Strategies for fostering students' reading comprehension while they solve modelling problems

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A profound level of reading comprehension is essential for solving modelling problems, as a problem solver has to understand the real-life situation presented in the task in order to construct an adequate situation model, real model, and mathematical model. The aim of this paper is to present the theoretical grounds and a sample learning environment for fostering reading comprehension. In the first part of this article, we summarise research on reading comprehension while solving modelling problems, report on research on two strategies (text highlighting and self-generated drawing) that can help learners comprehend a modelling problem, and describe ways to implement both strategies in the classroom. In the second part, we present a learning environment that can be implemented to foster reading comprehension in the lower secondary classroom.

Keywords: Modelling, strategies, reading comprehension.

Introduction

Reading comprehension is important for learning in school and for students' everyday lives. As each school subject offers students different kinds of texts, fostering subject-specific reading comprehension strategies is necessary for learning and problem solving. In math classes, students have to read different kinds of texts such as proofs or word problems. We address word problems, which, in order to be solved, require a demanding transition from the real world to mathematics as *modelling problems* (Blum, Galbraith, Henn, & Niss, 2007). To solve a modelling problem, students need a profound understanding of the task, as a superficial combination of the numbers given in the task is not sufficient for finding a solution to a modelling problem. In this paper, we characterise the role of mathematical reading competency for the solution of modelling problems, summarise research on two strategies (highlighting and self-generated drawings) that might improve reading comprehension, and present a learning environment for fostering these strategies in the classroom.

Theoretical background for reading comprehension and modelling

Reading comprehension while solving modelling problems

Analyses of the cognitive processes that people engage in during mathematical modelling have often distinguished the following activities in the solution process: (1) The problem solver constructs a *situation model* on the basis of the information presented in the task and on his or her prior knowledge. The situation model reflects the learner's mental representation of the given situation. A profound level of reading comprehension is essential for constructing the situation model if a problem is partly or completely represented as text. (2) The situation model is then simplified and structured to obtain a *real model* that contains only the information necessary to solve the task. The problem solver needs a deep understanding of the problem to separate relevant from redundant information. (3) Mathematising the real model leads to the *mathematical model*. (4) Working mathematically then serves to produce a *mathematical result*, which is then used to refer back to the real situation given in the task by interpreting and validating the *real result*. In the following, we concentrate on understanding and simplifying/structuring, for which reading comprehension is critically important.

Understanding and simplifying/structuring are expected to be essential for solving the entire modelling problem. These expectations were supported by the results of the study by Leiss et al. (2010). In this study, the strategies that students used in the construction of an adequate situation model and real model were found to have a positive influence on their modelling competency. Therefore, we conclude that it is important to support learners' reading comprehension by the use of strategies. Two strategies suitable for fostering reading comprehension in modelling problems are presented in the following chapter.

Highlighting and self-generated drawings for reading comprehension

Highlighting. Highlighting is a cognitive strategy that aims at directing the learner's attention to the specific words or sentences in a text. To highlight important information, the learner has to identify and select the relevant information.

The prompt "Use the highlighting strategy" is not sufficient by itself to support the learner's reading comprehension, as the quality of strategy use has been found to be important for effects on students' performance. The use of highlighting has to be regulated by students in order to be efficient, otherwise the learner might highlight too many words in the text or might forget to highlight some important information. In a study of college students, Leutner et al. (2007) examined the effects of training the highlighting strategy with expository texts. They compared three groups: The first group did not receive any training and served as a control group, the second was given a strategy training in highlighting, and the third was given a combination of strategy and self-regulation training. The findings revealed that the third group outperformed the others in text comprehension. Thus, self-regulation seems to be an important factor for strengthening the appropriate and purposeful use of cognitive strategies such as highlighting. The combination of strategy and self-regulation training included the following elements: First, the learning goal was presented. Second, the participants observed how a fictive person applied the highlighting strategy. Third, they were given a strategy training in which the steps of the highlighting strategy were presented and then applied. Fourth, a self-regulation training followed, in which the steps of the highlighting strategy were recalled, and then the steps of the metacognitive strategy (monitoring, self-evaluation, and reaction) were introduced and applied.

Highlighting strategies can also be expected to be useful in the domain of mathematical modelling, as modelling problems with reference to real life often contain redundant information. In order to construct a situation model and a real model, students have to identify relevant information. In an exploratory case study by Leiss et al. (2010), some difficulties in the use of the highlighting strategy while solving modelling problems were observed. An analysis of students' solutions showed that some students highlighted all numbers written in numeric form given in the task, including numbers that were not needed to solve the modelling problem. Further, some students did not highlight numbers written in word form, even when these numbers were essential for the solution. These observations demonstrate the limited quality of the highlighting strategy by students and emphasise the importance of improving the quality of the use of this strategy.

Self-generated drawings. Another important cognitive strategy that can be applied to support a learner's reading comprehension and problem solving is the creation of self-generated drawings. Whereas highlighting is aimed at selecting the most important information, drawings are aimed at organising and visually representing information given in the text.

A study by Leopold and Leutner (2012) revealed the advantages of drawing activities for the comprehension of science texts. Students in grade 10 were instructed to read text paragraphs and then make a drawing that represented the main ideas of the paragraphs. To train the students to use the strategy, they worked on an example that demonstrated how to process the text with a related drawing. The results showed positive effects of the drawing instructions on students' science text comprehension. Drawing activities encourage students to construct a mental model and seem to offer a useful strategy for facilitating students' deeper understanding. In the domain of mathematics, positive effects of drawing activities were found on 3rd-grade students' word problem solutions (Csíkos, Szitányi, & Kelemen, 2012).

Drawing activities might also support the construction of a situation model in the context of mathematical modelling (Rellensmann, Schukajlow, & Leopold, 2017). Strategic knowledge about drawing was found to have a positive effect on modelling performance. This effect was mediated by the accuracy of the *situational* and *mathematical drawings* and emphasised the importance of the quality of the strategy for solving modelling problems. In addition, the study revealed that the accuracy of mathematical drawings is a strong predictor of modelling performance, whereas the situational drawing had only indirect influences on performance by facilitating the construction of a mathematical drawing. These findings suggest that self-generated drawings offer a strategy that is useful for fostering modelling. The most promising was found to be the generation of accurate mathematical drawings. Thus, when instructing students how to generate a drawing, teachers should pay special attention to the accuracy of the mathematical drawing (namely that it contains correct relations and all relevant numbers). Learners should be encouraged to generate a situational drawing if they do not succeed in drawing a more abstract mathematical model in their first attempt.

Even though highlighting and drawing seem to be useful strategies for fostering reading comprehension during mathematical modelling, they need to be taught in rich learning environments. In the following chapter, we present some learning environments that are appropriate for teaching these strategies.

Highlighting and self-generated drawings in learning environments for improving modelling

Several studies have investigated the effects of different learning environments on students' modelling competency. In the following, we present two studies that integrated highlighting and drawing strategies (among other elements) in their learning environments to foster modelling competency.

Verschaffel et al. (1999) revealed the positive effects of a certain learning environment on 5th graders' modelling and problem-solving competency. The learning environment contained the acquisition of an overall metacognitive strategy that involved five stages in the planning of the whole solution process. Eight strategies (e.g., "Distinguish relevant from irrelevant data" or "Draw a picture") were embedded in the first two stages (Verschaffel et al., 1999, p. 202). The distinction between relevant and irrelevant data is related to the highlighting strategy, as appropriate highlighting aims to make this distinction. Another condition under which the results were acquired was the instructional technique used in this study. It consisted of systematic changes between whole-class discussions and small group work. In both phases, the teacher encouraged the use of strategies and encouraged the students to reflect on their purposeful use in order to stimulate the regulation of strategy use.

A learning environment for modelling that included strategic elements was examined by Schukajlow et al. (2015). A scaffolding instrument called the *solution plan* with four steps was used in this study to support students' modelling activities. Strategic prompts were assigned to each step. As a whole, the solution plan served as a metacognitive planning strategy that was designed to guide students through the process of solving a modelling problem. Fostering reading comprehension was not the sole focus of the solution plan, but it included cognitive strategies that were aimed at improving reading comprehension (e.g., strategies such as "Look for the data you need and, if necessary, make assumptions!" or the strategy "Make a sketch!"; Schukajlow et al., 2015, p. 1244). Although the highlighting strategy was not explicitly mentioned in the solution plan, it is closely connected to the strategy of looking for relevant data.

The student-centred *operative-strategic* learning environment used in this study is characterised by a systematic change between individual work in groups and whole-class discussions. The whole-class discussions included presenting solutions and reflecting on the solution processes (Schukajlow et al., 2015, p. 1243). The study found that an experimental group that was taught the solution plan outperformed a control group that was not taught the solution plan in solving modelling problems. Furthermore, students in the experimental group reported more frequently using self-reported strategies than the control group.

On the basis of the theoretical and empirical findings on the effects of highlighting and self-generated drawing, we developed a learning environment for fostering reading comprehension. We describe this learning environment which will be approved in the next step of the project in the following section.

Learning environment for fostering students' reading comprehension while they solve modelling problems

Based on the theoretical grounds presented in the first part of this paper, the following learning environment was developed to foster 9th graders' modelling competence with special regard to the beginning of the modelling process, namely understanding, structuring, and simplifying. The aim of the learning environment is to improve students' performance in these sub-competencies by fostering their reading comprehension via trainings in highlighting and the use of self-generated drawings. Similar to some other studies that implemented strategy trainings (see e.g., Leutner et al., 2007, or Schukajlow et al., 2015), the duration of the teaching unit will be five lessons with a total of approximately 225 minutes[1].

The modelling problems that will be used in the present learning environment include text and can be solved by applying the Pythagorean Theorem as a mathematical procedure. The Pythagorean Theorem was chosen because of the importance of this mathematical procedure for national and international curricula. Before the beginning of the teaching unit that was designed to foster reading comprehension, students are expected to know the Pythagorean Theorem and to practise it on intra-mathematical problems. A sample problem *Reaction time* is shown in Figure 1.

Reaction time

During the 2016 European Championship, Germany played against Slovakia in the round of the last sixteen. With goals by Boateng (minute 8), Gomez (minute 43), and Draxler (minute 63) the German team won with a score of 3 to 0.

In the 14th minute, Germany was allowed a penalty kick after a foul by Slovakia. A penalty kick is shot from a distance of eleven metres from the goal, which has standard measures of 2.44 m in height and 7.32 m in width. The German penalty taker was Mesut Özil, and Matus Kozacik was in the Slovakian goal.

Unfortunately, the penalty kick was stopped by Kozacik so that Özil missed the chance to have an early score of 2 to 0 for Germany. His penalty kick was shot a bit too feebly and flew just over the ground to the lower right corner where Kozacik was able to deflect it away from the goal. Although the penalty kick was not shot very hard, the goal-keeper didn’t have much time to react, as the football flew at a speed of about 80 km/h towards the goal.

Calculate how much time the goal-keeper had after Özil’s kick to reach the position where he stopped the ball just before the corner of the goal.

Figure 1: Sample modelling problem *Reaction time*

In line with the solution plan study by Schukajlow et al. (2015) and the study by Verschaffel et al. (1999), the learning environment that we chose for our teaching unit includes systematically changing between individual work, group work, presenting solutions, and reflecting on the solution process as a class (Schukajlow et al., 2015, p. 1243).

In the first lesson, both of the strategies of highlighting and using self-generated drawings are introduced. The students are given the modelling problem *Reaction time* (cf. Figure 1) and are requested to highlight important information and to generate a drawing while doing their own individual work in groups, but they are asked not to solve the problem. The task requires the application of both the highlighting and drawing strategies.

[1] Schukajlow, Kolter, and Blum (2015) measured effects after 205 minutes of total treatment. Leutner, Leopold, and den Elzen-Rump (2007) used a time of 150 minutes. Verschaffel et al. (1999) used 20 lessons to teach eight strategies, so five lessons for teaching two strategies seemed appropriate for our teaching unit.

After the individual work in groups, students present their highlighted texts and their drawings and describe how they proceeded in applying both strategies. To encourage a discussion about potential difficulties in the use of these strategies, the group that the teacher chooses to make the first presentation should be one that had difficulties with the generation of the highlighting or drawing. During the presentation and the subsequent reflection on the presented solution, the teacher should direct students' attention to typical problems that result from the misapplication of strategies. The teacher should then present the learning goal of the teaching unit to the students, namely to improve reading comprehension and the ability to solve modelling problems.

After that, the first two steps of the solution scheme (cf. Figure 2) are introduced. They save the results of the class discussion in written form and might also provide some advice that was not mentioned by the students. The solution plan by Schukajlow et al. (2015) was adapted to better fit the aim of fostering reading comprehension and to guide the application of both of the strategies of highlighting and producing self-generated drawings. The solution scheme with the reading strategies of highlighting (integrated in step 1) and the creation of self-generated drawings (integrated in step 2) is shown in Figure 2.

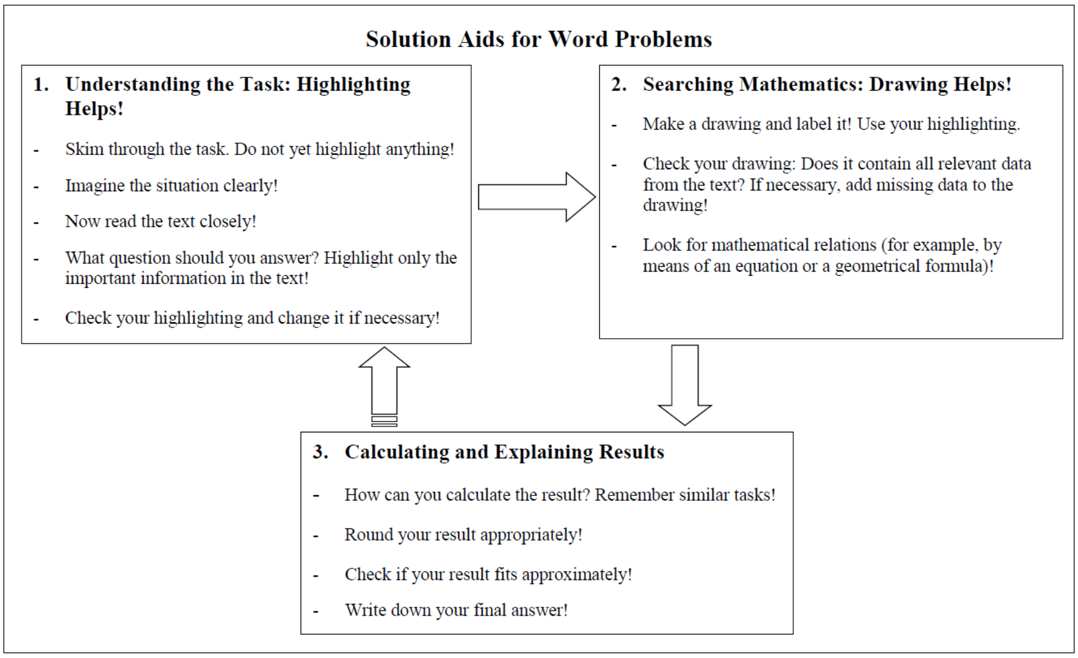

Figure 2: The solution scheme used in the learning environment

The entire solution scheme serves as a planning strategy for the whole solution process. In step 1, the highlighting strategy is presented. First, students are told to skim the task. This means they should get an overview of the task, which might contain a title, text, questions, pictures, tables, or diagrams. While skimming the text, students do not need to understand each word in detail. After skimming the text, the students are prompted to imagine the situation presented in the task. This might help to activate prior knowledge about the topic and facilitate reading the text in detail in the next step. These activities stimulate the *understanding* of the real situation. The highlighting is prompted after they read the question again, as the selection of relevant data depends on the question posed in the problem. After they finish the highlighting, students evaluate whether they highlighted only the most important information in the task and revise their highlighting if needed. The highlighting is aimed at helping students to *simplify* the given information.

In step 2 of the solution scheme, the students are asked to make a drawing and label it with relevant data from the text. These activities help them simplify and structure the information given in the text. During the following monitoring activities, students check whether their drawings contain all relevant data from the text and whether all mathematical relations are represented correctly. If students do not succeed in constructing an accurate mathematical drawing, they can first generate a less abstract situational drawing. At the end of step 2, students *mathematise* the information given in the drawing. In step 3, the students calculate a solution and obtain the mathematical result. They interpret, validate, and present the result in a final answer in step 3. The arrow pointing back towards step 1 indicates that the solution process might be restarted if the result does not fit.

In the first two lessons of the teaching unit, the students are familiarised with only the first two steps of the solution scheme in order to train their reading comprehension strategies and the sub-competencies of understanding, simplifying, structuring, and mathematising as part of the modelling process. In lessons 3, 4, and 5, students practise the entire modelling process by applying the entire solution scheme.

In line with Leutner et al.'s (2007) study, the current learning environment contains the same main elements to stimulate self-regulation. First, goal setting is realised in the first lesson. Students are confronted with a modelling problem that requires a profound level of reading comprehension. The teacher explains that the aim of the teaching unit is to learn strategies that support reading comprehension and to solve reality-related modelling problems. Second, instead of observing the application of the strategy by a fictive person, students analyse their classmates' highlighted texts and drawings in both the work done in small groups and the presentations involving the whole class. These practises are implemented in order to stimulate students' activities in the classroom. Third, the strategy training begins with a presentation of the steps that are necessary for highlighting and drawing and is followed by an application of the strategies while solving modelling problems. In contrast to Leutner et al.'s (2007) study, the self-regulation training is integrated in the strategy training. If requested, the teacher gives strategic advice by referring to the relevant steps of the solution scheme and stimulates reflection on the use of strategies during the individual work in groups and during the whole-class discussion when solutions are presented and reflected on. This process helps to encourage the use of strategies and to establish the solution scheme as a scaffold for solving modelling problems. In order to stimulate the self-regulation of strategy use, we included the prompts "Check your marks and change them if necessary" in step 1 and "Check your drawing" in step 2 of the solution scheme. Further, the validation of the results of solution problems is stimulated by the prompt "Check if your result fits approximately" in step 3 (cf. Figure 2).

Summary and future steps

In the first part of this paper, we discussed the theoretical background for reading comprehension and modelling. Based on the theory, we presented in the second part of the paper a learning environment to foster students' reading comprehension while solving modelling problems. This learning environment will be evaluated in a project for pre-service teachers. The pre-service teachers will obtain the material and instructions to implement the learning environment in their classrooms to gain practical experience in fostering reading comprehension in mathematical education. They will attend a seminar to prepare for the project and to reflect on the experiences they made while implementing the project.

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