PLANNING, MONITORING AND MULTIPLE SOLUTIONS WHILE SOLVING MODELLING PROBLEMS

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In a quasi-experimental study that was carried out in the framework of MultiMa-project, we compared two groups of 9th graders from six middle track classes. In one group modelling tasks, where the solutions do not demand making assumptions about the missing data, were treated for five lessons. In the other group students solved similar modelling problems, where different assumptions were possible and students had to develop two and more different solutions five lessons long. Before and after the teaching unit students answered questionnaires about planning and monitoring their activities while solving problems. The analysis showed the positive influence of treating and developing multiple solutions on students' planning and monitoring activities.

INTRODUCTION

Development of multiple solutions by students is an important part of curriculum in different countries (NCTM, 2000). However, we do not know much about the influence of treating multiple solutions on students' learning. In the MultiMa-project (Multiple solutions for mathematics teaching oriented towards students' self-regulation) the impact of teaching multiple solutions on students' performance, affect and metacognitive activities was investigated. In this paper we focus on treating multiple solutions in the classroom, on developing multiple solutions by students, and also on students' planning and monitoring activities while solving modelling problems.

THEORETICAL BACKGROUND

In this section we report on three theoretical issues: (1) planning and monitoring as metacognitive activities, (2) developing and treating multiple solutions, as well as (3) modelling problems and multiple solutions.

Planning and monitoring as metacognitive activities

Metacognition and cognition are important for the performance in cognitive tasks. The relationship between cognition and metacognition is explained by Garofalo and Lester (1985, p. 164) as "… cognition is involved in doing, whereas a metacognition is involving in planning and choosing what to do and monitoring what is being done". "'Metacognition' refers to one's knowledge concerning one's own cognitive processes and products or anything related to them …" (Flavell, 1979, p. 232). Metacognition includes among other things active monitoring, planning and consequent regulation of cognitive processes in order to achieve goals.

In the discussion about how to solve mathematical problems successfully, planning and monitoring are considered to be important activities. Polya's (1948) description of solving problems consists of four steps: (1) understanding the problem, (2) devising a

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plan, (3) carrying out the plan, and (4) looking back. The second and the fourth step refer mainly to the planning and monitoring activities. Garofalo and Lester (1985) also included planning solution and checking results in their list of persons' activities that help to solve complicated problems.

Most research results from correlational and interventional studies support the importance of metacognition for students' performance (see summary by Schneider and Artelt (2010)). German 15-year-olds from the high academic track not only outperform students from the low academic track, but also know more about metacognitive activities (Schneider & Artelt, 2010). A part of the program for improvement in metacognition (IMPROVE) that was developed and evaluated in Israel includes stimulation of planning and monitoring activities with a help of questioning (Kramarski, Mevarech, & Arami, 2002). The following questions were among others: What strategy, tactic, or principle can be used to solve the problem or complete the task and why? Does it (the result) make sense? How can I verify the solution? The analysis showed the positive impact of metacognitive instructions on low and higher achievers from the 7th grade (12 years old).

Developing and treating multiple solutions

In the last decades, the principle focus of research in the field of developing and teaching multiple solutions has mainly been on students' performance. Whereas in the high achievement countries, such as Japan, teachers demand to develop multiple solutions of a problem, German and American teachers are often highly satisfied with one solution only (Hiebert et al., 2003). Teachers believe that presentation of multiple solutions confuse students and do not stimulate their development in mathematics classrooms (Leikin & Levav-Waynberg, 2007).

In the domain of mathematics, several experimental studies showed positive effects of treating multiple solutions on performance and cognitive flexibility by students with sufficient prior knowledge (Große & Renkl, 2006; Rittle-Johnson & Star, 2007). In these studies the main principle for teaching multiple solutions was stimulation of connection between different solution methods. This principle is based on the constructivist theories of learning, which argue that developing different solutions and representations helps students acquire multiple representation of the subject matter and improve their performance.

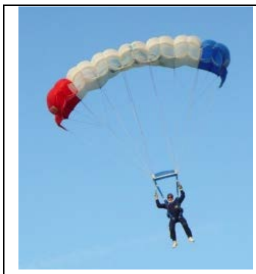
Conceivably, the treatment and development of multiple solutions stimulates planning activities. While developing multiple solutions of modelling problems, students might identify the missing data and think about possible assumptions that allow to develop two results before they begin with solving the task. Therefore, they might plan their solution. Further, they can compare the results and control their activities frequently if they have to develop more than one solution.

Rittle-Johnson and Star (2007) have investigated, whether comparing two solution methods of the same problem or presenting two solution methods using different problems effects students' procedural flexibility. As students of the first group were more flexible in the choice of the appropriate solution method, we assumed that metacognitive abilities, like planning and monitoring can be improved due to the treatment and development of multiple solutions. In this study, we aim to prove this assumption.

Modelling problems and multiple solutions

Students' improvement in ability to solve problems with close connection to reality is an important goal of mathematics education. The core of activities while solving modelling problems is the demanding transfer processes between reality and mathematics (Blum, Galbraith, Henn, & Niss, 2007).

We distinguish between three types of solutions while solving modelling problems. First, multiple solutions can be constructed due to the variation in mathematical solution methods. The second type of multiple solutions can be developed if students have to make assumptions about the missing data and, thus, get different outcomes/ results. The third one includes the variation in mathematical solution methods as well as in different outcomes/ results. In this paper we report on the study carried out in order to explore the effects of treating the second type of multiple solutions on students' learning.



Parachuting

When "parachuting", a plane takes jumpers to an altitude of about 4000 m. From there they jump out the plane. Before a jumper opens his parachute, he makes free fall of about 3000 m. At an altitude of about 1000 m the parachute opens and the sportsman glides to the landing place. While falling, the wind carries the jumper away. Deviations at different stages are shown in the table below.

Wind speed	Side deviation per thousand meters during free fall	Side deviation per thousand meters while gliding
Light	60 m	540 m
Middle	160 m	1440 m
Strong	340 m	3060 m

What distance does the parachutist cover during the entire jump?

Figure 1: Modelling task "Parachuting"

While solving the modelling task "Parachuting", among various assumptions, also those about the wind power in the respective falling stage have to be taken (see Fig. 1). Depending on the assumed wind power, students get different results using Pythagoras' Theorem as a mathematical solution method.

The influence of treating multiple solutions while solving modelling problems on students' self-regulation was investigated in the study by Schukajlow and Krug (2012). In this study planning and monitoring activities were used for the conceptualisation of self-regulation. The self-regulation of learning was measured on the basis of the statements that refer to setting goals, making a plan for attainment of these goals, and monitoring the attainment of the goals set. The results showed that the group, in which multiple solutions were treated, reported significantly more often on their self-regulation in post-test than the group, where students were instructed to develop one solution only, under control of self-regulation in pre-test. This finding points out that treating multiple solutions can have positive influence on students' metacognitive activities such as planning and monitoring ones. In order to prove this assumption, we analysed this study's data on the improvement in both activities.

Another important question is what role the number of solutions really developed by the students play in students' learning. A recent study showed that treating multiple solutions in the classroom does not always result in development of multiple solutions by all students: 4% of students could not find any solution, 38% found one and 58% two and more solutions (Schukajlow & Krug, 2012).

As the treatment and development of multiple solutions can stimulate students' reflection on the questions: *How can different solutions be developed?, Do the results of multiple solutions significantly differ from each other? and Do the results make sense?*, we expected the positive effects of both factors on planning and monitoring activities.

Research questions

The research questions of the study were:

- Do students' planning and monitoring activities differ according to the possibility to develop multiple solutions? In particularly, whether treating multiple solutions while solving modelling problems results in more frequent planning and monitoring activities?
- Does the development of multiple solutions influence students' planning and monitoring activities positively?

METHOD

Design and sample

138 German ninth graders (42.8% females; mean age = 15.2 years) were asked about their planning and monitoring activities while solving complicated word problems before and after a five-lesson period (see Figure 2). Three schools with two middle track classes each participated in this study. Each of six classes was divided into two parts with the same number of students in a way that the average achievements in the both parts did not differ and there was the approximately same number of males and females in each part. In one part of each class students were instructed to develop multiple solution of modelling problems (group "multiple solutions") and in the other part to develop one solution of these problems (group "one solution"). The students of groups "multiple solutions" and "one solution" were taught in different classrooms.

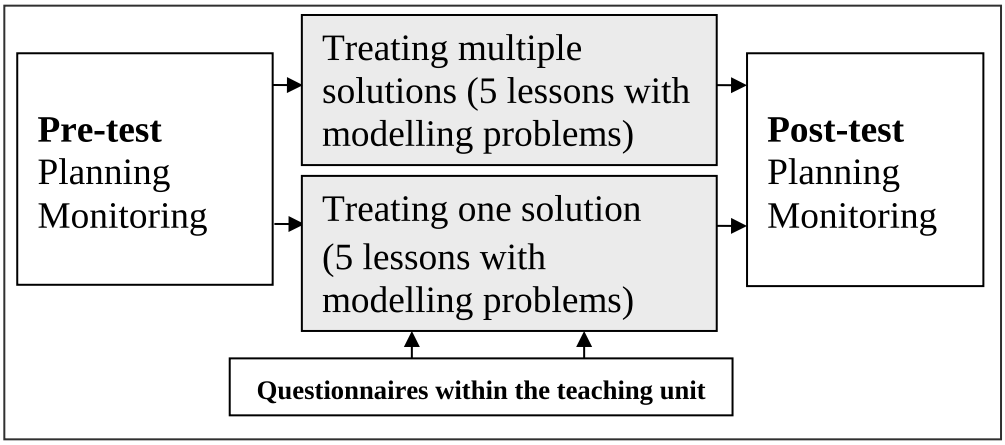


Figure 2: Overview of the study design

Four teachers that participated in this study received instructional manuals with all the tasks to be treated in each group, with the solutions of the problems, and with a detailed plan of the teaching unit. Further, all teachers were instructed about specific ways to promote the development of multiple and one solution while treating modelling problems. As each teacher instructed the same number of student groups in the "multiple solutions" and "one solution" conditions, the influence of a teachers' personality on students' learning did not differ between both conditions. In order to observe the implementation of the treatment, one member of the research group was present in each lesson.

Treatment

The student-centred learning environment from DISUM-project (c.f. Schukajlow et al., 2012) was taken as a base of the teaching method that we applied in the recent study. Elements of "directive" instruction complemented this teaching method. In both experimental groups the same methodical order was used. Students solved a modelling task according to a special kind of group work (alone, together and alone) and then a teacher presented the solution (or different solutions) or otherwise students discussed their solution (or different solutions) in the whole group in the classroom. The teacher summarised a lesson and reflected on the key points of each experimental group. In the "multiple solutions" condition, the teacher emphasised the development of different results by estimating the missing data and made the connection between different solutions a subject of discussion. In the group "one solution", the teacher focused on the development of one solution only.

In order to stimulate the construction of multiple solutions in one experimental group and to prevent the development of more than one solution in the "one solution" condition, two similar versions of each task were developed. Each problem in the group "multiple solutions" required the construction of two solutions. In the task "Parachuting" (see Figure 1), the following question was posed: "What distance does the parachutist cover during the entire jump? Find *two* possible solutions". Students in the group "one solution" solved similar versions of the problems that had to be solved by the students from "multiple solutions" group. But unlike them, they had to deliver only one solution. The main data that are needed to solve these versions of the problems were specified. In the one-solution version of the problem “Parachuting” the main data were the wind velocity and altitude in which the jumper opens his parachute.

Measures

Students’ planning and monitoring activities while working on problems with connection to reality were measured using a 5-point Likert scale (1 = not at all true, 5 = completely true) before and after a five-lesson teaching unit (see Figure 2). The sample items were for scale “planning” (4 items) “If I solve a complicated word problem … I make a plan” and for scale “monitoring” (8 items) “If I solve a complicated word problem … I prove at the end, whether a result fits the problem approximately”. Both scales were adapted from the study that was carried out by Rakoczy et al. (2005) and already used in other studies (see e.g. Schukajlow & Leiss, 2011). The reliability values (Cronbach’s Alpha) were in both pre- and post-test .66 and .74 for planning and .82 and .84 for monitoring.

The number of solutions that were developed by students during the teaching unit was measured using students’ questionnaires. After every lesson, the students were asked about the number of solutions they developed for each modelling problem in this lesson. For example: “While solving the problem “Parachuting” I developed today … (0: no solution; 1: one solution; 2: two solutions; 3: more than two solutions)”. Students’ answers were summarised to the mean score, which we used for further analysis of the data.

RESULTS AND DISCUSSION

In order to control the implementation of the treatment, all lessons were observed by at least one member of our research group. The observations confirmed the correct implementation of instructions in both treatment conditions (c.f. also Schukajlow & Krug, 2012).

Impact of treating multiple solutions on planning and monitoring activities

First, the ANCOVA with “treatment condition” as independent measure, “planning” in post-test as dependent measure, and “planning” in pre-test as covariate was conducted (see the means in Table 1). The analysis shows that students of the group “multiple solutions” reported in post-test to have planned their activities while solving word problems more often than students of the condition, where only one solution was treated ($F(118, 2)=3.5$, $p=.07$, *effect size* $(\eta)^2=.03$). The similar analysis with “monitoring” in pre-test as dependent measure and “monitoring in post-test” as covariate also reveals a positive impact of treating multiple solutions on students’ monitoring activities ($F(118, 2)=9.8$, $p<.01$, *effect size* $(\eta)^2=.08$).

	Treatment of multiple solutions		Treatment of one solution	
	Mean	SD	Mean	SD
planning in pre-test	3.11	.74	3.08	.83
planning in post-test	3.48	.81	3.23	.83
monitoring in pre-test	3.56	.68	3.54	.81
monitoring in post-test	3.90	.65	3.54	.78
number of solutions within the teaching unit	1.72	.54	1.17	.34

Table 1: Students' planning, monitoring and number of solutions

These results confirm our assumption that treating multiple solutions has positive influence on students' metacognitive activities. If a teacher encourages students to develop more than one solution in a way applied in our teaching unit, students make plan and control their solutions more often.

Influence of the number of developed solutions on planning and monitoring

As the number of developed solutions is a continuous predictor variable, we have used a linear regression with two predictor variables to answer the second research question. The predictor variables for explanation of the variance in planning activities in post-test were planning measured in pre-test and the number of solutions developed by the students during the teaching unit. Apart from planning in pre-test ($\beta=.42$, $p<.01$), the number of solutions has a significant influence on students' self-reported planning activities in post-test ($\beta=.24$, $p<.01$). The analysis of students' monitoring activities was conducted in the same way and shows the similar result. The monitoring activities measured in pre-test and the number of solutions have a significant impact on monitoring activities in post-test (monitoring in pre-test: $\beta=.54$, $p<.01$; number of solutions: $\beta=.19$, $p=.01$). Students who developed more solutions during the teaching unit planned and controlled their solution more often than students who developed fewer solutions.

One important limitation of the recent study is using questionnaires in order to measure students' metacognitive activities and the number of developed solutions. A validation of these measures in future studies is essential.

The results of this study point out that treatment as well as development of multiple solutions have a positive impact on students' metacognitive activities. Thus, processing of the tasks that required the development of multiple solutions can foster not only performance (Rittle-Johnson & Star, 2007) but also other learning outcomes (Schukajlow & Krug, 2012, 2013). An open question is however, how metacognitive activities, performance and development of multiple solutions link to each other. Development and verification of theories which specify the impact of treating multiple solutions on cognitive and metacognitive variables are an important future research issue.

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