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Pictures in Modelling Problems. Does Numerical Information Make a Difference?

Marcus Schmitz
Department of Mathematics
University of Münster
Apffelstaedtstr. 21
48149 Münster
schmitz.marcus@uni-muenster.de

Stanislaw Schukajlow
Department of Mathematics
University of Münster
Apffelstaedtstr. 21
48149 Münster
schukajlow@uni-muenster.de

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Abstract Pictures are an important part of everyday life, and they often accompany modelling tasks. However, we do not know much about the role of pictures in modelling. To address this research gap, we randomly assigned students to three groups. In the experimental groups, in addition to the text, the problems included useful or superfluous numerical information in pictures, whereas the pictures that went with the problems in the control group did not include any numerical information. We assessed the picture-specific utility value and modelling performance of 110 students in upper secondary school. The picture-specific utility value reflects the perceived usefulness of a picture for understanding the problem. As expected, students assigned a lower utility value to the pictures that contained additional superfluous numerical information. However, we did not find differences in the students' modelling performance.

Keywords: pictures, Cognitive Load Theory (CLT), text-picture comprehension, utility value.

We hereby certify that this chapter is not being submitted for consideration for publication elsewhere and is original, unpublished work:

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1 Introduction

Improving students' ability to solve real-world problems by using mathematics is an important goal of mathematics education; thus, modelling competence is part of school curricula all over the world (Niss et al. 2007). In order to strengthen the extent to which modelling problems are linked to the real world, modelling problems often include pictures. In addition, being able to deal with the combination of pictures and text is important for professional and everyday life. Despite the importance of pictures for modelling, there is a large deficit in research on the effects of pictures in modelling tasks and on the processing of tasks that include pictures.

Multimedia theories such as the integrated model of text and picture comprehension (Schnotz 2014) have suggested that text-picture design influences mental processing and learning effects. One prerequisite for supporting students' understanding of problems is that the students notice the usefulness of pictures that accompany the problems. Prior studies have yielded inconsistent results concerning whether students perceive pictures as useful while solving real-world problems (Böckmann and Schukajlow 2018; Dewolf et al. 2015). Moreover, we did not find any research on the effects of different types of numerical information in pictures on the usefulness of pictures or mathematical performance.

On the basis of these considerations, we aimed to gain more knowledge about the role of additional numerical information in pictures on modelling performance and the perceived usefulness of additional numerical information for understanding the task. When we refer to 'additional numerical information' in this study, we mean information contained in pictures that is also described in the text. For example, this additional information may refer to distances with the given length drawn in the picture.

2 Theoretical Framework

2.1 Pictures in modelling problems

At the core of mathematical modelling, there is a demanding process by which information must be translated between the real world and mathematics. There are several activities that are part of the solution process that are often described in a cycle that begins with the student's understanding of the real-world situation and ends with the validation of the results (e.g. Blum and Leiß 2007). More specifically, students need to construct a model of the situation that they will then simplify and idealize before constructing a mathematical model. At the end of the solution process, students need to interpret and validate their results.

In order to strengthen the extent to which modelling problems are linked to the real world, modelling tasks that are presented in the classroom should and often do include text and pictures. We assume that pictures can support certain modelling activities and thus influence students' modelling performance. For example, certain pictures can be particularly helpful for understanding and creating a model of the situation. The extraction of the necessary information from the text represents a potential barrier for students when they solve modelling tasks. Further, superfluous information in the text increases the difficulty of the task. Pictures can potentially help students organise information, simplify the situational model, and mathematise the information.

Pictures used in combination with text can serve different functions. Elia and Philippou (2004) developed a taxonomy of pictures for problem solving. There are *decorative* pictures that are irrelevant to the contents of the corresponding text. The picture does not refer to events or information in the text. Pictures with a *representational* function 'represent the whole or a part of the content of the problem' (Elia and Philippou 2004, p. 328). *Informational* pictures present information that is essential for solving the modelling problem. All pictures used in this study have a representational function. In our study, we used photos as the pictures because they are closely connected to reality. Such realistic pictures are two-dimensional simulations of objects from a specific perspective with a great deal of potential to support mental model construction (Schnotz and Cade 2014). Figure 1.1 shows an example of a modelling task used in our study.

Kite

Lucas got a new kite as a birthday present. The kite is 1 m in length and 50 cm in width. Lucas flies the kite with his friend Susan (see picture). They are standing at a distance of 80 m from each other. The kite's string has a length of 100 m. Susan is right under the kite and 20 m from the sea.

How high is the kite flying at this moment?

Fig. 1.1 Kite task with no additional numerical information in the picture

Comprehension of the *Kite* task results in a model of the situation that includes Lucas and Susan, a kite, a piece of string, and the positions of the two people and the kite. To calculate the desired height of the kite, students can use the Pythagorean theorem and add an estimate of Lucas' height. The picture in the task can help the problem-solver organise the information and construct a model of the situation. In the modelling process, the picture can be used as an easily accessible external representation of the situation.

2.2 Text and picture comprehension

Several studies have shown that students generally learn more deeply from text when it is combined with pictures than from text alone (Mayer 2009). Models such as the cognitive theory of multimedia learning (Mayer 2009) or the integrated model of text and picture comprehension (Schnotz 2014) describe this positive multimedia effect. They assume that a multimedia effect occurs only under certain conditions. One assumption is that the text and the picture can only be processed into a joint mental model if they are closely semantically connected. This conforms to the *coherence condition*. According to the *contiguity condition*, the text and the picture can only contribute to the construction of a joint mental model if they are presented closely together in space or time.

Furthermore, in multimedia theories, working memory plays a central role and determines to a large extent whether the multimedia presentation leads to a positive learning effect through the optimal use of working memory or whether it hinders learning through overloading. The different loads on working memory and the resulting effects are described in John Sweller's Cognitive Load Theory (CLT) (Sweller 1994), which distinguishes between the cognitive structures in long-term and working memory. Long-term memory can store large amounts of information in *schemas.* Schemas refer to cognitive structures that incorporate multiple elements into a single element with a specific function. Schemas can be retrieved from long-term memory into limited working memory in which all conscious cognitive processing occurs. Thus, working memory can perform complex cognitive activities despite its limited capacity by retrieving these schemas. CLT therefore represents learning as the process of acquiring schemas.

According to CLT, there are three types of cognitive load on working memory that occur during the processing of new and already stored information: intrinsic, extraneous, and germane cognitive load. *Intrinsic load* describes the load on working memory caused by the complexity and difficulty of the learning content. Intrinsic load is characterised by the number of interacting learning elements kept in working memory for processing. The amount of load depends on the learners' individual level of expertise since the number of processed elements depends on the schemas stored in long-term memory. Thus, all instruction has an inherent difficulty associated with it, and this inherent difficulty, which produces intrinsic load, cannot be altered by an instructor.

The manner in which learning material is designed can also produce cognitive load. When such load is unnecessary and thereby interferes with building schemas, it is referred to as *extraneous load*. Thus, extraneous cognitive load is generated by the manner in which information is presented to learners and is under the control of instructional design.

The third source of cognitive load is *germane* cognitive load. Whereas extraneous cognitive load interferes with learning, germane cognitive load enhances learning. So germane load is related to information and activities that foster processes of schema construction and automation. Thus, when pictures support modelling activities such as understanding or structuring, they produce germane cognitive load.

A central assumption of CLT is that the three types of cognitive load can be accumulated into the total cognitive load. If this total cognitive load exceeds the capacities of working memory, learning cannot occur (Sweller 1994). This hypothesis is only valid if the intrinsic load is sufficiently complex. A high intrinsic load combined with a high extraneous load can lead to an overload of working memory resources and prevent germane load. However, if the learning content (intrinsic load) is very low, an unfavorable design style (extraneous load) will not lead to an overload of working memory. These ideas must be considered when designing learning material and are therefore also important for the use of pictures in modelling tasks.

2.3 *Picture-specific utility value*

The expectancy-value theory links expectancies and personal values and describes utility (or extrinsic) value as one of four components of values that influence task performance, task choice, and motivation (Eccles 1983). A task's utility value refers to the importance of a task or its parts (e.g. pictures) for extrinsic indicators of success such as an accurate solution, grades, or career. In this study, we analysed utility value of pictures for understanding modelling problems. We investigated different types of pictures and the picture-specific utility value to determine whether the pictures facilitated students' understanding of the modelling problems and thus supported the solution process.

A positive relation between values and students' performance was confirmed for problems with and without a connection to reality (Schukajlow 2017). Further, students usually realise that decorative pictures are less helpful for understanding and solving than pictures with representative or essential functions (Böckmann and Schukajlow 2018). Otherwise, students often do not use information from representative pictures in p-problems (Dewolf et al. 2015) or essential pictures in arithmetic word problems (Elia and Philippou 2004) for their solution process.

The extent to which additional numerical information in pictures influences picture-specific utility value or modelling performance has not yet been investigated, and thus, we aimed to address this research gap in the present study.

2.4 *Research questions*

We conducted this study to address the following research questions:

(1) How do students rate the utility value of representative pictures that contain additional useful or superfluous numerical information?

(2) How does additional useful or superfluous numerical information in representative pictures affect students' modelling performance?

Prior research has shown that students rate the utility value of representative pictures higher than pictures with a decorative function. Thus, we expected that students would assign a higher utility value to pictures with additional useful numerical information and would assign a lower utility value to pictures with additional superfluous numerical information than to pictures with no additional numerical information.

The integrated model of text and picture comprehension (Schnotz 2014) describes the concept that the positive multimedia effect depends on the text-picture design. We expected that pictures with additional useful numerical information would result in higher modelling performance and pictures with additional superfluous numerical information would result in lower modelling performance compared with pictures without additional information.

3 Method

3.1 Design

110 students from five upper secondary schools in grades nine and ten (mean age = 15.26, SD = 0.89; 47.8% female) participated in the study. The students in each class were randomly assigned to one of three groups: a control group with no numerical information in the pictures (CG), an experimental group with additional useful numerical information in the pictures (EG-U), and an experimental group with additional superfluous numerical information in the pictures (EG-S). Students first estimated the picture's specific utility value for understanding problems that described six modelling tasks in a questionnaire. The instructions in the questionnaire were: 'Read each problem carefully and then answer some questions. **You do not have to solve the problems!**' Then students read each problem and answered the question about utility value. After completing the questionnaire, the students solved the tasks.

In the present study, we used six modelling problems on the topic of the Pythagorean theorem. The tasks were developed and tested in prior studies (Böckmann and Schukajlow 2018; Schukajlow 2017). Unlike in the prior studies, all tasks included representational pictures in all three groups. In this study, the pictures representing the tasks differed across the three groups in the additional numerical information given in them. In the experimental groups, in addition to the text, the problems included useful or superfluous numerical information in the pictures, whereas for the control group, the pictures that accompanied the problems did not include any numerical information. The pictures from a sample problem (i.e. the Kite task) with three different types of additional information are shown in Fig. 1.2.

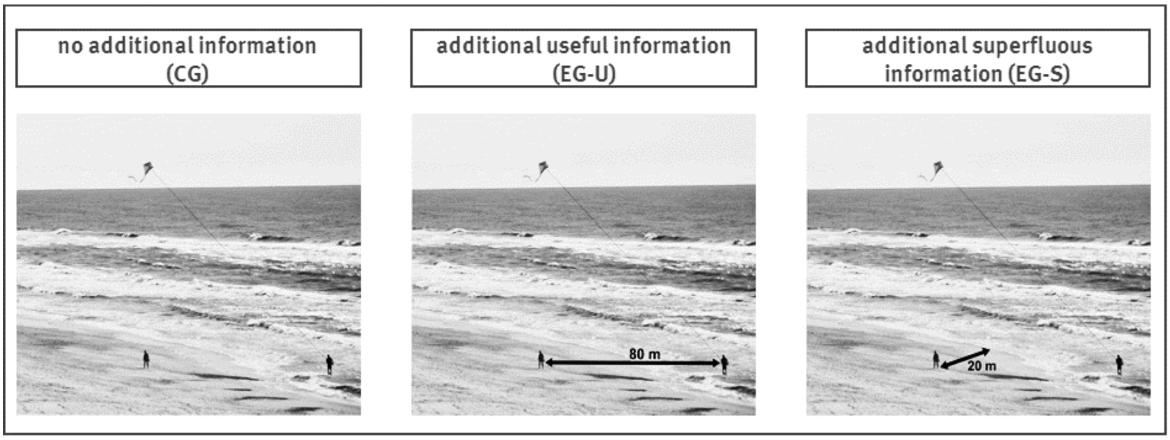

Fig. 1.2 Different pictures of the Kite task with no additional information (CG), additional useful information (EG-U), and additional superfluous information (EG-S)

3.2 Utility value

To measure the utility value of pictures with different kinds of additional numerical information for each modelling problem, we used the statement 'The picture helps me to understand the problem'. The students rated the item on a 5-point Likert scale (1=not at all true; 5=completely true). Cronbach's alpha as a measure of reliability for the picture-specific utility value was satisfactory (.73).

3.3 Modelling performance

To assess students' modelling performance, we estimated the accuracy of their solutions to the problems on a 3-point scale. Students achieved 0 points for a task if they used an incorrect mathematical model. If students used a partially accurate mathematical model, they received 1 point. Students received 2 points for their modelling performance if their mathematical model was completely accurate. Figure 1.3 shows an exemplary solution for the kite task that received a score of 2 points. After calculating the leg, the student added 1.65 m because of the height of Lucas who is holding the kite. This is why we gave the solution 2 points for modelling performance.

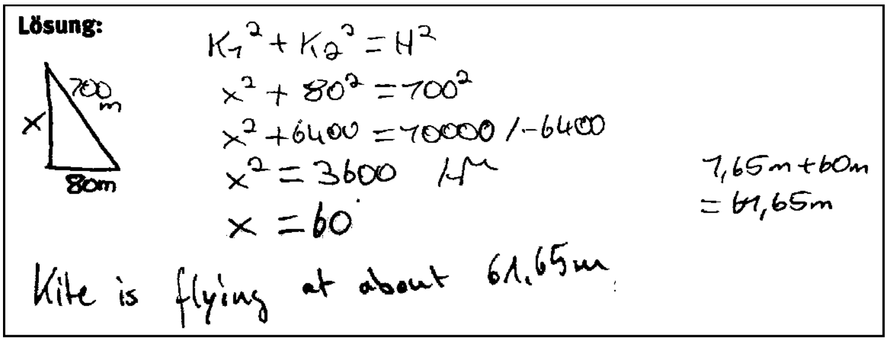

Fig. 1.3 Exemplary student solution for the kite task with 2 points for modelling performance

To test the inter-rater reliability of the coding procedure, more than 15% of the solutions were coded by two members of the research team. The inter-rater reliability resulted in a good match between the two coders (Cohen's Kappa=.98).

4 Results

The comparison of school grades in mathematics with an ANOVA indicated that the experimental groups and the control group did not differ in their mathematical abilities, $F(2,105) = 1.57, p = .212$. To compare the groups, we calculated arithmetic means for utility value and modelling. A one-way ANOVA with a post-hoc Bonferroni Correction was used to analyse group differences.

4.1 Picture-specific utility value

To answer the first research question, we compared the utility value for the students who rated the pictures that contained additional superfluous numerical information (EG-S), additional useful numerical information (EG-U), and no additional information (CG). Table 1.1 shows that the utility value means differed across the three groups.

Table 1.1 Means and standard deviations for picture-specific utility value

Additional information in pictures		
superfluous (EG-S)	useful (EG-U)	control group (CG)
3.03 (0.70)	3.65 (0.59)	3.5 (0.73)

As expected, students gave the lowest utility value ratings to the pictures that contained superfluous additional numerical information and the highest to the pictures that contained useful additional numerical information. There were significant differences in picture-specific utility value between the three groups, $F(2,107) = 8.41$, $p < .001$. A post hoc analysis using a t-test confirmed significant differences between the EG-S and CG ($t(71) = 2.83$, $p = .006$, $d = 0.66$) and between the EG-U and EG-S, $t(71) = 4.11$, $p < .001$, $d = 0.96$. No significant difference was found between the EG-U and CG, $t(72) = -0.97$, $p = .337$, $d = 0.23$.

4.2 Modelling performance

The second research question referred to the comparison of modelling performance in the three groups. Table 1.2 shows that the means and standard deviations of the modelling performance scores differed across the three groups.

Table 1.1 Means and standard deviations for modelling performance

Additional information in pictures		
superfluous (EG-S)	useful (EG-U)	control group (CG)
6.30 (2.57)	5.97 (2.24)	5.02 (3.05)

In contrast to our expectations, students' modelling performance in the EG-S and EG-U were close to each other and slightly higher than in the control group. The ANOVA showed that there was no statistically significant difference in the three groups' modelling performances, $F(2,105) = 1.43$, $p = .244$.

5 Discussion

5.1 Additional useful numerical information

According to the Integrated Model of Text and Picture Comprehension (Schnotz 2014), the conditions needed to create a positive multimedia effect are that the text and the picture are semantically connected to each other and that the text and the picture must be presented close together in time or space. With regard to the temporal and spatial presentations, the pictures that contained additional useful numerical information (EG-U) and no additional numerical information (CG) met the conditions equally. In this study, we expected that the coherence between the text and the picture and thus the picture-specific utility value would be higher in the EG-U and that a stronger multimedia effect would increase the modelling performance results. However, against our expectations, students assigned similar utility value to the pictures the contained additional useful numerical information (EG-U) and the pictures that did not contain additional numerical information (CG) with respect to understanding the task. A similar finding was revealed for modelling performance. One reason for these results might be that the difference in coherence between the pictures for the EG-U and the control group was too small in our study.

5.2 Additional superfluous information

In line with our expectations, the EG-S showed the lowest utility value in this study. Further, we expected that the additional superfluous information in the picture would increase the extraneous cognitive load and overload working memory for some students and decrease their modelling performance. Contrary to what we expected, the EG-S showed the highest modelling performance, even though it did not differ significantly from the other groups.

It is possible that the superfluous information may have led the students to study the pictures more intensively, thereby supporting their overall understanding of the situation. According to this view, recognising that the information was superfluous would be an example of one kind of cognitive load required to understand the learning material. Thus, the pictures that contained the additional superfluous information would result in an increased germane cognitive load, which could have a positive effect on learning and would explain the slight increase in modelling performance in the EG-S.

5.3 Overall discussion and implications

The results provide initial indications of the effect of different types of numerical information in pictures that accompany modelling tasks. Students assign higher utility value to pictures that provide additional useful numerical information than to pictures with additional superfluous numerical information. However, a higher perceived utility value of pictures with additional useful numerical information did not result in an increase in modelling performance. One possible explanation for this result is that assigning numerical information to the appropriate object in the picture might not be the main barrier to solving modelling problems. Other modelling activities such as noticing that information is missing from the problem or making assumptions about missing information were found to prevent students from finding realistic solutions and solving modelling tasks (Krawitz et al. 2018). Another explanation might be that the effects of the pictures that provided useful information depended on what the numerical information referred to. In our study additional numerical information drawn into pictures were obvious because you can find the same information in the text. If additional numerical information drawn into pictures were not obvious because you cannot find it in the text (e.g. the height of Lucas who is holding the kite) it could help students notice this information and include it in their mathematical model.

The results of our study offer initial implications for the design of pictures in modelling tasks. The findings on utility value indicate that students noticed pictures while solving modelling problems. Thus, numerical information that is included in the pictures can influence the modelling process to a considerable extent. We therefore suggest that teachers should think about designing pictures in modelling tasks and prepare them conscientiously.

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