulty of constructing a situation model for mathematical tasks (in the following named as *situation value*)?
(1b) How does the degree of real-life relationships of tasks (in the following named as *modelling level*) relate to the situation value?
(1c) How does the empirical difficulty of tasks relate to the situation value?

2. Students

(2a) How do general reading competence, mathematical reading competence, and intra-mathematical competence ("working formally") correlate with modelling competence? (cf. Sect. 1.2)?
(2b) How much variance in the modelling competence can be explained by both factors: mathematical reading competence and competence of solving intra-mathematical tasks?

3. Teachers (the following questions will be discussed only on an explorative level with qualitative methods)

(3a) How do teachers support students in constructing a situation model?
(3b) What are the effects of situation-related interventions by teachers on the students' solution processes in class?
(3c) What are the effects of situation-related interventions by teachers on the individual student's test performance?

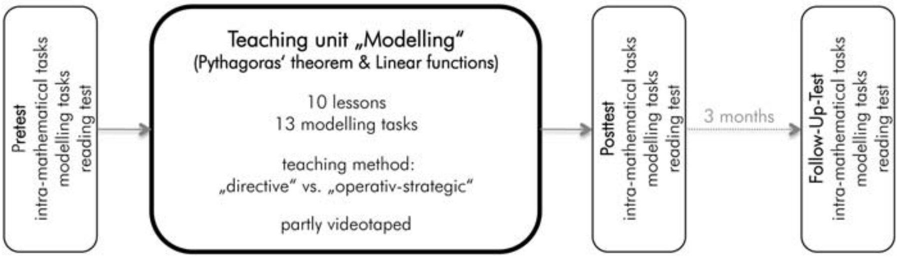


Fig. 4 Design of the DISUM-study

3 Method

3.1 Design and Samples

The present study is based on an analysis of data that were produced as a specific extension of the data from the so called *Main Study 2* of the *DISUM*[2] *Project* (cf. Leiss et al. 2007, pp. 227/228). The *Main Study 2* was a classical intervention study with a pre-, post- and follow-up-test that can be illustrated in Fig. 4.

Two different teaching methods[3] were implemented all in all in 21 German 9th grade classes within a ten lessons teaching unit on modelling, and studied with respect to their effects on students' achievement and attitudes by means of various tests and questionnaires. Some of the classes were reduced in size in order to homogenize the classes with respect to their mathematical ability.

3.2 Research Instruments and Methods

3.2.1 Videography

Two of the 21 classes were constantly videotaped, plus two lessons in 12 classes and supplementary single lessons in individual classes depending on organizational and technical possibilities. One flexible camera was there to videotape the teacher's activities; in addition, in the independence-oriented lessons where students worked in groups of four, one group of students that had been chosen at random at the beginning of the lesson was videotaped. Consequently, a total 76 of the 210 lessons were videotaped and also transcribed in most cases.

[2]"**D**idaktische **I**nterventionsformen für einen **s**elbständigkeitsorientierten aufgabengesteuerten **U**nterricht am Beispiel **M**athematik", in English: "Didactical intervention modes for mathematics teaching oriented towards self-regulation and directed by tasks". The project has been funded by the German Research Foundation (DFG) since 2005. It is an interdisciplinary project directed by W. Blum (mathematics education), R. Messner (pedagogy both University of Kassel) and R. Pekrun (educational psychology University of Munich).

[3]The teaching methods were on the one hand a "directive", highly teacher regulated method with students' working individually on exercises similar to those developed earlier; and, on the other hand, an "operative-strategic" method where students mostly work independently in groups, supported by the teacher with the help of strategy oriented interventions, and where retrospective reflection takes place with the whole group.

3.2.2 Educational Standards Test

All the students had to take a general mathematics test aimed at receiving a non-subject specific mathematics score for each student. The results were also used to homogenize some of the classes as mentioned before. The test was composed of 23 tasks taken from the standardization process for the German national Education Standards (Cronbachs $\alpha = 0.61$). For pragmatic reasons, only multiple-choice tasks that allow a fast evaluation were used.

3.2.3 General Reading Test

The reading speed and comprehension test LVGT 6–12 standardized for PISA 2000 was used as a measuring instrument for the students' reading competence. The special advantage of the test is that it fulfills one of the main quality criteria (including re-test reliability for the "reading comprehension" scale $\alpha = 0.87$). In addition, it also enables a valid measure of reading comprehension in only six minutes (cf. Schneider et al. 2007). The reading comprehension scale in this study had a satisfactory reliability of $\alpha = 0.75$.

The students were given a coherent text to read. There were words in parentheses to complete the sentences in several places in the text. Only one of the three given words fit the context of the text and the students were to underline that one. The right answer postulated reading comprehension; example:

> *The giraffe is one of the largest mammals in the world. It can be up to six meters tall. It has two to three horns on its head depending on whether it is from the north or the south. These (heads, spots, horns) are covered with a skin and end with a tuft.*

Due to the chance level related to the multiple choice format, a random corrected score was calculated according to the following formula for every item i, based on Lienert and Raatz (1994, p. 69):

$$Score_i = 2 \cdot \left(number\_correct\_answers_i - \frac{number\_false\_answers_i}{number\_alternativ\_answers_i - 1} \right)$$

For each of three possible choices within this test this means that a student receives two points for a correctly underlined word, -1 point for an incorrectly underlined word and 0 points when nothing is underlined. The total value reached on adding the individual values is principally open upwards as the length of the text was chosen so that it is hardly possible to thoroughly complete the text in the given amount of time. This means it is possible to measure a broad spectrum of achievement from poor readers to very good readers.

3.2.4 Mathematical Achievement Test

According to the aims of the project, it was necessary to have specific test instruments available which allow taking sensitive measures within the aspired mathematical fields of *Pythagoras' theorem* and *Linear functions* and for the competencies of

The price is right

Maren had worked on the following task in a class test:

Boat hire

Mr Eichdorf wants to hire a sailing boat at the "Edersee" (a lake in Germany) with his girlfriend. They can chose between the following two offers:

Maritim:	***Nemo:***
basic fee: 10 €	*no basic fee*
price per hour: 16 €	*price per hour: 20 €*

Which boat should they hire?

Maren wrote as an answer: I would choose "Nemo" because you have to pay a basic fee for "Maritim".

Her teacher gives her only a few points for that answer. Try to find an answer for that reaction: Why is the teacher not satisfied with her solution? You do not have to write down your own solution for the task Boat hire!

Fig. 5 Item for the sub-competence of validating

Working formally and *Modelling*, using the Rasch model. For this purpose, a performance test was developed, drawing on numerous pre-studies, with a special rotation design, with PISA anchor items and mainly new task formats. The reliability of the whole test is $\alpha = 0.68$, still at a satisfactory level. Besides a one-dimensional Rasch scaling which acts on the assumption of merely one latent ability *mathematical competence*, the test was also scaled with respect to the three dimensions *modelling tasks* (reliability $\alpha = 0.67$), *dressed-up tasks* (reliability $\alpha = 0.68$) and *intra-mathematical tasks* (reliability $\alpha = 0.62$)—with the aim of gaining differentiated results, e.g. concerning the task specific influence of mathematical reading competence.[4] Besides this differentiation concerning how close the tasks were to reality which was desirable per se, the psychometric quality criteria AIC and BIC (cf. Weakliem 2004) showed that this is a model that describes the construct better than a merely one-dimensional test model.

Dimension 1: modelling tasks A basic idea of the construction of the modelling test was to test, on the one hand, the complex competence of "mathematical modelling" as a whole; an example is the introductory task "Diapers" (cf. Fig. 1). On the other hand, tasks have been constructed aiming at single sub-competencies shown in the modelling cycle (cf. Sect. 1.2). An example for the sub-competence of validation is the following task "The price is right" (see Fig. 5). The first five steps of the modelling process have to be done only "passively" and only the validating step has to be performed "actively".

The advantage of this approach for the present study was that the usually high level of difficulty (e.g. solution frequency "Diapers": 12.5%) for modelling tasks could be lowered for these tasks (e.g. solution frequency "The price is right": 40%) without

[4] In the following, this is called modelling level 2 (modelling tasks), 1 (dressed-up tasks) and 0 (intra-mathematical tasks).

reducing the text intensity or the closeness to reality. All in all, 27 complex and 23 sub-competence related modelling items with different answer formats were used in the test. The level of difficulty covered was between −1.76 and 2.42 logits.

Dimension 2: Intra-mathematical tasks This subtest which mainly refers to the competence of “working symbolically/technically/formally” (“symbolisch/technisch/formalesArbeiten” in the German Education Standards) almost exclusively includes tasks where the general reading competence only plays a minor role and intra-mathematical abilities take center stage. An example is the following task “Intersection point” (see Fig. 6).

All in all, 18 items were used for this dimension. The level of difficulty of these tasks is with −1.53 to 2.95 logits a bit higher than the level for the modelling tasks, whereby the technical tasks requiring the calculation of the intersection point of two straight lines, in particular, exhibited a high level of difficulty.

Dimension 3: Dressed-up tasks This dimension covers tasks which are dressed-up in the language of everyday life or another discipline. The main demand is not to deal with the real context but to extract the intra-mathematical essence by “undressing” the task (Niss et al. 2007). Whereas this type of task can be separated quite easily from purely intra-mathematical tasks of dimension 2, the boundary to the modelling tasks of dimension 1 will be a following one. An example of such a task is the following task “Tablecloth” from PISA (see Fig 7).

All in all, 11 of these items were used in the test. The level of difficulty came out between −1.63 and 2.56 logits. Thus, there are tasks in all three dimensions that cover the level of difficulty relevant for educational purposes quite well. Further tasks would have been desirable merely in the lower level between −3 to −1.5 logits.

3.2.5 Mathematical Reading Test

In order to check to what extent it is possible to speak about a mathematics-related reading competence as part of the competence of mathematical communication (cf. NCTM 2000), three tasks which explicitly aim at the sub-competence of constructing a situation model[5] have been integrated into the achievement test described earlier (Sect. 3.2.4).

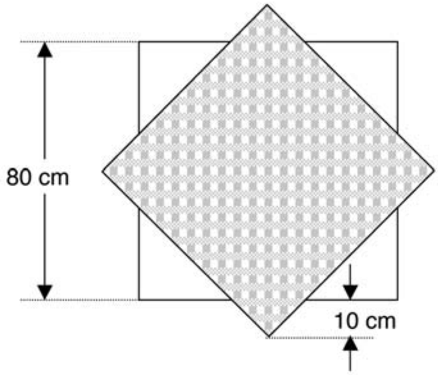
> Intersection point
>
> Determine the intersection of the two following straight lines.
>
> $y = x + 110$
>
> $y = 5 \cdot x - 10$

Fig. 6 Intra-mathematical test item

[5] Additional to the construction of a reading based situation model a partial construction of a real model is necessary for solving the task “Sloths” (cf. Fig. 8). But a small case study with 8-year-old primary school pupils ($N = 10$), skilled enough in reading to construct the situation model but not skilled enough mathematically to construct an adequate real model, showed that it is possible to solve this mathematical reading

Tablecloth

A square tablecloth lies on a square table. The tablecloth hangs over 10cm for all four edges as the graph shows.



What are the dimensions of the tablecloth? Define its surface area.

Fig. 7 Dressed-up test item

On the basis of these three mathematical reading-tasks, a math reading score between 0 and 3 has been ascribed to each student, according to the sum of the correct solutions.[6]

4 Results and Discussion

4.1 Task Related Results

In this section, we will refer to the following research questions 1a, 1b and 1c (cf. Sect. 2.4).

Research question 1a: Is it possible to reliably rate the theoretical difficulty of constructing a situation model for mathematical tasks (in the following named as *situation value*)?

Research question 1b: How does the degree of real-life relationships of tasks (in the following named as modelling level) relate to the situation value?

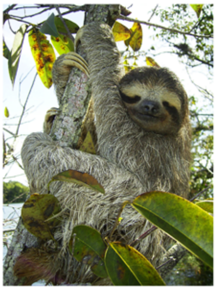
Research question 1c: How does the empirical difficulty of tasks relate to the situation value?

task "correctly" without having a correct conception of how the relevant aspects are mathematically connected. This can be regarded as a hint for a sufficient validity of the introduced measurement instruments (reliability $\alpha = 0.67$).

[6]It can be rightly criticised that the investigation of the score was not uninfluenced by the intervention because the three items for each student had been distributed over three measuring periods (cf. Fig. 4). The students' results show a slight increase of the solution frequency for these items within the post-test, though there are no significant differences between the two interventions.

Sloths

The sloth that is pictured on the right-hand side sleeps 15 hours a day. It eats during the remaining 9 hours. In between it goes swimming in the river 45 metres away. It goes there about 3 times a week. It comes down the tree fairly quickly, but then it only walks at a snail's pace: The sloth needs 1.5 minutes to take one step and moves forward 0.5 metres.



Circle all numerical data in the text that are required to be able to answer the following question. You do not have to answer the question!

How much time does the sloth need to reach the river for a swim?

Fig. 8 Mathematical reading task

4.1.1 Characterizing the Difficulty to Construct a Situation Model[7]

In order to determine the difficulty of forming an adequate situation model (named as *situation value*) a high inference rating was carried out for the 68 tasks used. A coding manual which describes how each task is to be evaluated with regard to the following three categories is the basis for these ratings:

1. Superficial characteristics (Average of the sub-categories a to e) + f:
   (a) Complexity of the concept (1 to 3 points)
   (b) Sentence structure/text coherency (1 to 3 points)
   (c) Numerical data/variables (1 to 3 points)
   (d) Context familiarity (1 to 3 points)
   (e) Length of text (1 to 3 points)
   (f) Alleviation due to graphics (0 or -1 points)
2. Complexity of task without its question (1 to 4 points)
3. Question complexity (1 to 2 points)

The value S for the difficulty of forming the situation model was calculated according to the following standardized formula[8]:

$$S = \big((\text{superficial characteristics} + \text{task complexity}) \times \text{question complexity} - 1\big) \times 111/14)$$

Using this procedure, a task can be given a value between 0 and 100. Empirically the values are between 1 and 87. Figure 9 shows, however, that a majority of the tasks can be found in the range of easy to intermediate difficulty and only relatively few situation model tasks present a greater problem in solving the task. This is consistent

[7] Further descriptions of tasks classifications may be found e.g. in Cohors-Fresenborg et al. (2004), Jordan et al. (2006).

[8] This procedure assumes that the problems are ones the students can—in principle—understand. In an authentic, real context, i.e. a context that was not specially prepared for the students, a completely incomprehensible term can lead to the value of the situation models having to be set at the highest level.

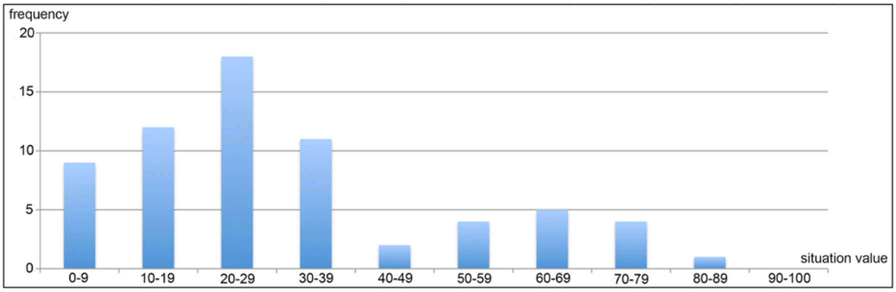


Fig. 9 Frequency scale for the tasks regarding the situation value

with the goals of the DISUM project in that it was not really the aim of conveying reading competence in the intervention described above but rather a matter of modelling competence. Accordingly the influence of the solution process related to the situation model was not to be dominating.

The inter-reliability in rating the tasks was, due to the highly inferred ratings (Cohen's Kappa = 0.70) in an acceptable range.

4.1.2 The Relationship Between the Modelling Level and the Situation Value

If the result of the previous section can primarily be seen as a methodological success, it is of greater contextual interest how the value *S* behaves, especially with respect to the reference to reality and the empirical difficulty of the tasks.[9] A correlation of 0.54 (significant at the 0.01 level) indicates the task's close connection between the situation value and the reference to reality.[10]

4.1.3 The Relationship Between Task Difficulty and the Complexity of the Situation Model

The already assumed close connection is less remarkable—although empirically hardly confirmed—than the fact that there are—even if it is more seldom—intra-mathematical problems with a relatively high situation value as well as modelling problems with a very low situation value. In the first case these are problems where a given intra-mathematical complex argumentation is to be verified. In the second case these are modelling problems with contexts so well-known to the students that it even helps them to understand the resulting intra-mathematical problem.

When the correlation (according to Pearson) between the situation value of a problem and its empirical difficulty (Logit)[11] is examined, then the result 0.12 (double page significance 0.36—cf. Bortz 2005, p. 111ff.) is disappointing at first glance.

[9]Only the pre-test data are used for the statistical analyses in Chaps. 4.1 and 4.2.

[10]The average situation value for problems with a modelling level of 0/1/2 was 10.8/32.6/38.3.

[11]There was no empirical connection between the modelling level and the difficulty strictly in terms of the test construction which allowed a distribution as even as possible throughout the entire field of difficulty for all problem variations.

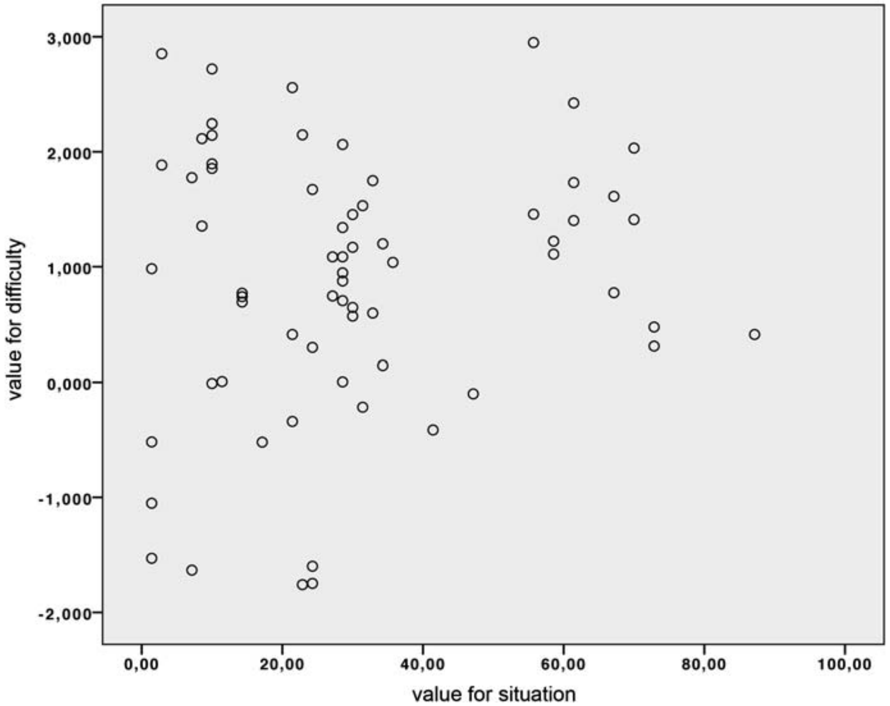


Fig. 10 Scatter diagram of the values for situation and difficulty

This low correlation can be explained in that a problem which has a low situation value, i.e. where understanding the problem is relatively easy, can still have e.g. high intra-mathematical demands. If we focus only on modelling tasks the correlation between the empirical difficulty and the situation value increases to 0.32 (double page significance 0.05). The following scatterplot (cf. Fig. 10) shows that the reverse case, namely a high situation value and a low level of difficulty, does not seem to be possible.

It also shows that even an extremely high situation value is not necessarily responsible for an extremely high problem difficulty, rather other factors are evidently necessary. These will be viewed more closely in the next section.

4.2 Student-Related Results

In this section, we will refer to the following research questions 2a and 2b (cf. Sect. 2.4):

Research question 2a: How do general reading competence, mathematical reading competence, and intra-mathematical competence correlate with modelling competence?

Research question 2b: How much variance in the modelling competence can be explained by both factors: mathematical reading competence and intra-mathematical competence?

Table 1 Correlation between reading and mathematical competencies

	Reading test	Math reading test	Educ. standards test	Intra-math. tasks	Dressed-up problems	Modelling tasks
General reading test	1	0.254**	0.178**	0.009	−0.006	0.111
Math. reading test	0.254**	1	0.192**	0.161**	0.183**	0.486**

**The correlation is significant at the 0.01 level (double page)

4.2.1 Connection Between (Mathematical) Reading Competence and Other Subject-Specific Competencies

Due to the close connection described in the previous section between forming a situation model and solving a modelling problem, it appears plausible if there were a close connection between students' reading competence and their modelling competence. Accordingly, Table 1 shows the correlation between the general and the mathematical reading test and the various achievement test data.

It shows that the results of both reading tests are related (0.25), yet not so strongly that they measure the same theoretical construct "reading competence". When the general reading competence is viewed first—as it was here (!)—then there is a lesser but more significant connection with achievement in the educational standards test. On the other hand, there is almost no correlation between general reading competence and the three dimensions of the DISUM test. This is surprising as especially the dressed-up and the intra-mathematical problems show a certain structural proximity to the Education Standards problems used in this context (average value 23.3) even when they are thematically less specific. An explanatory hypothesis that cannot be verified due to a lack of research is that—contrary to the more open format of the DISUM tests—the multiple choice format in the educational standards test used exclusively in these problems has a significant connection to general reading competence because of the selection of possible correct answers supported by general reading competence.

On the other hand, taking the mathematical reading test into consideration, this then seems to be sensitive to the situation value of a mathematical problem presented in Sect. 4.1.1. Correspondingly, there is a moderate connection between the mathematical reading value and the intra-mathematical or dressed-up dimension.[12] However, the clearly higher correlation of 0.49 shows that the competence of mathematical reading is especially a main prerequisite for successfully treating a modelling problem. Here it is clear once more that reading is not only to be viewed as a subject-specific content of German lessons, but rather that subject-specific problems require just such a reading competence training in subject-specific lessons.

Intra-mathematical competence also plays an important role in solving modelling problems in addition to mathematical reading competence highlights this with a

[12]There will be no more significant correlations when computing partial correlations between mathematical reading and the intra-mathematical ($r = 0.02/p = 0.73$) or the dressed-up dimension ($r = 0.04/p = 0.42$) (control variable: modelling).

relatively high correlation of 0.302 (significant at the 0.01 level) between intra-mathematical and modelling competence.

4.2.2 Mathematical reading competence + intra-mathematical competence = modelling?

To what extent modelling competence can, however, be reduced to just a connection between mathematical reading competence and intra-mathematical competence was tested with a regression analysis. When modelling competence is seen as a dependent variable that is determined by two independent variables (mathematical reading competence and intra-mathematical competence), then the result is that about 29% of the achievement variance is modelling problems can be explained by these two factors (standardized beta value $\beta_{mathematical\_reading} 0.45/\beta_{intra\text{-}mathematical} 0.23$—significant at the 0.01 level—cf. Sheskin 2004, p. 1007). This empirical result shows that modelling—strictly in terms of the modelling cycle—demands far more competence from the students than just the reading literacy undressing of reality-related contexts and the following processing of reality-related problems.

4.3 Teacher-Related Results: The Case Study of Mrs. R

In this section, we will refer to the following research questions 3a, 3b, and 3c (cf. Sect. 2.4):

Research question 3a: How do teachers support students in constructing a situation model?
Research question 3b: What are the effects of situation-related interventions by teachers on the students' solution processes in class?
Research question 3c: What are the effects of situation-related interventions by teachers on the individual student's test performance?

In the previous sections the importance of the two factors "mathematical reading" and "intra-mathematical skills" for students' modelling competence was emphasized. Now the question is how mathematical communicating, especially reading comprehension of mathematical problems, can be conveyed by the teacher and what influence this has on the students' achievements. Even when studies specifically directed towards answering these questions are needed, the following will take this into consideration in terms of an explorative case study in analyzing the class of Mrs. R. more carefully. We chose this class for a case study analysis because it was one of only two classes which were completely videotaped in the framework of the *DISUM Main Study II* (cf. Sect. 3.2.1).

4.3.1 Characterizing Instructional Interventions in Forming the Situation Model[13]

Seen qualitatively it is remarkable that the mathematics teacher, Mrs. R, uses a relatively broad spectrum of situation model related aids in supporting her students without any instruction concerning this matter from the DISUM intervention manual. All

[13]The term *situation strategies* is to be used in the following for those procedures which (are to) support the students in forming an adequate situation model.

those interventions were included with the goal of helping the students form an adequate situation model, independent of whether this happened with direct instruction or with more strategically oriented hints.[14] Correspondingly, the interventions diagnosed in Mrs. R's 10 lessons teaching unit can be described by the following four main categories and the ten sub-categories:

I. Collect information
   - I.1 Reading (text/graphics)
     Mrs. R: "*I would suggest your carefully reading the problem once more.*"
   - I.2 What data are/information is given—sought?
     Mrs. R: "*What are the given variables?*"
   - I.3 Explain unknown terms and/or unclear contents
     Mrs. R: "*Global means total, as much as desired, no extra calculation.*"

II. Select information
   - II.1 What is the most important part of the task?
     Mrs. R: "*And I will give you another hint. Think about which things are vital and important for the task, which are not.*"
   - II.2 Make labels
     Mrs. R: "*Read the text as if you have a magnifying glass and underline the facts for the printer.*"

III. Connect information
   - III.1 Connect the various sources of information
     Mrs. R: "*Look for the variables written in the sketch.*"
   - III.2 Connect the graphic and the written information
     Mrs. R: "*What's the radius? Underline it and draw it in the sketch.*"

IV. Process information
   - IV.1 Make a sketch of the situation
     Mrs. R: "*Perhaps it would be good for you to make a sketch of the situation.*"
   - IV.2 Formulate your own question/clarify the given question
     Mrs. R: "*...copy out what they want to know in the question. Write the question in your own words.*"
   - IV.3 Put yourself in the situation and play it through
     Mrs. R: "*Imagine you drive to Luxemburg to get petrol Why would you do that?*"

Whenever such aids were used in class, it was usually in the case of two characteristic situations. On the one hand, there was a reason for the intervention in that at the beginning of the solution process the students stated they had a fundamental problem in understanding the task. On the other hand, the teacher often decided to use a situation model related aid when the students had made a mistake because of the lack of a situation model. One focus of the intervention behavior was formed by those interventions that call for students to underline relevant information in the text. One

[14]The difference to real model related interventions is that the situation model related interventions were limited to not putting the solution of the problem in the foreground by a certain form of simplification or structuring, but rather the pure understanding of the demand/ real situation.

third of Mrs. R's help in the 10 lesson periods in the DISUM study related to situation models.

Considering the effects attained by this intervention behavior, it must be differentiated between two levels: the one is how the students act on these interventions in class and the other one is how this affects the individual modelling competence tested in the post-test.

4.3.2 The Effects of Situation-Related Interventions by Teachers on the Students' Solution Processes in Class

When the task solutions completed in class are analyzed with regard to externalized situation models and/or with regard to the help given by the teachers, it seems that this assistance often helped overcome the problem. After this generally supportive intervention by Mrs. R over 40% of the about 240 students' solutions in class contain strategies like underlining, making sketches, etc. It turned out, however, that the students needed these interventions during the entire unit so that they could consciously use the appropriate processing strategies. This led to the circumstance that some students regularly use appropriate processing strategies (like e.g. Mrs. R's student Anita who was called on five times in class to consciously deal with the task related situation model and who very successfully did that in 11 of the 15 tasks treated).

4.3.3 The Influence of Teacher Interventions Related to Situation Models on the Individual Student's Test Performance

Until now a corresponding re-coding of the pre- and post-test solutions has been done in randomly selected five of the classes from the *DISUM Main Study II* (cf. Sect. 3.1) including the one just mentioned. The aim was to catalogue the strategies for forming situation models observed in the students' solutions.[15] The average of 3.6 applied strategies per student post-test after the DISUM unit in Mrs. R's class (standard deviation 2.34) showed that in this class the strategies were applied more often or were more visible than in the other classes (average of 0.6–2.1 applied strategies per student post-test—standard deviation between 1.03 and 1.9). Furthermore, this was related to a significant increase in situation strategies between the pre- and post-test in Mrs. R's class (average pre-test 2.1).[16]

The analysis of the connection between the students' achievement data and the application of the situation strategies shows that 80% of the situation strategies in the post-test are allotted to modelling tasks. In the intra-mathematics tasks and the dressed-up tasks there were only a few exceptional cases where such strategies were used. The reason for this behavior might be found in the assumed accessibility of such tasks, but this cannot be clarified with the available data. Correspondingly we focus only on the connection between modelling competence and the use of situation strategies in the post-test.

[15]The coding mainly referred to such externalized actions that could be contextually described in the aspect (teacher intervention) depicted in Sect. 4.3.1.

[16]This increase indicates that Mrs. R. has especially broached these situation strategies within the DISUM unit.

Table 2 Correlation of the situation strategies with the individual achievement data

		Model post
Situation strategies in the post-test	Correlation	0.302**
	N	101

**The correlation is significant at the 0.01 level (double page)

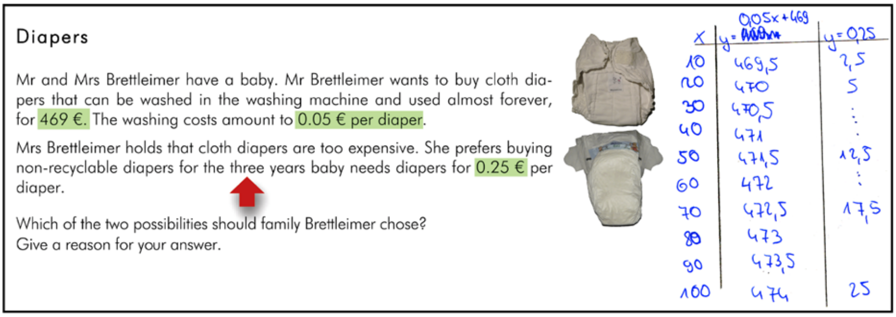


Fig. 11 Anita's situation strategies in the post-test

Table 2 shows that the ability to solve modelling tasks can positively be connected to the use of strategies to form situation models (research question 3b).

Focusing on this positive result on individual students in Mrs. R's class again, there are positive examples like Oliver for the use of situation strategies. With him a considerable increase in modelling competence coincides with a clear increase in the use of situation strategies ($3 \rightarrow 6$). Problematic—in terms of the positive effect of teacher interventions—is only that he did not use the underlining often emphasized in Mrs. R's lessons—instead he only made sketches. The student with the most increase in situation strategies between the pre- and the post-tests ($2 \rightarrow 9$), Anita, makes another problem concerning the teacher's intervention behavior mentioned earlier clearer. Mrs. R tells the class e.g. to make a sketch or to underline things. This leads to Anita's now using situation strategies in nine of the tasks in the post-test. However, she only solved one completely correct since she e.g. applied the marking strategy to all numerals, meaning she missed the numbers written as words in the tasks, e.g. Diapers (see Fig. 11) and came to an inadequate solution.

On a positive note, it may be said that Anita increased her individual modelling competence by repeated use of the situation strategies and that she is now more able to overcome this first obstacle.

5 Conclusion

The goal of the study at hand was to show which (task) factors depend on the construction of an adequate situation model, how this is related to students' modelling competence and to what extent this can positively be influenced by specific teacher intervention. It was shown that—even when there is no linear connection to the general task difficulty—the formation of an adequate situation model illustrates a specific characteristic which generates difficulty in the student's solution process, especially with modelling tasks. Having said this, the description of forming a situation model is also justifiable empirically as a sub-competence of modelling and the corresponding expansion of the modelling cycle (see Sect. 1.2). How much the situation model influences the solution process is underlined by the fact that intra-mathematical competencies also play an important role in modelling, but not more than students' mathematical reading skills do (see Sect. 4.2.2). This again shows the facts found in TIMSS (cf. Blum 1998) and PISA (cf. Neubrand 2004) and elsewhere that focusing on simply technical skills in class does not suffice to convey such complex competencies like mathematical modelling to the students. However even if the quantitative results of this study underline the importance of supporting mathematically related reading skills and situation strategies, the qualitative results show that there are numerous difficulties in this supporting process.

There are various programs supporting general reading skills with proven quality (cf. Artelt et al. 2005; Hattie 2007), yet there has been no empirical knowledge about the subject-specific promotion of reading competence in mathematics education. And even when the use of situation strategies to improve modelling competence appears to promise success, it has been shown that a conscious long-term training of such strategies is necessary.[17] Accordingly, the struggle with texts can no longer remain only a part of mother language lessons, rather it must also become an object of mathematics education based on tasks and texts. Situations with mathematical-specific structuring and verbalizing done by students help promote understanding in mathematics lessons. A positive "halo effect" of such training where students are asked to consciously construct and thus externalize their situation model is that students' difficulties become visible to the teacher at an early phase of development. This can form an important additional support in the diagnosis of student learning processes.

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[17] An example of a successful strategy training for problem solving are the studies of Perels et al. (2007).

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