UNCERTAINTY ORIENTATION, PREFERENCE FOR SOLVING TASK WITH MULTIPLE SOLUTIONS AND MODELLING

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In this study 235 ninth graders from ten German middle track classes were asked about their preference for solving tasks with multiple solutions, uncertainty orientation and treating tasks with multiple solutions in their everyday mathematical classes. Preference for solving tasks with multiple solutions and uncertainty orientation were assessed before and after a five-lesson teaching unit promoting modelling competency with either one solution or multiple ones as well as in the control group. The findings show that (1) preference for solving tasks with multiple solutions is connected with students' uncertainty orientation and treating these tasks in the classroom, (2) after the teaching unit, a group where multiple solutions were treated indicated the stronger preference for solving tasks with multiple solutions.

Key words: multiple solutions, affect, modelling, word problems, teaching methods

INTRODUCTION

The principles of high quality teaching mathematics include the use of cognitively demanding tasks, development of multiple solutions, reflecting on, comparing and discussing different solution methods (Silver, Ghousseini, Gosen, Charalambous, & Font Strawhun, 2005). For the recent 10 years, there have been first evidences that the comparison of different solution methods can improve students' mathematical competency, if students have a sufficient prior knowledge in the target domain (Rittle-Johnson, Star, & Durkin, 2009). This finding is in line with the outcomes of studies that compared teaching methods in different countries. Teachers in high-performing countries such as Japan demand from students to find more than only one solution and to discuss solution methods in the classroom. Teachers in the other countries often think that students will be lost in the variety of solutions and therefore do not practice them (Leikin & Levav-Waynberg, 2007).

There is still a lack of studies, which investigate emotions, attitudes and beliefs regarding to the use of multiple solutions. As students' improvement in the affect domain is an important goal of mathematics education, we focus in this study on students' uncertainty orientation, preference for solving problems with multiple solutions, and treatment of (or dealing with) these problems in the classroom. Further, we have investigated the possibilities to influence students' uncertainty orientation and their preference for solving problems with multiple solutions positively. A five-lesson teaching unit promoting multiple solutions while modelling was conducted and evaluated using a 3x1 experimental-control-group design.

This study was carried out in the framework of MultiMa-project (*Multiple solutions for mathematics teaching oriented towards students' self-regulation*) that has been funded by the German Research Foundation since 2011 (SCHU 2629/1-1). MultiMa aims to investigate students' dealing with multiple solutions while modelling, students' affect as well as the development of mathematical competency in learning environments oriented towards self-regulation (Schukajlow & Krug, in press).

THEORETICAL BACKGROUND AND RESEARCH QUESTIONS

Multiple solutions and modelling problems

The analysis of solving problems showed that there are three types of multiple solutions (see a similar approach by Tsamir, Tirosh, Tabach, & Levenson, 2010). The first type of multiple solutions can be conducted due to the variation in *mathematical* solution methods. The second type of multiple solutions can be developed if students solve problems with missing data. For solving these problems, they have to take assumptions about the missing data and thus, get different outcomes. The third type of multiple solutions includes the variation in mathematical solution methods as well as in different outcomes.

The missing data is one of typical features of modelling problems. The core of modelling activities are demanding transfer processes between reality and mathematics (Blum, Galbraith, Henn, & Niss, 2007). We illustrate different types of multiple solutions using the task "Parachuting". For calculation of hypotenuses while solving this problem, students can use as mathematical procedure either Pythagoras' Theorem or scale drawing. Further, in order to solve the problem, they have to take assumptions about some data such as wind power and can get different results using the same mathematical method.

Parachuting

When "parachuting", a plane takes jumpers to the altitude of about 4000 metres. From there they jump off the plane. Before a jumper opens his parachute, he free falls about 3000 metres. At an altitude of about 1000 metres the parachute opens and the sportsman glides to the landing place. While falling, the jumper is carried off target by the wind. Deviations at different stages are shown in the table below.

Wind speed	Side deviation per thousand metres during free fall	Side deviation per thousand metres while gliding
Light	60 metres	540 metres
Middle	160 metres	1440 metres
Strong	340 metres	3060 metres

What distance does the parachutist cover during the entire jump?

Affect and modelling

Stability (state vs. trait) and functionality (truths, feelings and preferences) are important characteristics of affective domain (Hannula, 2012). In this study, we focus on the personal trait "uncertainty orientation" and students' preference for solving tasks with multiple solutions.

The uncertainty orientation (Sorrentino & Roney, 1999) describes a person's typical ways of dealing with complexity, uncertainty, and abundant information (Hänze & Berger, 2007). Uncertainty-oriented persons are interested in complex situations and use these situations to gain the new knowledge. Persons with strong certainty orientation look for situations, which are already familiar. Huber, Sorrentino, Davidson, Eppler, and Roth (1992) found that students with uncertainty orientation learn more, if cooperative teaching methods were applied in the classroom. However, this result could not be confirmed in further studies (Hänze & Berger, 2007). One possible explanation of this inconsistence is that the level of structuring the learning material such as the type of tasks, influences the learning processes. The treatment of problems, in which all data are given, meets the needs of certainty-oriented students. The treatment of problems with missing data, whereas the development of multiple solutions is demanded, meets the needs of uncertainty-oriented ones. As the uncertainty orientation is a personal trait regarded to the dealing with uncertainty in different situations, we do not expect that solving the tasks with multiple solutions in the classroom can change this trait.

Preferences are closely connected with motivational constructs (Hannula, 2012). They can regard to the global objects such as learning, or to the specific situations, such as dealing with different kinds of tasks. The assessment of task-specific measures allows to collect information about students' affective dimensions and can help to answer questions that are specific for mathematics educations. Task-specific measures are more sensitive than traditional instruments and can be used for evaluation of short-term interventional studies, where significant changes in motivational traits using traditional scales could not be expected. In the study by Schukajlow et al. (2012), there was found that student-centred teaching method for fostering modelling competency improved students' *task-specific* enjoyment, interest and self-efficacy expectations. The nearby average relationship of $r=0.27$ between treating tasks with multiple solutions and students' preference for solving these tasks (Krug & Schukajlow, 2012) indicates that treating tasks with multiple solutions can influence students' preference in a positive way. One goal of this study is to replicate this result. Another goal is to investigate, whether treating multiple solutions while solving modelling problems influences students' preference for solving problems with multiple solutions positively. "Treating tasks with multiple solutions" means in this context that students work on problems, which demanded the development of more than one solution.

Research questions

The research questions of the study are:

1. How strong is a relationship between students' uncertainty orientation, preference for solving tasks with multiple solutions and dealing with this kind of tasks in the everyday classroom?
2. Does treating multiple solutions while solving modelling problems influence students' uncertainty orientation?
3. What is the impact of treating multiple solutions while solving modelling problems on students' preference for solving tasks with multiple solutions?

METHOD

Design and sample

235 German ninth graders (46% females; mean age=15.4 years, SD=0.62) were asked about their preference for solving tasks with multiple solutions and their uncertainty orientation before and after five-lesson teaching unit. Further, before the teaching unit the students were asked, how frequently they solved tasks with multiple solutions in their regular classes. In some German federal states, students, who completed the fourth grade, are assigned to the low, middle or high track classes depended on their performance level. The middle track classes were chosen for this study because in these classes the students of all three performance levels are to be found. Ten middle track classes (Realschule) from ten comprehensive schools (German Gesamtschule) and 8 teachers with at least two years' experience of teaching mathematics participated in the study. We assume that experienced teachers could better implement the instructions of the study.

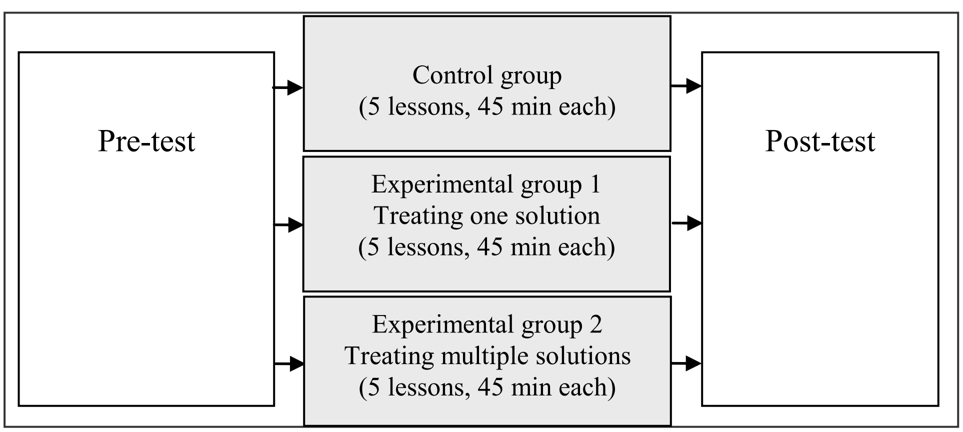

Figure 1: Overview of the study design

The sample consists of one control group and two experimental groups. The experimental groups needed a strong support by the research team, so six classes from three schools near the university were assigned to experimental groups. The remaining four classes were assigned to the control group. During the five-lesson period 105 students of the control group were not allowed to solve modelling problems – solving of which may lead to comparison of different solutions – as well as to the problems to the "Pythagoras' theorem". To control, whether teachers really used only these types of problems, all the tasks solved by the students of control group were collected and analysed. The task analysis showed that no problems that required multiple solutions were treated.

Three schools with two middle track classes each were assigned to the experimental groups. Each of six classes was divided into two parts with the same number of students in such a way that the average achievements in both parts did not differ and there was the approximately same ratio of males and females in each part. In one part of each class one solution of modelling problems (experimental group 1: "one solution") and in the other part multiple solution of modelling problems (experimental group 2: "multiple solutions") were treated. During the teaching unit, students of both experimental groups were taught using modelling problems about "Pythagoras' theorem". The topic "Pythagoras' theorem" had already been treated using mathematical tasks without connection to reality before the MultiMa teaching unit, in order to foster the use of this powerful mathematical procedure and to prevent applying different solution methods.

To implement the treating of modelling tasks with and without multiple solutions, two teaching scripts were developed. Four teachers, who had to give lessons in experimental groups, received these scripts with all tasks to be treated and a detailed plan of the teaching unit. Further, they were instructed about the specific ways to promote multiple solutions vs. one solution. Each teacher taught the same number of student groups in the experimental group "one solution", as well as in the experimental group "multiple solutions", so the influence of a teacher personality on students' learning did not differ between both groups. In each lesson that was provided in the experimental groups, one member of the research team was present to videotape and to observe the implementation of the instructions.

In both experimental groups the same methodical order was committed. Students solve a modelling task according to a special kind of group work (alone, together and alone again) (Schukajlow et al., 2012). A solution (or different solutions) of the first modelling task is presented in the first lesson by the teacher and in the following ones by the students. The teacher summarizes and reflects on the key points of each group. In the group "multiple solutions" the teacher emphasises the development of different outcomes by estimating the missing data.

In order to stimulate the development of multiple solutions in the group "multiple solutions" and to prevent the development of more than one solution in the other experimental group, two similar versions of each treated task were developed. In the group "one solution" students solved among other tasks the task "Parachuting", where the data, needed to solve the task, were given. These data were the wind velocity and the altitude, in which the parachute opens. The question posed in the task was: "What distance does the parachutist cover during the whole fall, if the wind speed is middle?” The similar task in the group “multiple solutions” demanded the development of two solutions. The question posed in the task was: “What distance does the parachutist cover during the entire jump? Find *two* possible solutions” (see sample tasks by Schukajlow & Krug, 2012b).

Measures

The main difference between experimental groups was the demand to develop one solution or multiple solutions. We proved this key using students’ questionnaires. After every lesson the students were asked about the number of solutions they developed for each modelling problem in this lesson. For example: “While solving the problem “Parachuting” I developed today (0: no solution; 1: one solution; 2: two or even more solutions)”.

Students’ self-perceptions were measured using a 5-point Likert scale (1=not at all true, 5=completely true) before and after a five-lesson teaching unit. A sample item was for uncertainty orientation (5 items) “I like unexpected surprises”. The scale uncertainty orientation was adapted from the studies by Dalbert (1996) and Hänze & Berger (2007). Students’ preference for tasks with multiple solutions (6 items) was developed using the theoretical background to multiple solutions. This scale consists of questions about three issues: (1) multiple mathematical solution methods (“While working on mathematical problems, I like different calculations leading to success.”), (2) multiple outcomes (“While working on mathematical problems, I like to get different results”) and (3) multiple solutions in general (e.g. “While working on mathematical problems, I like to use different solution methods.”) Treating multiple solutions in the everyday mathematics classroom was measured using 6 items that are similar to the scale “preference for tasks with multiple solutions” (e.g. “In mathematics, we often work on problems that offer different solution methods”). Using of 6 items for measurement of each scale is one limitation of this study. The reliability values (Cronbach’s Alpha) for uncertainty orientation were 0.86 and 0.87, those for the preference for tasks with multiple solutions 0.77 and 0.85 in the pre- and the post-test respectively, and the reliability value for treating multiple solutions in mathematics classroom was 0.81 in the pre-test. More information about the validity of the measures should be collected in the future studies. All measures were a part of the longer questionnaires. Pre- and post-tests took 30 minutes each.

RESULTS

Preliminary analysis

First, we compared the number of solutions developed by both groups, “multiple solutions” and “one solution” (Schukajlow & Krug, 2012a). We analysed students’ answers using the t-test. The analysis shows that there are significant differences between the numbers of solutions that were developed in the respective groups ($T(138)=6.7$; $p<0.001$; effect size Cohen’s $d=1.16$). Whereas the majority of the students in the group “multiple solutions” developed two and more solutions (mean=1.55, standard deviation SD=0.39), students in the group “one solution” reported on the development of one solution only (mean=1.14, SD=0.33). The analysis of the number of students’ solutions in the experimental groups and that of the tasks treated in the control group indicates that it was possible to realise the instruction conditions as it was intended in the study.

Uncertainty orientation, preference for solving tasks with multiple solutions and treating multiple solutions in the everyday mathematics classes

To answer the first research question, we analysed the relationship between students’ perceptions in the pre-test. There are low to middle statistically significant correlations between respective measures (cf. Tab. 1). Uncertainty-oriented students more frequently report on treating problems with multiple solutions in the classroom and on their preference to solve these problems than students without uncertainty orientation. The middle correlation of 0.46 was measured between students’ preference for solving tasks with multiple solutions and the treatment of the tasks with multiple solutions in the classroom. As a correlation between treating multiple solutions in the classroom and uncertainty orientation is weak, it is possible that the treatment of such problems foster students’ uncertainty orientation, but not significantly.

	UncO	**PrMS**	**MSC**
UncO	1	0.26*	0.17*
PrMS		1	0.46*
MSC			1

* The correlations are at least significant at the 5% level

Table 1: Pearson correlations between uncertainty orientation (UncO), preference for problems with multiple solutions (PrMS) and treating multiple solutions in the classroom (MSC)

The second research question pertains to the effect of treating modelling problems with multiple solutions on students’ uncertainty orientation. Is it possible to change students’ uncertainty orientation after the five-lesson teaching period and are there any differences between control and experimental groups? In order to test, whether the factor “type of intervention” influences students’ uncertainty orientation in post-test, ANCOVA (covariate: uncertainty orientation in pre-test) was conducted. The ANCOVA revealed a significant effect of the pre-test score ($F(1, 213)=84.60$, $p<0.05$), but no effect of “type of intervention” on the uncertainty orientation ($F(2, 213)=1.53$, $p=0.22$). This result shows that treating multiple solutions while solving modelling problems does not influence students’ uncertainty orientation.

The effect of the “type of intervention” on students’ preference for problems with multiple solutions (research question 3) was also computed using ANCOVA (covariate: PrMS in pre-test). The ANCOVA showed effects of PrMS in pre-test on the PrMS in post-test ($F(1, 215)=56.43$, $p<0.05$) and a significant impact of the factor "type of intervention" on students' preference for problems with multiple solutions in post-test ($F(1, 215)=6.87$, $p<0.05$). Students in the group "multiple solutions" report in post-test with respect to pre-test on stronger preference for problems with multiple solutions compared with the students in the group "one solution" or in the control group. No differences in this scale were found between the group "one solution" and the control group.

		Control group	**One solution**	**Multiple solutions**
		M(SD)	M(SD)	M(SD)
UncO	pre	3.66(0.89)	3.61(0.94)	3.50(0.92)
	post	3.32(0.99)	3.49(1.03)	3.41(1.03)
PrMS	pre	3.22(0.85)	3.29(0.90)	3.35(0.73)
	post	2.98(0.92)	3.03(1.02)	3.56(0.87)
MSC	pre	2.96(0.78)	3.05(0.78)	3.04(0.77)

Table 2: Students' uncertainty orientation (UncO), preference for problems with multiple solutions (PrMS) and treating multiple solutions in the classroom (MSC) in the control and experimental groups

DISCUSSION

In the study reported here, students' uncertainty orientation, preference for solving tasks with multiple solutions and dealing with this kind of tasks in the everyday classroom were assessed. Between uncertainty orientation and students' preference for solving tasks with multiple solutions, a small statistically significant relationship was found. This relationship shows that students who were looking for unfamiliar situations like to solve the tasks that demand taking assumptions, allow choosing different mathematical methods and having different mathematical results. So, the results of the study support the assumption by Hänze & Berger (2007) that the level of structuring the material can influence students' learning. However, we did not gain any results, whether uncertainty-oriented students prefer solving well-defined tasks more than certainty-oriented students. It is possible that uncertainty-oriented students have more positive perceptions of solving any kind of mathematic tasks than certainty-oriented students. One important future research field is therefore to compare students' preference for tasks with multiple solutions with students' preference for clear-structured, well-defined tasks that have one right solution only, taking in account students' personal traits such as uncertainty orientation. Another important question concerns the connection between uncertainty orientation and the construct of identity. Uncertainty orientation may be a part of identity which influences our actions in learning situations.

The average correlation between the dealing with tasks required multiple solutions and preference for solving these tasks means that students who frequently report on solving the tasks with multiple solutions like to solve this kind of tasks more than students that solve them rarely. This finding confirms our previous results (Krug & Schukajlow, 2012) and indicates that treatment of tasks with multiple solutions can improve students' preference for solving this type of problems.

In order to prove the direction of the assumed connection between both measures, an experimental study was conducted. In the experimental groups modelling problems with and without multiple solutions were treated five lessons long. The analysis of the data showed that students of the group, where multiple solutions were treated, liked the tasks with multiple solutions more than the students who solved modelling tasks with one solution only, or than the students of the control group with respect to their pre-test. As no differences between the group "one solution" and the control group were found, the changes in the students' preference could not be attributed to the special kind of student-centred teaching method applied in the experimental groups. Also, we showed that a students' task-specific motivational construct regarding to multiple solutions can be changed positively. The positive changes in students' preference for tasks with multiple solutions can promote students' involvement in the content activities and, as a result, improve their performance. Investigation of relationships between students' preferences, emotions, beliefs and performance concerning different type of problems is an important future research field. Finally, we showed that the personal trait uncertainty orientation did not differ between the experimental groups and the control group. For changing this stable trait, a special long-term training program regarding different domains is needed.

As in this study we used new scales with a limited number of items, and as there are only few studies in the domain of mathematics that investigated students' preferences regarding to multiple solutions or their uncertainty orientation, a replication of our results in future research is essential.

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