CHARACTERIZING EXTERNAL VISUALIZATION INTERVENTIONS: A SYSTEMATIC LITERATURE REVIEW

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In this systematic literature review, we applied a qualitative content analysis to 13 research articles, comprising 17 external visualization (EV) interventions, published between 2018 and 2022. Our aim was to gain insights into the efficacy of EV interventions and to identify the EV intervention characteristics that might mediate the effect of the intervention on students' learning. We found that most EV interventions reported a positive impact on school students' understanding and problem solving. On the basis of our analyses and the explanations provided by the authors, we hypothesized nine characteristics of effective EV interventions, including the visualization process, technology use, multiple EVs, visual interaction, metacognitive reflection on EV, scaffolding of EV, and EV transfer.

INTRODUCTION

Mathematics as a discipline often deals with abstract objects and concepts, such as rational numbers or change. External visualization (EV; i.e., the use of physically embodied depictional representations (Schnotz, 2005), such as a tape diagram, a drawing, or a graph of a function) can be an important medium that supports students' learning (Arcavi, 2003). As two examples, exploring the relationships between the graph of a function and its derivative in an interactive learning environment can help students understand the concept of change, and translating a Bayes problem into the EV of a tree diagram might help them solve the problem. Consequently, mathematics instruction should provide students at all educational levels with learning environments that focus on EV to support their learning and to develop their abilities in constructing, using, and interpreting EV (OECD, 2019). In this systematic literature review, we analyze the extent to which recent mathematics education research has addressed EV interventions, and we synthesize the characteristics of the interventions and the studies' findings to gain insights into the efficacy of these learning environments.

THEORETICAL BACKGROUND

EV in mathematics education

In a previous scoping review (Schoenherr & Schukajlow, under review), we identified visualization components, tools, and purposes as three key characteristics of EV in recent mathematics education research: First, EV includes two visualization components (Arcavi, 2003). The process component includes all physical and mental activities and processes related to selecting, constructing, using, and interpreting EVs. The product component describes the resulting visual depiction (e.g., type, appearance, and accuracy of EV). EV interventions can focus on one or both components. For

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example, Kobiela and Lehrer (2019) had students experience and reflect on the process of generating a rectangle by asking students to sweep paint on a ceramic tile with a squeegee. Second, students made use of tools to interact with EVs, including paper-pencil, hands-on objects, gestures, or technology. EV interventions can give students opportunities to use one or several tools. Third, students use EV for diverse purposes (e.g., understanding, problem solving, and applying proofs in various mathematical content domains). Although EV seems to be most obvious in the domain of geometry, as geometry often relies on spatial reasoning, students can use EV in other domains, such as calculus and algebra (Arcavi, 2003). An open question is: To what extent does the efficacy of EV interventions depend on the intended purpose and content domain?

Learning environments that focus on EV

We define an EV intervention as any school, classroom, or learning environment in which students are provided with a set of activities aimed at promoting mathematics learning with or through the use of EV in an empirical study (e.g., a teaching sequence given to a class of Grade 12 students on the concept of change using an interactive graphing tool). Due to the variety of different kinds of EVs and different ways of implementing these interventions in learning settings, EV interventions can differ greatly. To the best of our knowledge, previous findings on the impact of EV interventions have not yet been synthesized. By systematically synthesizing recent intervention studies and their findings in the current review, we aim to describe learning environments that have focused on EV in recent mathematics education research and offer insights into their impact on student learning.

Characteristics of learning environments

Previous reviews in other research areas have revealed various mediating characteristics of powerful learning environments. For example, in a meta-analysis of 84 studies, Dignath and Büttner (2008) identified *duration of training* and *metacognitive reflection on learning* amongst others as characteristics of effective learning environments targeting self-regulated learning. As another example, Duijzer et al. (2019) compiled characteristics of embodied learning environments from 44 research articles, including the *real-world context*, *multimodality*, *multiple representations*, *student control*, and *attention capturing*. Besides these EV-unspecific characteristics of learning environments, little is known about the characteristics that are specific to EV for learning mathematics. As one example, Fiorella and Zhang (2018) discussed the *scaffolding of EV* and *metacognitive reflection of EV* as characteristics that potentially influence the efficacy of self-generated drawing for STEM learning. As another example, Presmeg (1986) concluded from an analysis of teaching styles that the generalization of specific EV is important for students to be able to learn with EV. An open question is: Which EV-specific and EV-unspecific intervention characteristics mediate the effects of recent EV interventions on student learning in mathematics?

RESEARCH QUESTIONS

We systematically reviewed recent empirical studies that were published in the last 5 years and investigated EV interventions that were designed to support school students' mathematics learning. Our aim was to address the following research questions: (a) What does the research literature on EV interventions report on their efficacy? (b) Which EV-specific and EV-unspecific characteristics might mediate the impact of these interventions on students' learning?

METHOD

Literature search and selection of studies

On April 26, 2022, we searched the high-ranked data bases Web of Science Core Collection, Scopus, Eric, PsycInfo, and Taylor & Francis Online Journals for the search terms diagram*, draw*, visual*, image*, sketch*, representation*, or graph* in the title, and math* in the whole text. In addition, we searched for peer-reviewed articles published between 2018 and 2022 in the English language for reasons of topicality and accessibility to the international community.

Our search identified 3,128 potentially relevant articles. To be included in the review, articles had to meet the following inclusion criteria: (a) the study focused on school mathematics learning or teaching with or through EV, (b) the study investigated an EV learning environment, (c) the study used a (quasi) experimental pre-posttest design or a posttest-only design to analyze the impact of an EV intervention on school students' mathematics learning. By screening Titles, Abstracts, and Keywords for inclusion criteria, we excluded 267 duplicates and 2,521 articles. Screening of the remaining full texts resulted in the exclusion of another 239 articles. We identified 41 articles that examined an EV intervention, out of which 12 articles met all the inclusion criteria. In the 12 articles, the authors investigated 17 EV learning environments by contrasting them against conventional learning or another EV learning environment.

Data extraction and analysis

We applied a qualitative content analysis to the full texts to systematically extract data on (1) reported efficacy, (2a) EV-unspecific characteristics of the learning environments, and (2b) EV-specific characteristics of the learning environments.

To describe the reported efficacy, we deductively coded *cognitive dependent measures* with the characteristics *understanding* and *problem solving* (including mathematical modelling) and inductively added *perception*, *interpretation*, and *mental rotation*. In addition, we coded whether taking part in the EV intervention had a *positive*, *zero*, or *negative* effect on learning compared with the comparison condition.

To extract EV-unspecific characteristics of the interventions, we first deductively applied the categories *school level*, *content domain*, and *intervention duration.* As EV-specific characteristics, we coded the *visualization component* (with the characteristics *process* and *product*) and *tool use* (with the characteristics *paper-pencil*, *technology*, *gestures*, and *hands-on objects*), on the basis of a previously developed coding scheme

(Schoenherr & Schukajlow, under review). To identify further characteristics, we applied a deductive-inductive procedure by first recording and then clustering characteristics mentioned by the authors into categories and assigning them to the EV-specific or EV-unspecific type. In this way, we added the EV-unspecific characteristics *metacognitive reflection on learning*, *scaffolding of learning*, and *student control* and the EV-specific characteristics *multiple EVs*, *visual interaction, metacognitive reflection on EV*, *scaffolding of EV*, and *EV transfer*.

As an indicator of coding reliability, two coders independently coded 25% of the included EV interventions on reported efficacy and EV-unspecific and EV-specific characteristics with a substantial percentage of agreement between 67% and 100%.

RESULTS

Efficacy of EV interventions

Comparing EV interventions with conventional learning, seven out of 12 EV interventions had a positive effect on student understanding and problem solving (e.g., Bernard & Senjayawati, 2019; Chen, 2019; Ke, 2019). For example, sixth and seventh graders who played an architecture simulation game including schematic EV outperformed students who were exposed to conventional learning in a problem-solving test on ratio, proportion, and area. Five EV interventions did not increase student learning compared with conventional learning (Ott, 2020; Rellensmann et al., 2021; Schoevers et al., 2020). As one example, providing students with an EV intervention on characteristics of accurate drawings did not result in increased modelling performance in geometry (Rellensmann et al., 2021). In studies comparing different EV interventions, findings were mixed with two studies reporting a positive effect (Aldalalah et al., 2019; Liang & She, 2021), one study reporting a positive effect for high-achieving students (Lee et al., 2018), and two studies reporting a null effect (Rellensmann et al., 2021; Soni & Okamoto, 2020). For example, Soni and Okamoto (2020) found that using number lines in a digital math game or in a paper-pencil workbook were equally effective at helping students learn fractions. Regarding visual perception, one study reported a positive effect after geometry training (Schoevers et al., 2020). No effects were found for non-geometry graphic interpretation (Lowrie et al., 2019) and mental rotation tasks (Ke, 2019; Ke & Clark, 2020).

Characteristics of effective EV interventions

On the *school level*, the majority of EV interventions addressed secondary school students ($n = 10$). The *duration* of EV interventions differed widely from four sessions of 15 min (Soni & Okamoto, 2020) to nine sessions of 60 to 90 min (Schoevers et al., 2020). The predominant *content domain* targeted in the EV interventions was geometry ($n = 8$), but other topics—for example, algebra ($n = 3$), probability ($n = 1$), and fractions ($n = 1$)—were also addressed. As we found positive and null effects across these characteristics, we cannot develop a conclusive hypothesis about the significance of the EV-unspecific characteristics educational level, intervention duration, and content domain for the efficacy of the EV interventions in this review.

A coding of the *visualization component* indicated that most studies (*n* = 9) included *visualization processes*. As one example, Lowrie et al. (2019) encouraged students in their EV intervention to mentally transform and manipulate 2D and 3D objects. Two studies that exclusively addressed *visualization products* did not find a positive effect on student learning (Ott, 2020; Rellensmann et al., 2021). For example, Ott (2020) addressed the product component by encouraging third-grade students to reflect on ready-made drawings in class. This led us to derive the hypothesis that addressing the visualization process component in EV interventions (i.e., all physical and mental activities and processes related to selecting, constructing, using, and interpreting EVs) is an important EV-specific characteristic for their efficacy.

In this review, seven studies used (amongst others) *technology* as a *tool* to construct or use EVs, two studies used hands-on objects, and four studies used paper-pencil only. Examples of technology used are Augmented Reality learning on mobile devices (e.g., Chen, 2019), an architecture simulation game (e.g., Ke & M. Clark, 2020), and dynamic geometry software (e.g., Lowrie et al., 2019). All studies using technology reported a positive effect on student learning, indicating that technology use might be an important EV-specific characteristic of effective EV interventions.

In addition, we extracted three EV-unspecific and six EV-specific characteristics that were considered potentially effective: The EV-unspecific characteristics consisted of *scaffolding of learning* (*n* = 3; e.g., Soni & Okamoto, 2020), *student control of learning* (i.e., individual learning pace and difficulty levels; *n* = 3; e.g., Chen, 2019), and *metacognitive reflection on learning*, including reflection on mathematical content, procedures, knowledge, and skills (e.g., Schoevers et al., 2020).

One frequently mentioned EV-specific characteristic (*n* = 7) was that the learning environment forced students to transfer information between *multiple EVs* (e.g., Bernard & Senjayawati, 2019; Liang & She, 2021), including concrete (Lowrie et al., 2019) and symbolic representations (Liang & She, 2021). Another EV-specific characteristic was *visual interaction* (*n* = 5), that is, the learning environment enabled students to visually observe, elaborate, explore, manipulate, and transform EVs (e.g., Ke, 2019). In addition, the authors proposed *metacognitive reflection on EV* (*n* = 2; Ott, 2020; Rellensmann et al., 2021), *scaffolding of EV* (*n* = 3; Ke & Clark, 2020), and *transfer of EV* across tasks (*n* = 3; e.g., Rellensmann et al., 2021) as promising factors that might increase the efficacy of EV interventions.

DISCUSSION

Of the 130 studies on EV in mathematics education research, a small proportion of studies (*n* = 12) used experimental designs to investigate EV interventions in schools. Our review of these studies showed a mixed—but mostly positive—impact on student learning in different mathematical topics, underlining the theoretically assumed benefit of EV as a medium for mathematics thinking and learning (e.g., Arcavi, 2003). The small number of experimental intervention studies indicates that more experimental studies are needed to obtain evidence for the effects of EV interventions on student learning. Still, evidence of efficacy depends strongly on the choice of control condition and outcome measures. Findings indicate that EV interventions might be particularly effective in comparison with conventional learning (i.e., without EV) with respect to near transfer tasks that measure students' understanding of the learning topic (e.g., Soni & Okamoto, 2020).

Mixed findings on the efficacy of EV interventions have highlighted the need to gather information on the characteristics of effective EV interventions. An important novel contribution of this review is that we identified three EV-unspecific and six EV-specific intervention characteristics that might influence the efficacy of EV interventions.

Theoretically, our findings on the EV-specific intervention characteristics contribute to the framework of EV in mathematics education, as they point to key characteristics of EV in learning environments. As such, the EV process component (i.e., learning how to construct, generate, use, and interpret EV; Arcavi, 2003) seems to be important for learning with or through EV.

Empirically, the characteristics we identified confirm and add to previously identified characteristics from EV research and other research areas. For example, our analyses supported the previously identified characteristics *metacognitive reflection on learning* (Dignath & Büttner, 2008), *student control of learning and multiple EVs* (Duijzer et al., 2019), *metacognitive reflection on EV* and *scaffolding of EV* (Fiorella & Zhang, 2017), and *transfer* of EV to different tasks to help students generalize characteristics of specific EV tasks (Presmeg, 1986). An important new contribution is that our analyses also uncovered the EV-specific characteristics *visualization process*, *technology use*, and *visual interaction*. This means, for example, that we hypothesize that providing students with opportunities to visually explore, manipulate, and transform EVs will increase student learning in learning environments that focus on EV. Further research is needed to determine the differential impact of the characteristics in order to contribute to a better understanding of how they influence the efficacy of EV interventions. As one example, technology use appeared to be positively related to the intervention's efficacy, a finding that might be explained by the use of individual learning sessions that provided students with scaffolding and opportunities for visual interaction.

Practically, as most of the characteristics have been supported by prior research, the EV-specific and EV-unspecific characteristics we identified can help practitioners design EV learning environments.

Limitations

In this review, we applied an extensive automatic search strategy to identify a wide range of studies that investigated EV interventions. Still, we might have missed some relevant studies that may have been framed differently. Our analysis of the learning environments was based on the information provided in the papers. To increase the objectivity of our coding, we relied on the terms used by the authors whenever possible (e.g., problem solving). However, different interpretations by the authors might bias this review's findings. Also, we analyzed a wide variety of learning environments to gain insights into EV intervention research and to determine the extent to which different interventions support student learning. For these reasons, it is difficult to generalize our results, and more research is needed on the benefits and boundaries of learning with or through EV. In this review, we focused on intervention characteristics. In addition, recent research has pointed to learner characteristics that influence the impact of EV interventions (e.g., mathematical abilities; Lee et al., 2018). More research is needed on learner characteristics and their interplay with intervention characteristics in promoting students' learning in EV learning environments.

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