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Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations

Abstract

In this study which was part of the DISUM project, 224 ninth graders from 14 German classes from middle track schools (Realschule) were asked about their enjoyment, interest, value and self-efficacy expectations concerning three types of mathematical problems: intra-mathematical problems, word problems and modelling problems. Enjoyment, interest, value and self-efficacy were assessed before and after a ten-lesson teaching unit promoting modelling competency related to the topics "Pythagoras' theorem" and "linear functions". The study aimed to answer the following research questions: (1) Do students' enjoyment, value, interest, and self-efficacy expectations differ depending on the type of task? (2) Does the treatment of modelling problems in classroom instruction influence these variables? (3) Are there any differential effects for different ways of teaching modelling problems, including a "directive", teacher-centred instruction and an "operative-strategic", more student-centred instruction emphasizing group work and strategic scaffolding by the teacher? The findings show that there were no differences in students' enjoyment, interest, value and self-efficacy between the three types of tasks. However, teaching oriented towards modelling problems had positive effects on some of the student variables, with the student-centred teaching method producing the most beneficial effects.

Keywords: affect, enjoyment, self-efficacy, modelling problems, word problems, self-regulation, teaching methods

1 Introduction

In mathematics education there has been a strong plea during the last few decades for treating modelling problems in mathematics classrooms (for an overview cf. Blum & Niss, 1991; Niss, Blum, & Galbraith, 2007; Verschaffel, Greer & De Corte, 2000). In addition to intensifying the learning process, an increased motivation, an activation of positive emotions and more student interest in mathematics are expected from providing references to reality. However, do students really attribute more importance to reality-related modelling problems than to intra-mathematical problems? Do they like them more? Are they more interested in solving these problems? All of these questions have not been investigated sufficiently so far, and the same holds for the question of how classroom instruction using modelling problems influences students' task-specific affective dimensions. In the present study, these questions are addressed in relation to the topics "Pythagoras' theorem" and "linear functions". These topics were selected because of the important role they play in mathematics curricula in Germany as well as in other countries.

The present study is part of the research project *DISUM* ("*D*idaktische *I*nterventionsformen für einen *s*elbständigkeitsorientierten aufgabengesteuerten *U*nterricht am Beispiel *M*athematik", in English "Didactical intervention modes for mathematics teaching oriented towards self-regulation and directed by tasks"). This is an interdisciplinary project between mathematics education (W. Blum), pedagogy (R. Messner, both University of Kassel) and educational psychology (R. Pekrun, University of München) which started in 2002. It aims at investigating how students and teachers deal with cognitively demanding modelling tasks, in particular what kinds of teachers' diagnoses and interventions are appropriate for supporting students solving such tasks independently, and what effects different learning environments for modelling have on the development of students' competencies and affect. The focus is on grades 8-10 (14-16-year-olds). The main result of DISUM so far is that the learners in the student-centred, operative-strategic" group outperformed the learners in the teacher-centred "directive" group in modelling competency significantly, both in the post-test and three months later in the follow-up-test (see a description of both teaching methods below). Students' self-regulation was improved strongly in the operative-strategic condition, and it was positively related to performance, students' self-reported enjoyment, effort, and use of learning strategies (cf. Schukajlow, Blum, Messner, Pekrun, Leiss & Müller, 2009). Another DISUM study deals with the connection between reading competency and the competency to solve intra-mathematical tasks while solving modelling problems. The results show, among other things, a key role of these two competencies for the performance of students as well as the relevance of students' strategies to construct a situation model (cf. Leiss, Schukajlow, Blum, Messner & Pekrun, 2010).

2 Theoretical Background and Research Questions

2.1 Importance of students' affective processes for the improvement of learning and performance

Learning is always accompanied by feelings and motivational, volitional and other affective processes that – at first sight – have only little to do with the construction of knowledge structures and students' achievement. This may be the reason why research in mathematics education was clearly focused on the content material until the 1970s. A limited amount of work in the affective area addressed selected attitudes towards mathematics and the emotion anxiety (cf. Zan, Brown, Evans, & Hannula, 2006). The article by McLeod, which called for a stronger linkage between studies in the affective field and research into instruction and cognition, can be seen as a turning point in mathematics education (McLeod, 1992). In mathematics education, the affective domain is traditionally divided into three essential groups: „emotions", „attitudes" and „beliefs". Emotions (joy, panic etc.) are momentary reactions to important events and objects., whereas beliefs and attitudes are thought to be relatively stable over time (McLeod 1992; Zan, Brown, Evans, & Hannula 2006).

However, although much empirical work has been done in recent years with regard to the affective domain, there still is a lack of research on students' emotions (for recent exceptions, see Efklides & Volet, 2005; Linnenbrink, 2006; Linnenbrink-Garcia & Pekrun, 2010; Pekrun, Goetz, Titz, & Perry, 2002a; Schutz & Pekrun, 2007), In addition, there is a recognizable lack of knowledge in the following two areas of research. With few exceptions such as the study by Gläser-Zikuda et al. (2005), subject-oriented intervention studies which investigate and compare the effects of various types of instruction on students' emotions are missing. Furthermore, most of the instruments available to measure students' emotions, attitudes and beliefs are not sufficiently grounded in perspectives of mathematics education (see for more recent research approaches Hannula, 2007; Leder, Pehkonen, & Törner, 2002; Pekrun, et al., 2007). These perspectives, however, are essential for evaluating and further developing theories in the field.

This paper addresses the emotion enjoyment as well as interest, value and self-efficacy expectations related to tasks with and without a reference to reality. As such, the focus is on specific affective constructs. These constructs can also be seen as parts of global constructs like, for example, beliefs/attitudes towards oneself or mathematics (cf. Op 't Eynde, De Corte, & Verschaffel, 2006). However, distinguishing between these specific constructs allows a more differentiated analysis of their relevance for learning processes and can provide more detailed information about how to create learning environments that promote adaptive affective processes.

The emotion enjoyment

Information about the quality of interactions between the environment and humans is conveyed with the help of emotions (Lewis, Haviland-Jones & Feldmann Barrett, 2008). Emotions prepare our actions, accompany these actions, and influence reflection about their outcomes. Enjoyment, anxiety, anger, and boredom are among the emotions most frequently experienced during classroom instruction (Frenzel, Pekrun, & Goetz, 2007; Larson & Richards, 1991; Pekrun et al., 2002a, in press).

There are few findings about the effects of enjoyment on learning and achievement. Specifically, field studies have found that students' enjoyment of learning correlates positively with their academic achievement (Pekrun, Goetz, Titz, & Perry, 2002b), although null findings have occasionally been observed as well (Linnenbrink, 2007; Pekrun, Elliot, & Maier, 2009).

Values and interest

"Value" characterizes the perceived importance attributed to objects, contents and actions (Eccles & Wigfield, 2002). A person can, for example, be convinced that mathematical skills are valuable for obtaining a job and for everyday life. Values play an important role in theories of human motivation. From the perspective of these theories, it can be assumed that students' motivation to learn is influenced by the importance attributed to learning and its objects (Wigfield & Eccles, 1992; Wigfield & Eccles, 2000). Metallidou and Vlachou (2010) show that students with high value beliefs in mathematics are described by teachers in self-regulated learning environments as better learners in the cognitive, metacognitive and motivational domains.

A second motivation construct of specific relevance for learning is interest. "An interest represents or describes a specific relationship between a person and an object in his or her "life-space"" (Krapp, 2000, p. 111), such as the relationship between a person and mathematics. Interest plays an important role in the learning process. Interested learners use understanding-oriented strategies more often and are less satisfied with superficial strategies such as memorizing (Schiefele & Schreyer, 1994). When students regard their work as meaningful and interesting, they engage more frequently in the self-regulation of their learning process (Pintrich, 1999). However, despite these findings, research has not yet succeeded in establishing a causal relationship between students' interest in a subject and their academic performance. The meta-analysis by Schiefele, Krapp, and Schreyer (1993) showed an average correlation of $r = .30$ between interest and performance across subjects. Although this correlation is higher than between other motivation constructs and performance (average correlation of 0.12 in the meta-analysis by Fraser, Walberg, Welch, & Hattie, 1987), it strongly decreased when prior domain-specific knowledge was controlled for. Traditional ways of schooling in which students are given little room for realising their own interests is probably responsible for this counter-intuitive finding. Studies on the relationship between interest and performance in learning environments in which students can choose contents or subjects show a closer linkage between the two variables (Köller, Baumert, & Schnabel, 2001).

Further, value as well as interest – independent of their importance for other constructs – are seen as goals in their own right in mathematics education. Various aspects of the usefulness of mathematics for teachers and students are elaborated, for example, in research about beliefs (see e.g. Leder, et al., 2002).

Self-efficacy expectations

Self-efficacy expectations are defined as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (Bandura, 2003, p. 3). One characteristic feature of self-efficacy expectations is a variable degree of specificity. As self-efficacy expectations are defined by actions, this construct can be related to learning in general, to learning in mathematics, to sub-areas such as algebra, or to specific competencies such as proving and arguing.

Self-efficacy is most important in educational, psychological and subject-oriented research. Students with a high level of self-efficacy profit in different ways concerning their academic learning and performance. They use understanding-oriented strategies more often (Pintrich & DeGroot, 1990), regulate their learning process more intensively (Zimmerman & Martinez-Pons, 1990), invest more effort (Schunk, 1989), and show better performances in problem solving (Hoffman & Spatariu, 2008) and mathematics (Malmivuori, 2006; Pietsch, Walker, & Chapman, 2003; Skaalvik & Skaalvik, 2008).

2.2 Teaching methods and students' affect

When learning environments are differentiated according to the degree of self-regulation allowed for the students, the advantage of more student-oriented forms of teaching and learning are often seen in the affective domain. A strongly teacher-directed form of teaching can, for example, diminish the interest in the subject (Seidel, Rimmele, & Prenzel, 2003). Other affective components like motivation and positive emotions can also develop negatively in such learning environments (Brophy & Good, 1986). On the other hand, student-oriented learning environments providing possibilities for cooperative exchange seem to create more favorable pre-requisites for positive developments in the affective domain (see Gläser-Zikuda, et al., 2005; Hänze & Berger, 2007; Hattie, Biggs, & Purdie, 1996). Reviews of studies on the impact of cooperative learning environments as compared to "traditional" or individual approaches show the benefits of cooperative learning for students' affective dimensions (Slavin, Hurley, & Chamberlain, 2003; Webb & Palincsar, 1996). These positive effects of cooperative group work on the motivation of students can be explained by the increased possibilities to set individual goals which is an important part of self-regulated learning (Schunk & Zimmerman, 2003); this aspect also plays a prominent role in motivational theories such as self-determination theory (Deci & Ryan, 2000).

Marcou & Lerman, (2007) have shown an impact of self-regulated student learning of problem solving on students' affect and performance in a quasi-experimental pre-post design. The teachers of the experimental group redeived two hours of instruction and a booklet including all the details and guidelines concerning the intervention. The control group did not get any support. 640 students from 4th, 5th and 6th grade participated in this study and were tested at the beginning and at the end of a school year. The main result of this study was that self-regulated teaching improves primary students' motivation, task values and performance. This result indicates that student-centred teaching can have a substantial impact on students' attitudes and beliefs. The study conducted by Panaoura, Gagatsis & Demetriou (2009) demonstrated that prompting of self-monitoring in students' behaviour improves their perception of abilities to solve problems.

2.3 Mathematical problems with and without a connection to the real world

Mathematics problems are often divided into three groups (cf. a.o. Niss, et al., 2007, S. 12): modelling problems, ("dressed up") word problems and intra-mathematical problems. Differences among these three groups of problems can be explained in the mental activities which are necessary for solving these problems. We will begin the description of these three groups of problems with the modelling problems because their treatment causes the most diverse demands.

Modelling problems

Demanding transfer processes between reality and mathematics are the core of modelling activities (Blum et al., 2007; Pollak, 1997). One of the process models to describe modelling activities is the modelling cycle proposed by Blum and Leiss (2007). In an idealized form, the solution process for a modelling problem can be characterized by a seven-step sequence of activities: (1) understanding the problem and constructing an individual "situation model"; (2) simplifying and structuring the situation model and thus constructing a "real model"; (3) mathematising, i.e. translating the real model into a mathematical model; (4) applying mathematical procedures in order to derive a result; (5) interpreting this mathematical result with regard to reality and thus attaining a real result; (6) validating this result with reference to the original situation; if the result is unsatisfactory, the process may start again with step 2; (7) exposing the whole solution process.

From this point of view, the modelling process is made up of seven steps. Distinguishing between these steps is helpful for reconstructing the modelling processes used by students when solving mathematical problems. However, students' actual processes are typically not linear but rather jump back and forth several times between mathematics and reality (see Borromeo Ferri, 2007; Leiss, 2007).

There are several other modelling cycles that can also be used to describe students' activities while solving modelling problems (e.g.,Galbraith & Stillman, 2006; Pollak, 1979; Verschaffel, Greer & De Corte, 2000). One characteristic advantage of the seven-step modelling cycle is the separation between constructing a situation model, a real model, and a mathematical model. This allows for distinguishing between difficulties in understanding the given situation, in simplifying and structuring the information extracted from the situation, and in choosing a suitable mathematical description of the situation during students' solution processes, and thus helps teachers in choosing appropriate, well-aimed and adaptive interventions especially in the critical translation phase at the beginning of the modelling process. Generally speaking, the seven-step cycle described above is

both sufficiently detailed to capture the essential cognitive activities taking place in actual modelling processes and sufficiently simple to guide the necessary observations and analyses in a parsimonious way.

"Dressed up" word problems

"Dressed up" word problems are related to reality as well. However, in these problems the reality-related mental activities are much simpler than in modelling problems since the simplified real model is already given from the beginning by the description of the problem. When constructing such problems, a certain mathematical topic is "dressed up" with reference to reality, and all the data that are necessary for finding the solution are given in the text (and no other data). So students do not need to make assumptions about missing data or about selecting relevant data while solving such problems. However, it is well-known (see the survey Blum, 2011) that making assumptions is one of the most relevant cognitive barriers for students when solving modelling problems. The validation of the real result is also much easier since it is mainly limited to checking the mathematical part, and several "modelling loops" are unnecessary here. These characteristic differences in the solution processes make it possible to perceive differences between both kinds of problems both for teachers and for students.

Intra-mathematical problems

The third type of problem consists of those without any connection to reality. The beginning of the solution process is a situation model that refers to a mathematical situation. The problem is solved on this basis by using suitable mathematical procedures.

Importance of problems with and without a connection to the real world for the development of mathematical competence

The distinction of problems according to their closeness to reality is not meant to be a valuation. All three types of problems are important for learning mathematics and for students' mathematical competence. Which problems ought to be used in class depends on which mathematical competencies are to be acquired during classroom instruction. As learning is often situated and a transfer of knowledge between the various areas of knowledge can only be attained to a limited degree (Greeno, 1989), it makes sense when developing, e.g., the competence "symbolic/formal/technical work" (cf. Leiss & Blum, 2006) to work on related intra-mathematical problems. This competence is also needed for solving modelling or "dressed up" word problems and can successfully be trained when working on modelling problems (Leiss et al., 2008). Well established skills in the other two types of problems are helpful in the development of modelling competencies as long as these skills are not impaired by non-mathematical heuristics, e.g. "key word strategies" when solving "dressed up" word problems (Reusser & Stebler, 1997; Verschaffel, Greer & De Corte, 2000). While applying "key word strategies" students do not construct an appropriate situation and real model, but translate only some words (e.g. more or less) directly into mathematical operations. By using only intra-mathematical or word problems it is, however, not possible to acquire modelling competency (see also Blum, 2007). The important role of teaching modelling competency is shown, e.g., by the study of Panaoura, Demetriou, & Gagatsis (2009). After an interventional program based on the model of Verschaffel et al. (2000), students' mathematical performance and their use of self-regulation strategies were significantly improved.

2.4 Students' affect towards modelling, word and intra-mathematical problems

Task specificity is an essential part of definitions of some motivational constructs such as self-efficacy or interest (see section 2.1) and plays an important role in their measurement (see for review Murphy & Alexander, 2000). A detailed analysis of measurement instruments shows that task specificity was often established by just adding a reference to mathematical problems to self-reported items, but without showing a real mathematical task. Typical item examples are "I feel that, to me, being good at solving problems which involve math or reasoning mathematically is (not at all important, ..., very important)" for perceived task value (Eccles & Wigfield, 1995) or "How much do you like doing math-related tasks at school?" for task motivation (Nurmi & Aunola, 2005; see also Greene et al., 1999 and Usher & Pajares, 2009). As yet, mathematical tasks have rarely been used for the measurement of emotions, attitudes and beliefs (but see Betts & Hackett, 1983; Pajares & Graham, 1999) and it may be an important step in mathematic education to construct task-specific instruments for measuring affect in mathematics.

A second important question relates to the state-like versus trait-like nature of task-specific measurements. While state-like scales measure constructs at a specific point of time (e.g. I enjoy mathematics teaching *today*), the trait-like scales collect data about the same constructs in general, that is, across time (e.g. I enjoy mathematics teaching). Some studies, such as investigations of the influence of learning environments on students' goal orientations (Ames, 1992), showed that task-specific measures may be sensitive to change. These findings suggest that task-specific constructs and measures may have a state-like nature (Murphy & Alexander, 2000).

In recent years, there were several pleas to elaborate the available educational and psychological measuring instruments by taking into account subject-specific aspects (Pekrun et al., 2007; Zan, et al., 2006), to develop new instruments on a theoretical basis (Ma & Kishor, 1997), and to study students' emotions, beliefs and attitudes in a domain-specific, subject-oriented way, especially by using tasks (Goetz, Frenzel, Pekrun, Hall, & Lüdtke, 2007; Kuntze & Reiss, 2006). In recent studies in mathematics education, new instruments to measure affect and new models of interaction between cognitive and affective variables were developed. For example, Hannula, Pantziara, Wæge & Schlöglmann (2009) speak about a "multimethod approach to a multidimensional affect". Measures conducted with task-specific instruments show the connection between task-specific measures (e.g. self-efficacy) and performance of students in mathematics (Pajares & Graham, 1999; Bong, 2002).

There is currently only one study about students' emotions towards mathematical problems with and without reference to reality. In the Project for the Analysis of Learning and Achievement in Mathematics (PALMA; see Pekrun et al., 2007), enjoyment of seventh graders was studied using two groups of problems which corresponded to "dressed up" word problems and intra-mathematical problems. One "dressed up" word problem was, e.g.:

Thomas would like to by a new backpack. His Father says: "I will pay half of the price, you should pay the rest." How much can the backpack cost if Thomas has saved up 60 €?

An intra-mathematical problem was, e.g.:

Solve the equation and find x: $66 - x = 12 \cdot \frac{480}{100}$

Using a 5-point Likert scale, 2509 students from the 7th grade had to indicate whether they enjoyed working on these problems. The statement was "I would enjoy solving this problem". The authors found that word problems were more enjoyable than pure number problems (Pekrun, et al., 2007).

2.5 Research questions

The present study was designed to answer the following research questions:

Research question 1. Do students' enjoyment, boredom, value, interest and self-efficacy expectations differ according to the type of problem (intra-mathematical problems, "dressed up" word problems, modelling problems)?

Research question 2. What are the effects of classroom instruction using modelling problems on students' enjoyment, value, interest and self-efficacy expectations related to these types of problems?

Research question 3. Are there any differences in the effects of different teaching methods, including "directive", teacher-centred instruction and "operative-strategic", student-centred instruction involving student group work and strategic scaffolding by the teacher?

3 Method

3.1 Design and sample

224 German ninth graders (50% females; mean age = 15.1 years, SD = 0.64) were asked about their enjoyment, value, interest and self-efficacy regarding various types of problems before and after a ten-period teaching unit. In this unit, only modelling problems in the sense described in section 2.3 were treated. Students did not get any instructions about differences between modelling problems and other problem types. 14 middle track classes (Realschule) from ten comprehensive schools (German Gesamtschule) participated in the study. Based on an initial mathematics achievemen test, all classes were reduced to 16 students so that class-average achievement did not differ between classes. The 14 teachers (50% female) were between 25 and 58 years of age (mean age = 46; SD = 10.4).

During the teaching unit, students were taught using modelling problems about "Pythagoras' theorem" and "linear functions". Half of the students (i.e., seven classes) were taught according to a teacher-centred ("directive") form of teaching and learning. This form of teaching and learning was operationalized as a highly teacher-regulated method comprising two components, namely direct instruction by the teacher and individual work by the students during regular classroom time. The most important guiding principles for "directive" teaching were the following (cf. Blum, in press):

- Development of common solution patterns by the teacher.
- Systematic change between whole-class teaching, oriented towards a ffictitious "average student", and students' individual work in exercises.

The modelling tasks were usually solved in the following way. First, one of the students read the task aloud to the class. Then the teacher developed, in a dialogue with some students, ideas on how to solve it. The teacher and the students developed a solution together, and the teacher wrote it down on the blackboard. After that the students solved a similar task on their own. The teacher supported each student individually while solving this task.

The other half of the students worked on the same modelling problems in the same order as the teacher-centred group, but they were taught by using a student-centered "operative-strategic" form of teaching and learning. Most of the time, students worked independently in groups of four according to a fixed cooperation script (for detailed information see Schukajlow et al., 2009). Group work was supported by the teacher with the help of strategy-oriented interventions and followed by reflection phases in the whole class while the solutions were presented. Thus, the essential guiding principles for this teaching unit were the following (cf. Blum, in press):

- Teaching aiming at students' active and independent knowledge construction (realising a permanent balance between teacher's guidance and students' independence). Teachers first use strategic interventions (e.g. "read the task again", "draw a sketch") before giving direct hints to the students if necessary ("It's a right-angled triangle in your sketch.").
- Systematic change between independent work in groups (scaffolded by the teacher) and whole-class activities (especially for comparison of different solutions and retrospective reflections).
- Group work consisting of three phases: (1) individual work (reading the text and getting a first idea of how to solve the problem), (2) cooperative work (exchanging the ideas with other students in the group), (3) individual work (writing down an individual solution)

The two topics "linear functions" and "Pythagoras' theorem" had already been treated with intra-mathematical tasks before the DISUM teaching unit.
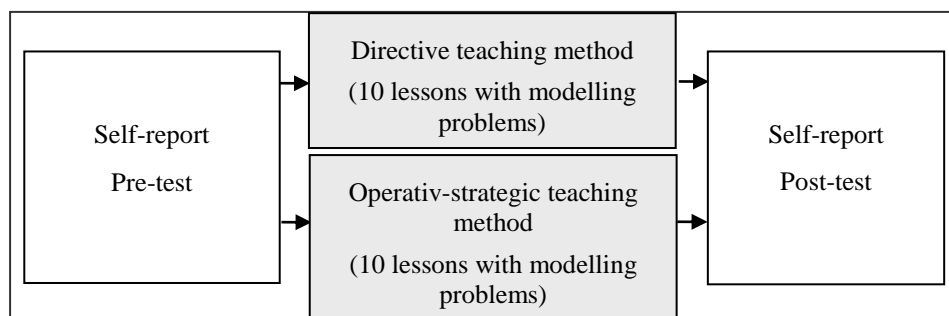

Figure 1: *Overview of the study design*

The teacher's main task in the "directive" form of teaching and learning was to mediate a unified and clear solution structure for every modelling problem. In the student-oriented treatment, the teachers were to intervene in such a way that the students' independence was optimally preserved. Prior to the intervention, all teachers received two-day training session in the teaching method they were supposed to practice during the intervention, and they were given a detailed written script for all lessons, including the solutions of all problems. In each lesson, at least one person from the research group was present in order to observe the implementation of the treatment. A treatment control using questionnaires, observations and video analyses showed that the teachers applied the two types of instruction very accurately.

3.2 Measures

The three types of problems constructed for the topics "Pythagoras' theorem" and "linear functions" differ in their degree of relation to reality. A new self-report instrument was developed

to measure students' affect about these three types of problems. First, five modelling problems, four "dressed up" word problems and four intra-mathematical problems are given. Following each of the problems, a number of self-report items are presented. These items ask respondents to report about their enjoyment, value, interest and self-efficacy expectation related to the problem. For the pre- and post-test assessment, the same problems and scales are used, in order to make it possible to analyse change in student's affective constructs.

3.2.1 Sample problems

Four word and four intra-mathematical problems (half of them for the topics "Pythagoras' theorem" and "linear functions", respectively) as well as five modelling problems (two problems for "Pythagoras' theorem" and three problems for "linear functions") were constructed. In what follows, we present one sample problem for each of the three problem types (topic: "Pythagoras' theorem"). One modelling problem for the topic "linear functions" is presented in the appendix.

Playground
The Traudt family is vacationing on a farm near Schwandorf in the Bavarian Forest. There is a playground there for the children, Lina and Maria. The playground's greatest attraction is the rope slide. The steel cable of the rope slide is stretched horizontally between two posts 10 meters apart. When Lina reached the middle, the steel cable stretched so much because of her weight that it dropped by 45 cm.

How long is the steel cable now?
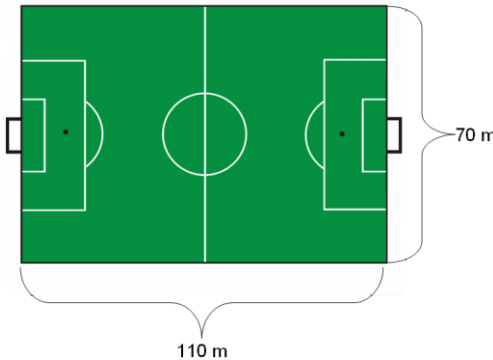
Figure 2: Modelling problem "Playground"

The playground problem is classified as a modelling problem because it calls for use of all of the modelling steps in order to solve it. An individual mental model of the described situation has to be constructed when the problem is read and the picture is viewed. This model includes various facts – in part irrelevant for the solution, for example, the geographical location of the playground. To build the real model, all irrelevant facts have to be sorted out from the situation model. The problem solver imagines an idealized cable ten meters long tightly stretched and then one that is hanging down 45 cm in the middle. This image represents the real model that has to be mathematised in the next step. The parts of the cable are seen mathematically as sides of an isosceles triangle. Using Pythagoras' theorem the length of the two sides is calculated and afterwards interpreted as the length of the entire hanging cable. Finally the result is validated and documented. For example, the question may be asked whether the cable is really vertically stretched so tightly and what effect deviations in this assumption would have on the result. This may stimulate some new modelling steps.
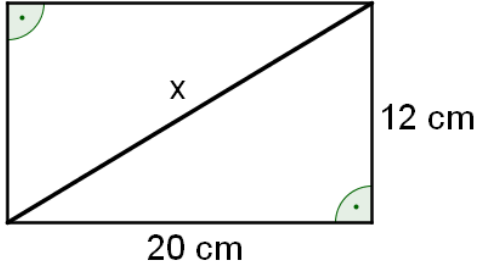
Figure 3: "Dressed up" word problem "Football Pitch"

The problem "football pitch" is classified as a "dressed up" word problem since the situation is so precisely pre-structured and simplified that the second step of the modelling cycle is almost completely skipped. By recognizing a rectangle in the football pitch shown, the situation can be immediately translated into a mathematical model. The diagonal is calculated by using Pythagoras' theorem and the result is interpreted and documented.

Figure 4: Intra-mathematical problem "Length x"

The solution of the intra-mathematical problem "Length x" requires the same mathematical activities as the problem "football pitch". The unknown length x is calculated by using Pythagoras' theorem.

3.2.2 Affect scales

In the questionnaire, each of the 13 problems was followed by four statements about students' enjoyment, value, interest and self-efficacy related to the problem. The instructions were: "Read each problem carefully and then answer some questions. **You do not have to solve the problems!**" After each problem, the students were asked to what extent they agreed or disagreed with each of four statements (enjoyment: "I would enjoy solving the problem shown"; value: "I think it is important to be able to solve this problem"; interest: "It would be interesting to work on this problem"; self-efficacy expectation: "I am confident I can solve the problem shown"; see Appendix), and were asked to use a 5-point Likert scale to record their answers (1 = *not at all true*, 5 = *completely true*). Although the four statements used represent main features of each construct, they do, of course, not completely cover all dimensions of each of the tested affective dimensions. The scores for the 52 statements were summed up to form 12 scales measuring the four constructs (enjoyment, value, interest and self-efficacy) for each of the three problem types (modelling, word and intra-mathematical problems; see Table 1). Given that there were five modelling problems, four "dressed up" word problems and four intra-mathematical problems, each of the four scales for affect towards modelling problems contained five items, and each of the eight scales for affect towards word and intra-mathematical problems contained four items. Scale reliabilities proved to be good or very good.

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Table 1: Reliabilities of the enjoyment, value, interest and self-efficacy scales

		Reliability (Cronbach's Alpha)		
		Modelling Problems	Word Problems	Intra-Math. Problems
Enjoyment	Pre-test	.90	.83	.83
	Post-test	.90	.87	.85
Value	Pre-test	.90	.89	.88
	Post-test	.88	.88	.86
Interest	Pre-test	.90	.85	.86
	Post-test	.90	.88	.87
Self-efficacy	Pre-test	.85	.87	.74
	Post-test	.86	.82	.78

As the modelling problems and the "dressed up" word problems are obviously similar in terms of their connection to reality, the number of words etc., differences between affect toward modelling problems versus word problems can likely only be recognized when the modelling problems are actually solved. Therefore, it is to be expected that the relationship between students' affect towards modelling problems and "dressed up" word problems tends to be stronger than the relationship between their affect toward modelling problems and intra-mathematical problems. This assumption is confirmed for all affective constructs tested in the present study (cf. Table 3). This finding also contributes to validating the instrument.

4 Results and Discussion

4.1 Data analysis

We used parametric tests (T-Tests, ANOVAs and MANOVA) to answer our research questions. There are several well-known assumptions when using these parametric tests. Some of these assumptions can be tested statistically (normal distribution of measures and homogeneity of variances). Levene´s tests showed that there was homogeneity of variances for all measures, so the parametric test could be applied in our case. Kolomogorow-Smirnow tests showed that the scores for some measures were not normally distributed. However, there were no outliers in the distributions, implying that these parametric tests could still be used (cf. Rasch & Guiar, 2004). The effect sizes which we will report in this paper do not depend on the distribution or the sample size and provide direct evidence on the practical significance of the results.

4.2 Preliminary analysis

Means, standard deviations and observed range of the scale scores as well as effect sizes (Cohen's d) and T values for the differences between pre- and post-test measures (df) are shown in Table 1. (Comment: Do you mean Table 2?) For all scales, the possible range of scores was 4.

Table 2: *Students' enjoyment, value, interest and self-efficacy expectations at pre-test and post-test*

		Pre-test	**Post-test**	**Observed**	**Cohen's** *d*	**T (df=209)**
		M (SD)	M (SD)	**Range**		
Enjoyment	ma**	2.82 (1.07)	3.10 (1.07)	4	0.26*	5.02
	w**	2.83 (1.07)	3.18 (1.07)	4	0.33*	6.46
	mod**	2.84 (1.06)	3.19 (1.08)	4	0.33*	5.98
Value	ma	3.51 (1.07)	3.63 (.95)	4	0.12*	2.06
	w	3.53 (1.08)	3.61 (.97)	4	0.08	1.23
	mod	3.53 (1.07)	3.64 (.95)	4	0.11	1.84
Interest	ma	2.96 (1.02)	3.17 (.98)	4	0.21*	3.94
	w	2.95 (1.01)	3.20 (1.00)	4	0.25*	4.51
	mod	3.02 (.99)	3.19 (.96)	4	0.17*	3.10
Self-efficacy	ma	3.37 (.84)	3.48 (.77)	4	0.14*	2.16
	w	3.29 (.85)	3.47 (.80)	4	0.22*	3.63
	mod	3.39 (.83)	3.52 (.80)	3,8	0.16*	2.52

*Notes. *p < .05. **Ma – intra-mathematical problems, w – "dressed up" word problems, mod – modelling problems.*

Correlations among the pre-test scales are shown in Table 3. As can be seen from the table, there is a high correlation across problem types within each affective dimension.

Table 3: *Pearson correlations among enjoyment, value, interest and self-efficacy expectations in the pre- and post-test.*

		Enjoyment			Value			Interest			Self-efficacy		
		ma*	w*	mod*	Ma	W	mod	ma	w	mod	ma	w	mod
Enj	ma	1	.84	.78	.44	.38	.37	.84	.76	.64	.56	.44	.34
	w	.76	1	.85	.42	.39	.39	.75	.80	.68	.47	.49	.39
	mod	.69	.84	1	.44	.42	.49	.71	.76	.80	.39	.38	.44
Val	ma	.55	.44	.35	1	.86	.84	.54	.50	.51	.41	.39	.30
	w	.41	.48	.38	.88	1	.87	.46	.49	.50	.38	.41	.31
	mod	.38	.46	.48	.82	.88	1	.45	.47	.58	.38	.38	.42
Int	ma	.78	.67	.56	.61	.50	.48	1	.89	.80	.50	.36	.30
	w	.62	.76	.65	.52	.58	.55	.83	1	.86	.42	.44	.32
	mod	.56	.69	.77	.44	.50	.59	.75	.86	1	.39	.38	.40
Self-eff.	ma	.54	.39	.31	.46	.35	.33	.41	.29	.22	1	.82	.72
	w	.33	.46	.34	.30	.36	.34	.24	.30	.23	.76	1	.78
	mod	.23	.34	.45	.24	.29	.38	.15	.23	.33	.63	.77	1

*Notes. * ma – intra-mathematical problems, w – word problems, mod – modelling problems. All correlation are significant (p<.05). The pre-test correlations are listed under and the post-test correlations over diagonal.*

4.3 Type of task and students' affect

The first question is whether students' task-specific affect differs for problems with and without references to reality. When arithmetic means for pre-test enjoyment and interest are compared across the three types of problems, it can clearly be seen that there were almost no differences among students' enjoyment, value, interest and self-efficacy towards intra-mathematical problems, word problems and modelling problems (cf. Table 2). None of these differences was significant. Consequently, teachers cannot assume that it is sufficient to simply select reality-related problems for triggering students' positive emotions and interest. Value, that is the importance of the ability to solve a problem, did not differ as a function of problem type. This suggests that it is of similar importance for students to be able to solve modelling as compared to other types of problems. The only significant difference was found for self-efficacy expectations related to word problems versus modelling problems ($T[209] = 2.44$, $p = 0.015$). Students were more convinced of being able to solve modelling problems than word problems. In view of the complexity of the modelling problems (see section 2.3), this finding is surprising and may have been due to students' lack of experience with modelling problems. This finding means that students tend to overestimate their ability to solve modelling problems and adjust their self-efficacy expectations after experience with these kinds of tasks (cf. similar findings by Panaoura, Demetriou & Gagatsis, 2009).

4.4 Effects of classroom instruction with modelling tasks on students' affect

The second research question pertains to the effect of treating modelling problems on students' task-specific affect. Can it be assumed that a short, ten-lesson period may change students' enjoyment, value, interest, and self-efficacyformed over several school years? Will addressing intra-mathematical problems and word problems for just three weeks have any effect? For example, are students more interested in the "new" modelling problems, and less interested in intra-mathematical and word problems, after the teaching unit? A comparison of post- and pre-test means shows that treatment of modelling problems did have positive effects on students' affect with regard to *all three* types of problems. Students' enjoyment, interest and self-efficacy expectations increased significantly. Value was the only variable for which the pattern of findings was mixed, with insignificant differences for word and modelling problems and an increase for intra-mathematical problems. The effect size of the differences varied between weak ($d = 0.20$) and medium ($d > 0.50$; (Cohen, 1992)).

However, one limitation is that the study design did not include a control group (classroom instruction not using modelling problems), implying that it is not possible to exclude testing effects in interpreting these findings. Also, one possible interpretation is that the carefully designed structure of the teaching units produced the positive changes found, irrespective of the type of problem addressed in the unit. Teaching with modelling problems implemented in the DISUM Project as the reason for the observed positive effects has to be seen as a whole. Furthermore, the positive changes in affect for all problem types could be explained by the fact that the teaching

unit and test addressed problems in the same mathematical content areas (Pythagoras' theorem and linear functions). The extent to which attitude change is transferred to other content areas is still an open question.

4.5 Differential effects of teacher-centred versus student-centred teaching on students' affect

The third research question asks if there were any differences between the effects of teacher-centred ("directive") versus student-centred ("operative-strategic") teaching. As documented in the last section, teaching of modelling problems caused positive changes in students' affect. Yet it could well be that the observed effects were caused by just one of the two kinds of treatment.

In the "operative-strategic" form of teaching, a clear increase of enjoyment and interest was observed (Table 4). Furthermore, self-efficacy scores for word problems and modelling problems increased significantly. Only the perceived value of solving problems remained unchanged. When differences between students' affect concerning problems with and without reference to reality are compared in the "operative-strategic" form of teaching, it is evident that the means for enjoyment and interest in intra-mathematical problems tended to increase less than in the other types of problems. A similar tendency can be seen for self-efficacy. As noted earlier, the smaller increase of self-efficacy for modelling problems, as compared to efficacy for word problems, may be explained by students' lack of prior experience with modelling problems. Students may, at first, overestimate their abilities to solve modelling problem (see also Panaoura, Gagatsis, & Demetriou, 2009) and need sufficient experience in solving such problems to adjust their self-efficacy judgments.

Table 4: Students' enjoyment, value, interest and self-efficacy expectations in the "operative-strategic" and "directive" forms of teaching

		"Operative-strategic"				"Directive"			
		Pre	Post			Pre	Post		
		M(SD)	M (SD)	T $(df=105)$	d	M (SD)	M (SD)	T $(df=104)$	d
Enj	ma	2.89(1.06)	3.23(1.01)	4.17	0.33*	2.74(1.07)	2.97(1.12)	2.91	0.20*
	w	2.92(.99)	3.33(.99)	5.16	0.41*	2.73(1.13)	3.03(1.12)	3.94	0.27*
	mod	2.85(1.06)	3.33(1.02)	5.79	0.46*	2.83(1.07)	3.05(1.13)	2.71	0.20*
Val	ma	3.61(1.01)	3.66(.98)	0.58	0.05	3.43(1.13)	3.62(.93)	2.30	0.18*
	w	3.58(1.04)	3.65(1.00)	0.96	0.07	3.49(1.12)	3.56(.95)	0.87	0.07
	mod	3.56(1.08)	3.62(.99)	0.80	0.06	3.51(1.06)	3.66(.92)	1.80	0.15
Int	ma	2.99(.98)	3.25(.95)	3.15	0.27*	2.93(1.07)	3.10(1.01)	2.37	0.16*
	w	2.97(.92)	3.29(.97)	4.15	0.34*	2.93(1.10)	3.11(1.02)	2.24	0.17*
	mod	2.97(.95)	3.29(.91)	3.90	0.34*	3.06(1.04)	3.10(1.01)	0.45	0.04
Self-eff	ma	3.39(.84)	3.53(.81)	1.86	0.17	3.35(.83)	3.43(.73)	1.15	0.10
	w	3.36(.79)	3.61(.84)	3.56	0.31*	3.22(.90)	3.34(.76)	1.63	0.15
	mod	3.43(.81)	3.60(.79)	2.27	0.21*	3.34(.86)	3.43(.80)	1.25	0.11

*Notes. * the effects are at least significant at the 5 % level. Enj, Int, Val, S.-eff. exp, Pre, Post and d. are abbreviations for enjoyment, interest, value, self-efficacy expectations, pre-test, post-test and the effect strength of Cohen.*

Clearly, the "directive" teaching unit produced weaker positive changes in students' enjoyment and interest. The effect sizes are about 0.20 and can thus be seen as weak. Moreover, effects were weak for value and self-efficacy as well, with most of the differences not being significant. For the "directive" instruction, there were no differences in students' affect towards different problems. The enjoyment scores showed a similar increase for intra-mathematical and modelling problems, and a slightly stronger increase for word problems. Finally, there is a comparable increase in students' interest in intra-mathematical problems and word problems, and the interest in modelling problems unexpectedly remained stable after the teacher-centred instruction.

For a direct comparison of the changes of students' affect in the two implemented forms of instruction, the difference of pre- versus post-test change of enjoyment and interest scores between the two treatments was checked for significance using repeated measures ANOVA. The analysis showed a significant influence of teaching method on students' *modelling problem-related* enjoyment and interest ($p<.05$; $F_{fr,m}(1,208)=4.87$, $(\eta_{fr,m})^2=0.023$; $F_{int,m}(1,208)=6.27$, $(\eta_{int,m})^2=0.029$). The operative-strategic teaching method for modelling problems thus tends to bring more advantages with regard to students' affect toward this type of problem. The affect toward other types of problems are in part also promoted.

This result is consistent with other studies showing that effects were stronger when the measuring instruments were more closely related to the intervention (cf. the meta-analyses by Hattie, et al., 1996; Seidel & Shavelson, 2007). On the other hand, related constructs (see, for example, studies about self-concept by Fennema, 1989) may profit from the intervention as well, as affect related to other types of problems did in the present study.

In order to test whether the factors "type of problem" (modelling, word or intra-mathematical problems), "type of intervention" ("operative-strategic", "directive") and "time of testing" (pre-test, post-test) and their interactions significantly influenced students' affect, additional repeated measures factorial ANOVAs were conducted. For enjoyment, there was a significant effect for time only ($p<0.001$, $F(1,208)=49$, $(\eta)^2=0.19$). The influence of the two other factors and their interactions with the variable "time" was not significant. For interest ("time": $p<0.001$, $F(1,208)=20$, $(\eta)^2=0.09$) and self-efficacy ("time": $p=0.002$, $F(1,208)=10$, $(\eta)^2=0.05$), a similar picture emerged. For value, the effect of the intervention was marginally significant only ("time": $p=0.051$, $F(1,208)=3$, $(\eta)^2=0.018$). Type of problem, type of intervention, and the interaction of these factors with time did not show any statistically significant effects for any of the affect variables.

5 Summary and Conclusion

In the study reported here, a new self-report instrument was developed to measure students' enjoyment, interest, value and self-efficacy expectations regarding problems with and without reference to reality. One specific characteristic of the scales is that they are directly anchored in mathematical problems, thus providing high sensitivity to change.

Using the instruments developed in this study, we found that students' enjoyment, value, interest and self-efficacy expectations are essentially identical for modelling problems, "dressed up" word problems and intra-mathematical problems. However, another study comparing enjoyment related to intra-mathematical problems versus word problems (Frenzel, et al., 2006; Pekrun, et al., 2007) found that students enjoyed word problems more than intra-mathematical problems. One practical implication could be that students do not automatically show more interest if a modelling problem is presented instead of an intra-mathematical or a word problem. Task-specific affect of students may, however, change in different ways if different problem types are actually solved and discussed.

The specific teaching unit with modelling problems developed in DISUM had positive effects on students' enjoyment, interest, and self-efficacy expectations for all three types of problems addressed. This finding is in line with some other studies. In contrast, the perceived value of solving such problems was not affected by the teaching unit. For changing perceived value, an intervention program specifically tailored to impact value would likely be necessary. Other studies indicate that positive changes in task values can improve affect and academic achievements in mathematics in self-regulated learning environments (e.g. Metallidou & Vlachou, 2010).

Further, two alternative forms of teaching with modelling problems were compared with each other. The data analyses show that the student-oriented, "operative-strategic" form of teaching tended to have stronger effects on students' enjoyment, value, interest and self-efficacy, as compared with "directive" teaching. These results correspond to findings of previous studies that investigated student-oriented and teacher-oriented forms of teaching and learning. Cooperative learning environments, like the "operative-strategic" one applied in this study, have often had a positive influence on students' affect (Slavin, et al., 2003; Webb & Palincsar, 1996). Significant advantages of the "operative-strategic" teaching method were found especially regarding interest and enjoyment in modelling problems. It seems that enjoyment and interest really profit from the divergent solution structure of modelling problems. So student-centred teaching methods seem to be better suited to improve both students' achievements (Schukajlow et al, 2009) and their affect while dealing with modelling problems.

One limitation of using task-specific questionnaires is the choice of rather specific topics and problems and the use of only one statement summarized across four or five problems within scales. In our study we have chosen two typical mathematical topics for 15-year-olds, "Pythagoras' theorem" and "linear functions". Task-specific affect regarding modelling, word and intra-mathematical problems ought to be investigated for other mathematical topics and for other age groups as well. The limitations resulting from using one statement only to measure the complex affect constructs are also an important research question. Future research needs to focus also on common features and differences of various methods of measurement, to clarify what parts of affect constructs can be adequately measured with what instrument. Another limitation is the way the questionnaire was applied. Students were asked to answer the questions without actually

solving the problems. That is why their perceptions are based only on the first impression of a problem. Moreover the students did not get any information about differences or common features of the three kinds of problems when they were asked about their perceptions. The responses may change if students are asked to solve the problems first and answer the questions about their affect afterwards or if information about differences in modelling, word and intra-mathematical problems are a part of the teaching unit. The investigation of differences between these approaches is an interesting research question, because it can provide evidence on a change of task-specific affect while working on different problem types.

Another limitation of this study is the use of problems from only two mathematical content areas for the measurement of affect, so a potential transfer to problems from other mathematical content areas is an open issue. Also, we do not know how sustainable such changes are and how easy they can be adopted in everyday teaching practice. It likely depends on the frequency with which teachers apply such cooperation scripts and use modelling problems in their lessons. Observing teaching practices in the classroom for a longer period after the intervention combined with a measurement of affect a few months later would be an important task for future research. Another interesting question is how effective an "operative-strategic" teaching style can be when applied to other problem types. Further research studies are needed to answer all of these questions.

Finally, one open field of research is the comparison of traditional instruments assessing affect in mathematics with these new task-related scales. We must also clarify how such specific task-related emotions, attitudes and beliefs are related to each other and to more global constructs of affect, whether such task-specific emotions, attitudes and beliefs explain more variance in students' performance than traditional instruments, and whether they mediate relationships between motivational traits and performance (Seegers & Boekaerts, 1993). A validation of the quantitative questionnaire instruments concerned with cognitive validation methods appears desirable in this connection.

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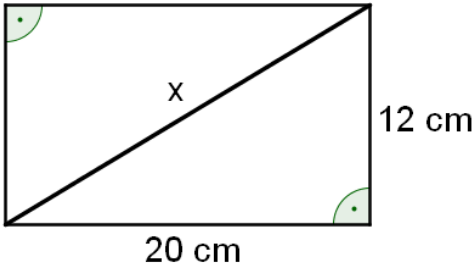
Appendix

Modelling problem (topic "linear functions")

Apples

Ms. Meier would like to cook an apple purèe on the Weekend

She can buy fresh apples either in a supermarket besides of her flat for 0.90 € each 500 g Box or drive a car 5 km to the next apple plantation. On the plantage she can pick the low-prized apples (see promotional poster on the right side)

Is it worthwhile for Ms. Meier to drive to apple plantation?
Give reasons for your answer.

Apples to pick up!
Right now, daily 8.00-18.00,
On Sunday 9.00-13.00
By farmer Newman in Borken
t. 01777-34328998

1kg   1,50 €

You drive on the autobahn A 49,
departure Edersee in direction of Borken

A part of test-book

Task 1

Length x

Calculate length x.

x

12 cm

20 cm

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	not true at all	is hardly true	is true in part	is true for the most part	is completely true
1. I would enjoy solving this problem ……………..……	O	O	O	O	O
2. I think it is important to be able to solve this problem ....…	O	O	O	O	O
3. It would be interesting to work on this problem………....…	O	O	O	O	O
4. I am confident I can solve the problem shown………………	O	O	O	O	O