Multiple solutions for real-world problems, and students' enjoyment and boredom

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Emotions are important for learning. In a previous study, we found that students who constructed more solutions for real-word problems with vague conditions reported higher enjoyment and lower boredom (Schukajlow & Rakoczy, 2016). In the present study, we had students construct multiple solutions by applying multiple mathematical procedures to real-world problems, and we investigated effects on the enjoyment and boredom. 307 students were assigned to the experimental or control group. Students in the experimental group applied two mathematical procedures, and students in the control group applied one mathematical procedure to solve real-world problems. During the lessons, they were asked to report their enjoyment and boredom. Contrary to our expectations, the results revealed no effects of the intervention on students' enjoyment or boredom.

Keywords: Emotion, affect, modelling, word problems, multiple solutions.

Introduction

Emotions are important for learning (Zan, Brown, Evans, & Hannula, 2006). Although students' academic emotions are prerequisites, mediators, and outcomes of the learning process in mathematics (Schukajlow & Rakoczy, 2016), they were neglected for decades. Thus, except for the emotion of anxiety, we do not know much about students' emotional development. Moreover, there is a lack of research on how teaching methods influence emotions. As there have been several calls for intervention studies, we decided to conduct a study that was aimed at clarifying the impact of constructing multiple solutions for real-world problems on cognitive and affective outcomes. We chose this teaching method and this kind of problem because constructing multiple solutions and solving real-world problems are emphasized in curricula in different countries. In the present paper, we taught students to construct multiple solutions by applying different mathematical procedures to solve real-world problems, and we investigated how this process affected enjoyment and boredom.

Theoretical framework and hypotheses

High-quality mathematics teaching implies that students should develop multiple solutions and compare these solutions in the classroom. Empirical evidence for the effects of constructing multiple solutions on cognitive outcomes comes from international comparative studies (Hiebert et al., 2003) and from experimental studies (Levav-Waynberg & Leikin, 2012; Schukajlow, Krug, & Rakoczy, 2015). However, the impact of constructing multiple solutions on affect is an open issue. For high-quality mathematics teaching, both cognitive and affective outcomes have to be taken into account. As we determined in the project MultiMa[1] (Multiple Solutions for Mathematics Teaching

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Oriented Toward Students' Self-regulation Learning), apart from students' achievements and strategies, there is also a need to consider their self-regulation, interest, motivation, and emotions.

Multiple solutions and real-world problems

Previous research on multiple solutions was conducted for the most part on intra-mathematical problems in different content areas such as geometry (Levav-Waynberg & Leikin, 2012) or early algebra (Star & Rittle-Johnson, 2008). Students' ability to solve real-world (or modelling) problems was not previously the focus of research on multiple solutions so far. Solving real-world problems first and foremost involves demanding transfer processes between reality and mathematics (Niss, Blum, & Galbraith, 2007). As real-world problems often include vague conditions and allow students to construct different mathematical models and apply different mathematical procedures, we distinguished between three categories of multiple solutions (Schukajlow & Krug, 2014b). The first category of multiple solutions are typical of real-world problems with vague conditions. In solving this type of problem, students make different assumptions about vague conditions and therefore arrive at different outcomes or results. Another type of multiple-solution problem occurs as a result of applying different mathematical procedures or strategies, a process that typically leads to the same mathematical outcome. The third category combines the first two categories. In the current paper, we explored the effects of applying multiple mathematical procedures while solving real-world problems for the topic of linear functions. We would like to illustrate this type of multiple-solution problem with the sample problem "BahnCard" (cf. Figure 1), which was developed in the framework of the MultiMa Project.

BahnCard

Mr. Besser lives in Hamburg. His parents live in Bremen. For the outward and return journeys with the "Deutsche Bahn" (German Rail), Mr. Besser has to pay 100 €. There are two special offers, the so-called "BahnCard 25" and the "BahnCard 50." The prices for each year and the prices for the outward and return journeys from Hamburg to Bremen for owners of the "BahnCards" are listed below.

BahnCard 25	BahnCard 50
Price per year: 59 € Price for a round-trip journey: 75 € Number of customers: 3.1 million	Price per year: 240 € Price for a round-trip journey: 50 € Number of customers: 1.6 million

Mr. Besser is going to buy a "BahnCard." When is it worth buying the "BahnCard 25" and when the "BahnCard 50"? Write down your solution.

Figure 1: Real-world problem "BahnCard" (Achmetli, Schukajlow, & Krug, 2014)

The problem solver is asked to read the problem "BahnCard," and identify the important values: price per year for each card and the amount of a round-trip journey that would be paid with each card. After mathematizing the problem, different mathematical procedures can be applied.

One mathematical procedure that can be applied is called "differences." In order to solve the "BahnCard" problem by using differences, students first have to calculate differences in the prices per year and for each round trip for owners of each card. Whereas the "BahnCard 50" is 181 € (= 240 € - 59 €) more expensive than the "BahnCard 25," each round trip with the "BahnCard 25" is 25 € (= 50 € - 25 €) more expensive than with the "BahnCard 50." The open question is how often Mr. Besser has to take a trip with the more expensive "BahnCard 50" until the cheaper prices for the journeys pay off. This is exactly after 7.24 (= 181 € ÷ 25 €) journeys per year. This result has to be rounded up, interpreted—for example, “For up to 7 journeys per year, the ‘BahnCard 25’ is cheaper”— validated, and the recommendation has to be wrote down.

Another way to solve this problem is to apply a mathematical procedure “table.” To apply this procedure, students must compare the costs for owners of the “BahnCard 25” and the “BahnCard 50” for different numbers of journeys per year (e.g. 1, 3, 6…). By performing this comparison systematically, they can identify that the “BahnCard 25” is cheaper for up to 7 journeys. If the owner makes 8 or more journeys, the “BahnCard 50” is preferable for him/her. Finally, students need to validate their result and write down their recommendation.

Enjoyment and boredom as achievement emotions

Emotions are typically defined as complex phenomena that include affective, cognitive, physiological, motivational, and expressive parts (Pekrun & Linnenbrink-Garcia, 2014). In the academic context, researchers are interested in achievement emotions, which occur in learning settings and are related to epistemic processes. Research on emotions in mathematics education has emerged from different philosophical traditions (Hannula, 2015) and has categorized emotions according their value (positive or negative), level of activation (activated or deactivated), or other characteristics. For example, enjoyment is one of the positive activating emotions (Pekrun, 2006). Students who enjoy problem solving are expected to report pleasant feelings. Moreover, when students enjoy mathematics, they feel activated excitement while working on a problem. The opposite behavioral and cognitive patterns are expected for the emotion of boredom. Boredom was suggested to be a negative deactivated emotion because boredom is accompanied by unpleasant feelings, and if students feel bored, they experience a state of deactivating relaxation. Following these considerations, a positive relation between enjoyment and performance and a negative relation between boredom and performance were hypothesized and confirmed in two empirical studies in the domain of mathematics (Schukajlow, 2015; Schukajlow & Krug, 2014a). Moreover, enjoyment but not boredom was found to predict students’ performance in a longitudinal interventional study (Schukajlow & Rakoczy, 2016).

According to the control-value theory of achievement emotions (Pekrun, 2006), emotions are strongly determined by control and value appraisals, which arise in learning situations. In order for a positive emotion such as enjoyment to emerge, students should (1) perceive their problem solving activities as controllable and be confident that they can influence the learning situation and (2) ascribe the problem solving activities a high value. If students think that they do not have any influence over their problem solving activities, or if they view these activities as meaningless, negative emotions will emerge. For example, boredom arises if students ascribe a low value to their activities. The relation between boredom and control appraisals is complex and is proposed to be a curvilinear U-shape. This relation implies that boredom occurs when perceived control is very high (i.e. task demands are very low) or when perceived control is very low (i.e. task demands are very high). However, in the context of problem solving activities, students do not have to deal with routine tasks. Thus, a negative linear relation between control appraisals (e.g. assessed via students’ performance or self-efficacy beliefs) and boredom was expected and confirmed in most empirical studies (e.g. Schukajlow, 2015).

Enjoyment, boredom, and multiple solutions for real-world problems

On the basis of theoretical considerations from control-value theory, we expected to find that constructing multiple solutions would increase students' control appraisals when solving real-world problems. Higher appraisals should increase students' enjoyment and decrease their boredom. Positive effects of constructing multiple solutions on enjoyment and negative effects on boredom were confirmed in our previous study. Students who constructed more solutions enjoyed their classes more and were less bored (Schukajlow & Rakoczy, 2016). In the current study, we sought to confirm these findings for the other type of multiple-solution problem and investigated the effects of applying multiple mathematical procedures for real-world problems on enjoyment and boredom.

Hypotheses

The hypotheses we addressed were: 1) Constructing multiple solutions by applying multiple mathematical procedures for real-world problems has a positive effect on students' enjoyment of mathematics; 2) Constructing multiple solutions by applying multiple mathematical procedures for real-world problems has a negative effect on boredom in mathematics.

Method

Sample and procedure

Three hundred seven German ninth graders from four schools with three middle-track classes each (48.26% female; mean age=14.6 years) participated in the present study. Before and after the teaching unit, students were asked about their enjoyment and boredom. The teaching unit consisted of two sessions with two 45-minute long lessons each. Each of twelve classes was divided into two parts with the same number of students in each part in the way students' mathematical achievements did not differ between the parts. Further, the number of males and females was approximately the same in each part. Eight of twenty-four groups were randomly assigned to the one-solution condition "differences" (OS1), eight groups to the one-solution condition "table" (OS2), and eight to the multiple-solutions condition "differences + table" (MS), taking into account that in each school, there had to be the same number of groups assigned to each condition, and the students in each class had to be assigned to different conditions (more details about the procedure can be found in Achmetli, Schukajlow, & Rakoczy, manuscript submitted for publication). Each group was taught separately by one of six teachers (three female, age: 27 to 60) who participated in the present study. The teachers taught the same number of groups in each condition in order to minimize the differences between conditions that might result from the influence of teacher personality on students' learning. All of the teachers received instruction manuals that included the lesson plans, problems for the students, and the solutions to these problems.

Treatment

The three treatment conditions implemented in the present study (OS1, OS2, and MS) were based on the positively evaluated student-centered learning environment for teaching modelling problems (Schukajlow, Kolter, & Blum, 2015). This student-centered learning environment was complemented by direct instruction at the beginning of the teaching unit. For the purpose of maintaining comparability between the conditions, the same order was implemented for all three treatment conditions. In the first lesson, the teacher demonstrated how real-world problems could be solved by applying one mathematical procedure (in the OS conditions) or multiple mathematical procedures (in the MS condition). In the three lessons that followed, the students solved real-world problems by applying the demonstrated procedures according to a special procedure for group work (alone, together, and alone), presented their solutions, and discussed these solutions with the whole group in the classroom. At the end of each lesson, the teacher summarized the key points of each treatment condition. In the multiple-solutions condition, the teacher encouraged the students, further, to compare and contrast the two mathematical procedures and the mathematical results.

Students first solved four similar tasks in the one-solution conditions and in the multiple-solutions condition. The only difference between these four problems was that students in the one-solution conditions were required to apply one mathematical procedure (“table” or “differences”), whereas students in the multiple-solutions condition were required to apply both mathematical procedures (“table” and “differences”). The sample problem “BahnCard,” which was given in the one-solution conditions, is presented in Figure 1. In the multiple-solutions condition, the problems were modified by adding the following sentence: “Use two different mathematical procedures to solve this problem.” As the discussion of the connection between mathematical procedures required additional time in the MS condition, one additional task was offered in each OS condition. Thus, in sum, students in the MS condition solved six and students in the OS conditions solved seven problems.

Measures

Enjoyment and boredom during the teaching unit were measured after the second and fourth lessons with a 5-point scale ranging from 1 (not at all true) to 5 (completely true). Both scales included three items each (see Table 1).

Scale	Item
Enjoyment	I enjoyed task processing. I was happy during task processing. Task processing was great fun for me.
Boredom	Task processing was boring. I got so bored during task processing that I had problems staying alert. I did not want to continue my work because it was so boring.

Table 1: Items used in the study to assess enjoyment and boredom

The scales were adapted from the well-evaluated Achievement Emotions Questionnaire (Pekrun, Goetz, Frenzel, Barchfeld, & Perry, 2011). The Cronbach’s alpha reliabilities were .80 and .79 for enjoyment and .81 and .83 for boredom for Sessions 1 and 2, respectively.

Treatment fidelity

To ensure the fidelity of the treatment, we videotaped the teaching unit, observed the lessons, and analyzed the students’ solutions. The analysis confirmed the treatment fidelity (Achmetli et al., manuscript submitted for publication). For example, we found that students in all classes worked on the respective version of the problem (MS vs. OS) and all teachers implemented the intended methodical order in their lessons. More specifically, we found that students in the MS condition developed significantly more solutions than the students in the OS conditions (MS vs. OS1: effect size Cohen’s d=4.97; MS vs. OS2: d=3.61).

Results

Preliminary results

In order to simplify the analysis of the effects, we combined the OS1 and OS2 conditions into one OS condition. Combining the two conditions did not influence the results significantly, as our statistical analysis did not show a difference at the 10% level of significance between the two OS conditions for motivational variables such as self-regulation (Achmetli et al., 2014) or interest ($t(186) = 0.182$; $p = .856$). Further, in order to ensure that the two conditions were comparable, we compared interest between the MS condition and the combined OS condition as this construct is closely connected to students' enjoyment and boredom (Schukajlow & Rakoczy, 2016). The analysis of interest at pretest revealed no differences between the MS and OS conditions (MS: M = 2.39 (SD = .90), OS: M = 2.39 (SD = .96)). This result indicates that students' emotional prerequisites were similar in the MS and OS conditions.

Applying multiple mathematical procedures and students' enjoyment or boredom

We hypothesized that constructing multiple solutions by applying multiple mathematical procedures would increase students' enjoyment and decrease their boredom. We tested both hypotheses by computing t-tests. The crucial assumption when using a t-test is that the variances are equal in the two groups. Levene's test of equality of variances was significant for students' boredom measured after the second and third lessons, indicating that the assumption of equal variances in the two groups had been violated ($F(280) = 4.022$, $p = .046$; $F(279) = 4.851$, $p = .028$). Thus, we used the adjusted degrees of freedom, t-values, and *p*-values for students' boredom. The descriptive statistics are presented in Table 2.

	Enjoyment first session Mean (SD)	Enjoyment second session Mean (SD)	Boredom first session Mean (SD)	Boredom second session Mean (SD)
MS	3.42 (0.92)	2.96 (0.91)	2.24 (1.05)	2.53 (1.16)
OS	3.43 (0.88)	3.00 (0.96)	2.03 (0.90)	2.30 (1.00)

Table 2: Means and standard deviations for enjoyment and boredom

Against our expectations, students' enjoyment during the first and second sessions did not differ between the MS and OS conditions (first session $t(280) = 0.75$, $p = .940$, Cohen's $d = 0.02$; second session: $t(279) = 0.297$, $p = .767$, $d = 0.04$). Thus, the enjoyment of students who solved real-world problems by applying multiple mathematical procedures was similar to the enjoyment of students who applied one mathematical procedure.

We did not find support for the second hypothesis. Our analysis did not reveal benefits of constructing multiple solutions by applying multiple mathematical procedures for students' boredom during the first or second session (first session $t(165) = 1.67$, $p = .097$, Cohen's $d = 0.22$; second session: $t(156) = 1.62$, $p = .108$, $d = 0.22$). Moreover, there was a slight (but not significant) tendency for students in the multiple-solutions group to feel greater boredom than students in the one-solution condition.

Discussion

In this paper, we aimed to analyze how constructing multiple solutions by applying multiple mathematical procedures while solving real-world problems would affect students’ emotions. On the basis of theoretical considerations from the control-value theory of achievement emotions (Pekrun, 2006) and prior research that found that developing multiple solutions had positive effects on students’ enjoyment and negative effects on their boredom (Schukajlow & Rakoczy, 2016), we expected positive effects of the treatment on enjoyment and negative effects on boredom during learning. However, our analyses did not confirm these hypotheses. Enjoyment and boredom in solving real-world problems did not differ between the multiple-solutions and one-solution conditions. Moreover, boredom was slightly lower in the one-solution condition compared with the multiple-solutions condition. One explanation for this finding might involve students’ high control appraisals. In the previous study, the mean values for students’ experience of competence, which can be taken as an indicator of students’ control appraisals (sample item: “I felt confident about my knowledge of the topic today”; range from 1 to 5), were 3.85 and 3.65 in the MS and OS conditions, respectively (Schukajlow & Krug, 2014b). However, in the current study, the mean values for students’ experience of competence were nearly one standard deviation higher and close to the theoretical maximum of 5 (Achmetli et al., manuscript submitted for publication). As noted in the control-value theory, if students’ control appraisals are too high (or task demands are too low), they can have a negative influence on students’ emotions. Thus, a future research question might involve asking whether posing more demanding real-world problems that require students to apply multiple mathematical procedures can increase students’ positive emotions such as enjoyment and decrease their negative emotions such as boredom. Another research question that should be addressed in an experimental study is about the non-linear connection between control appraisals and students’ emotions. This assumption of the control-value theory needs more empirical evidence from randomized studies. More specifically, the corvilinear U-shape relation between control appraisals and boredom should be addressed in future longitudinal studies. Further, it might be the case that the type of multiple-solution problem makes a difference. Whereas students enjoy making different assumptions about missing information, constructing different solutions, and comparing their results, this enjoyment might not hold when they apply different mathematical procedures. Similar effects (low level of boredom for the first type of multiple-solution problem, but no difference in boredom for the second type of multiple-solution problem) were also found for students’ boredom. Another explanation for no effects of the intervention on emotions might be that students in the multiple solution condition were not offered to chouse their favorite procedure during three of four lessons. More efforts are needed to clarify the role of multiple solutions for affective measures and more generally, with respect to the effects of teaching methods on students’ affect.

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