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The power of emotions: Can enjoyment and boredom explain the impact of individual preconditions and teaching methods on interest and performance in mathematics?

Abstract We investigated students' emotions as intervening variables between teaching methods, motivational and performance prerequisites, and outcomes. 144 students from German schools were assigned to two conditions. In one condition, students were prompted to develop multiple solutions for modelling problems that were missing information. In the other condition, students had to find one solution for modelling problems that were not missing information. Students' interest and performance were measured before and after the 5-lesson teaching unit, and students' enjoyment and boredom were measured during the teaching unit. The path analyses revealed: (1) Students who developed more solutions enjoyed their mathematics lessons more and were less bored than students in the other condition; (2) Enjoyment affected students' interest and performance at posttest and mediated the effects of prompting them to find multiple solutions on interest at posttest; (3) Students' enjoyment during learning mediated the effects of prior interest on interest at posttest.

Keywords: interest, performance, multiple solutions, enjoyment, boredom

Highlights

- We examined the roles of enjoyment and boredom in mathematics instruction
- Prompting multiple solutions affected enjoyment positively and boredom negatively
- Enjoyment during learning positively affected interest and performance at posttest
- Enjoyment mediated the effects of prompting multiple solutions on interest
- Prior interest indirectly influenced interest at posttest via enjoyment

1 Introduction

Emotions are important for students' learning of mathematics (Goldin, 2014; Zan, Brown, Evans, & Hannula, 2006). In recent decades, a control-value theory of achievement-related emotions was proposed (Pekrun, 2006), and research confirmed that perceived control and the value of learning activities were antecedents of achievement emotions (Buff, 2014; Pekrun, Cusack, Murayama, Elliot, & Thomas, 2014). However, more research on the connections between emotional, motivational, and cognitive variables in mathematics education is needed with regard to the following issues: the predictors of emotions in the classroom, the influence of classroom emotions on students' motivation and achievement, and most important, the teaching methods that positively influence students' emotions (see overview by Pekrun & Linnenbrink-Garcia, 2014). In this study, we aimed to connect two research fields: the psychology of emotions and learning. Further, because students' performance and interest are both considered to be important for learning mathematics (Schukajlow & Krug, 2014b; Zan et al., 2006), we sought to contribute to the improvement of the practices applied in schools by examining the effectiveness of a teaching method for cognitive and motivational outcomes.

We assessed emotions during a teaching unit that aimed to promote students' interest in mathematics and their ability to solve modelling problems. Modelling problems are demanding tasks that are connected to reality, and the ability to solve these problem is important for students' current and future lives (Niss, Blum, & Galbraith, 2007). In the current study, we proposed and tested a theoretical model in which a teaching method that prompts students to find multiple solutions, the number of solutions developed, students' prior interest, and students' prior performance affected emotions in the classroom, which then influenced their interest and performance.

2 Prior research, theoretical model, and hypotheses

2.1 Enjoyment, boredom, interest, and performance: What do we know about their relations?

2.1.1 Enjoyment and boredom as achievement emotions

Emotions are defined as complex phenomena that include affective, cognitive, physiological, motivational, and expressive parts (Pekrun & Linnenbrink-Garcia, 2014). Enjoyment and boredom are important for human life as they are connected to career aspirations and career choice (Wigfield, Battle, Keller, & Eccles, 2002) and were identified as the second and third most prevalent reasons for not continuing with mathematics after graduation from secondary school (Brown, Brown, & Bibby, 2008). We conceptualized enjoyment and boredom in a domain-specific manner and assessed these emotions while students were engaged in task processing because task processing was found to be one of the main activities in mathematics classrooms (Hiebert et al., 2003). While enjoyment was conceptualized as "having fun," the focus of boredom was on having trouble remaining alert and continuing to work.

In the control-value theory of achievement emotions, appraisals of the value and control of learning activities and outcomes are assumed to be important for activating achievement-related emotions (Pekrun, 2006). In the context of non-routine problem solving, students with high appraisals of control and who value of their activities can be expected to enjoy learning and not be bored during learning activities. Positive changes in perceived control and value were found to lead to positive changes in school students' enjoyment of mathematics and college students' boredom (Buff, 2014; Goetz, Pekrun, Hall, & Haag, 2006).

2.1.2 Enjoyment, boredom, and interest

Interest represents a specific person–object relation and is characterized by a person engaging and reengaging with this object over time (Hidi & Renninger, 2006; Krapp, 2005). A positive relation between interest-related measures and enjoyment was found by students at school and at university (Pekrun, Goetz, Frenzel, Barchfeld, & Perry, 2011; Tulis & Ainley, 2011). The expectation that "… personal interest in the activity domain can give rise to appraisals of controllability and value – promoting students positive emotions, such as enjoyment of learning" (Ainley & Hidi, 2014, p. 217), among other effects, was confirmed in studies in the context of self-regulation, where effects of motivational beliefs, including interest, were found on positive emotions (Ahmed, van der Werf, Minnaert, & Kuyper, 2010; Winberg, Hellgren, & Palm, 2014). Hence, prior interest should have a positive effect on enjoyment. However, a lack of interest in combination with a requirement to solve problems during regular mathematics classes can trigger anger, anxiety, or other negative emotions but not obviously boredom (Pekrun, Hall, Goetz, & Perry, 2014). To our knowledge, the effects of prior interest on students' boredom have not yet been tested. On the basis of prior research, we expected that students' prior interest would predict enjoyment, whereas the effect of a lack of prior interest on boredom could not clearly be derived from prior research and was based on theoretical assumptions from the control-value theory of achievement emotions (Hypothesis 1).

Emotions during learning might not only predict interest-related measures but may also be influenced by them. Positive affective experiences, including enjoyment, during learning accompany situational interest and are important for triggering students' individual interest (Ainley, 2007; Krapp, 2006). Moreover, students' positive affective experiences predicted motivation in a longitudinal study in mathematics over one year (Buff, Reusser, Rakoczy, & Pauli, 2011) and supported the expectation of positive effects of enjoyment on interest (Hypothesis 2). The relation between boredom and interest-related measures is not yet clear. Most studies have found a negative correlation between boredom and interest (Pekrun, Hall, et al., 2014; Vogel-Walcutt, Fiorella, Carper, & Schatz, 2012). These results suggest negative effects of boredom on interest (Hypothesis 2). However, the effect sizes of correlations between boredom and interest-related measures were lower than correlations between enjoyment and interest-related variables (correlation of .45 between enjoyment and intrinsic motivation and correlation of -.23 between boredom and intrinsic motivation by Pekrun et al., 2011). One possible explanation for these weak relations may be that the learning process of highly interested students can be accompanied not only by positive but sometimes also by negative emotions (Ainley & Hidi, 2014).

As prior interest should affect enjoyment and boredom, and as enjoyment and boredom should influence interest at posttest, we expected prior interest to have an indirect effect on interest at posttest with emotions as intervening variables (Hypothesis 3).

2.1.3 Enjoyment, boredom, and students' performance

Students' prior achievements can predict students' control and value beliefs, and via these appraisals, their enjoyment and boredom as well (Pekrun, 2006). Empirical evidence for potential effects of prior performance on enjoyment can be found in the correlations between students' grades in mathematics at the beginning of the school year (fall, T1) (Ahmed, van der Werf, Kuyper, & Minnaert, 2013) and enjoyment or boredom measured during the school year (winter, T2; spring, T3). Further evidence comes from positive correlations between students' mathematical performance in grades 3 and 6 and enjoyment in grades 6 and 9, respectively (Hannula, Bofah, Tuohilampi, & Metsämuuronen, 2014). School students' boredom was not related to grades (Ahmed et al., 2013), but college students' boredom was predicted by prior grades in three of four estimated regressions in a study by Pekrun et al. (2014). On the basis of these findings, we expected positive effects of prior performance on enjoyment (Hypothesis 1). As no effects of prior achievements were found at school and partially found at university, we considered theory and expected a negative influence of prior performance on boredom (Hypothesis 1).

Not only can the impact of performance and interest on emotions be assumed but also the opposite direction. Students' enjoyment is positively related to grades at school and university (.22 and .46, respectively, Goetz, Frenzel, Pekrun, Hall, & Lüdtke, 2007; Pekrun et al., 2011), and positive changes in enjoyment produce positive changes in students' grades (Ahmed et al., 2013). The relation between enjoyment and performance assessed by competence tests has been investigated less often and has ranged from .15 to .45, depending on the type of questionnaire and the kinds of problems used to measure performance (Schukajlow & Krug, 2014a). An analysis of the reciprocal effects of enjoyment and students' performance in primary and early secondary school, however, did not confirm that mathematics performance could be predicted by students' enjoyment (Hannula et al., 2014). Because of the positive correlation between enjoyment and achievement (Goetz et al., 2007; Schukajlow & Krug, 2014a) and the positive effects of students' enjoyment on students' grades (Ahmed et al., 2013; Mega, Ronconi, & De Beni, 2014), we expected that enjoyment would positively influence students' performance (Hypothesis 2). A pattern similar to enjoyment has also been revealed for the effect of boredom on student achievement. College students' boredom was negatively correlated with their grades (Pekrun, Goetz, Daniels, Stupnisky, & Perry, 2010), and school students' boredom was negatively related to academic grades across different school subjects (Goetz et al., 2007). The correlation between 9th-graders' boredom and mathematics performance ranged from -.36 to .0 (Schukajlow, 2016) and thus had a slightly weaker effect size than the correlation between enjoyment and performance. Results of longitudinal studies have indicated negative effects of boredom on students' grades at school (Ahmed et al., 2013) and at university (Pekrun et al., 2010; Pekrun, Hall, et al., 2014). Specifically in the mathematical domain, an increase in boredom in mathematics by 7th-graders was connected to a decrease in students' mathematics grades across a school year (Ahmed et al., 2013). Hence, we hypothesized a negative effect of boredom on students' performance (Hypothesis 2).

Moreover, we expected indirect effects of prior performance on students' performance at posttest, with enjoyment and boredom as intervening variables (Hypothesis 3).

2.2 Prompting multiple solutions and students' enjoyment and boredom

Prompting students to find multiple solutions in the classroom is identified as a high-quality element of teaching in the principles and standards for teaching mathematics in different countries (e.g. National Council of Teachers of Mathematics, 2000). The effectiveness of the development and the comparison of multiple solutions for students' conceptual knowledge, flexibility, and creativity has been shown in a number of studies (Levav-Waynberg & Leikin, 2012; Rittle-Johnson & Star, 2009). However, it has yet to be determined whether this method offers benefits over prompting students to find one solution with respect to students' performance, emotions, and motivation (Schukajlow, Krug, & Rakoczy, 2015). In the present study, we focused on multiple solutions due to information that is missing from the modelling problem, thus resulting in different outcomes. In the sample problem "Half-Timbered House" (Figure 1), the diameter of the log (20 cm) is given in the task, and students are asked find the maximal length and width of the rectangular cross-section of a piece of timber that can be cut from this log. Students can assume that the length of the piece of timber can be 10 cm or 20 cm, construct a rectangle as a mathematical model for this problem, and calculate the widths using Pythagoras' Theorem as the mathematical procedure. With these assumptions, they can calculate the measurements of the rectangle, interpret them as the length and width of the piece of timber, and validate their results using their real-world knowledge (see more about modelling by Schukajlow et al., 2015; Schukajlow et al., 2012).

Half-timbered house

In Germany, there are more than one million half-timbered houses. You can see a scaffolding construction of a half-timbered house in the picture on the right. In such a construction, wooden beams forming rectangular areas are used. In a lumber mill, only one piece of timber is made from each log to minimize the amount of wood chips.

In the picture below, you see a log. What is the maximum length and width of a rectangular cross-section of timber that can be cut from a log with a diameter of 20 centimeters?

Figure 1. Modelling problem "Half-timbered house."

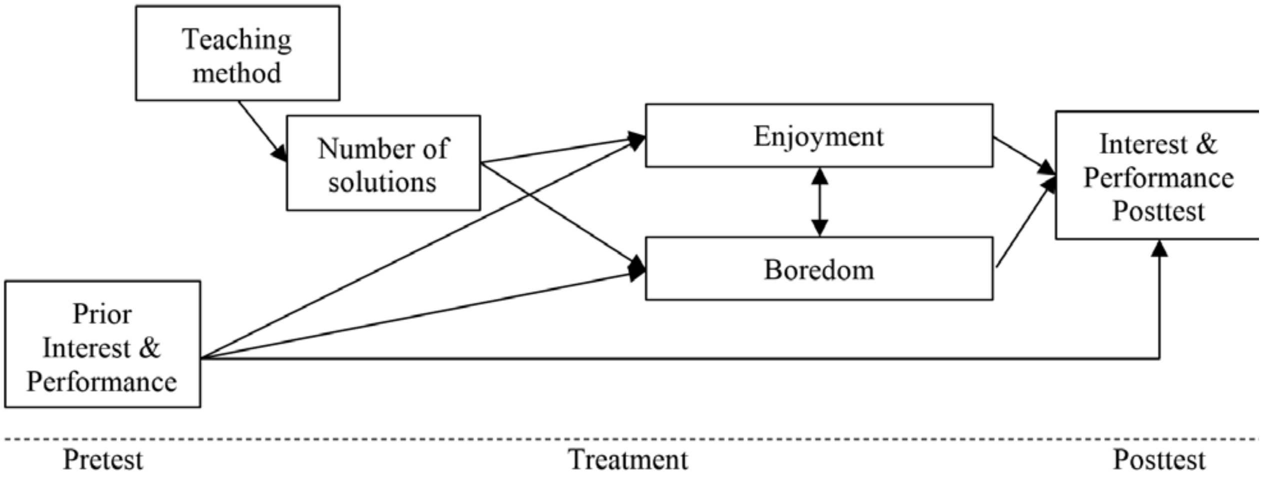
Two lines of evidence support the theoretical implications of the impact on students' emotions of prompting and constructing multiple solutions while solving modelling problems with missing information (Hypothesis 4). First, prompting students to find multiple solutions in the classroom enhanced the number of solutions developed, and the development of multiple solutions strengthened students' control appraisals such as perceived competency and autonomy (Schukajlow & Krug, 2014b) as well as their use of monitoring and self-regulation strategies (Schukajlow & Krug, 2012, 2013a). Higher appraisals of control, in turn, trigger enjoyment and decrease boredom. Second, modelling problems that are missing information offer students the opportunity to make their own assumptions and develop their own solutions. As prompting students to find multiple solutions for modelling problems that are missing information enlarges the number of personally relevant solutions, students' value appraisals of their problem-solving activities in the classroom are positively influenced. The increased value appraisals positively affect enjoyment and negatively affect boredom. Furthermore, we predicted indirect effects of prompting and constructing multiple solutions on students' interest and performance at posttest, with enjoyment and boredom as intervening variables (Hypothesis 5).

2.3 Research questions, path model, and hypotheses

The research questions of the present study consisted of the following: Do prior interest and performance predict students' enjoyment and boredom in the classroom (see Hypothesis 1)? How do these emotions influence students' interest and performance (see Hypothesis 2)? Does teaching multiple solutions affect enjoyment and boredom (see Hypothesis 4)? Via these emotions, do students' prerequisites and the teaching method also influence interest and performance at posttest (see Hypothesis 3 and 5)?

For an analysis of the links between the teaching method and students' emotional, motivational, and cognitive prerequisites and outcomes, we applied two path models to the data (one with interest and one with performance as the independent and dependent variables). This allowed us to test the direct effects on emotions, interest, and performance and additionally to examine whether enjoyment and boredom would transmit the effects of experimentally manipulated treatment conditions or motivational and cognitive prerequisites on the outcomes. Path models connect the treatment condition or individual prerequisites (e.g., the teaching method, prior interest, or performance) with the outcome (e.g., interest or performance after the intervention) via mediators or intervening variables (e.g., the number of solutions developed and students' enjoyment or boredom).

On the basis of theoretical considerations and prior research on the effects of prompting students to construct multiple solutions and students' motivational and cognitive prerequisites on students' enjoyment, boredom, interest, and performance, we hypothesized a path analytic model (Figure 2).



Note: The double-headed arrow represents a correlation between two measures, and the directed arrows (other paths) represent the direction of the regressions for the respective measures.

Figure 2. Hypothesized path-analytic mediation model.

The following primary hypotheses were tested:

Hypothesis 1 (Prior interest and performance -> Emotions). Prior interest and performance will positively affect enjoyment and will negatively affect boredom during mathematics classes.

Hypothesis 2 (Emotions -> Interest and performance at posttest). Enjoyment will positively affect students' interest and performance at posttest, whereas boredom will negatively affect students' interest and performance at posttest.

Hypothesis 3 (Prior interest and performance -> (via Emotions) -> interest and performance at posttest). Prior interest and performance will indirectly affect interest and performance at posttest with positive effects transmitted through enjoyment and negative effects transmitted through boredom during mathematics classes.

Hypothesis 4 (Number of solutions and teaching method -> emotions). Students who develop more solutions will enjoy mathematics classes more and will be less bored during problem solving; Prompting students to find multiple solutions as a teaching method will positively indirectly affect students' enjoyment and negatively indirectly affect boredom in the classroom with the number of solutions developed as the intervening variable.

Hypothesis 5 (Teaching method and number of solutions -> (via Emotions) -> Interest and performance at posttest). Prompting students to find multiple solutions will have positive indirect effects on students' interest and performance at posttest through the number of solutions developed and enjoyment and boredom during mathematics classes as intervening variables. The number of solutions developed will have positive indirect effects on students' interest and performance at posttest through enjoyment and boredom during mathematics classes.

3 Method

3.1 Participants and Procedure

144 ninth-graders (43% female; mean age 15.22 years (SD = 0.60)) from three comprehensive schools (German Gesamtschule) that each had two middle-track classes took part in this study. Students from each of the six classes were assigned to one of two parts of the class with the same number of students in each part in such a way that the average achievement level in the two parts did not differ, with students' achievements estimated by their final grades in math. The ratio of males and females was approximately equal in each part of the class, and each part was instructed in a separate room. One part of each classroom was assigned to the multiple-solution condition (students were prompted to

find multiple solutions to modelling problems that were missing information); the other part was assigned to the one-solution condition (students were prompted to find one solution to similar modelling problems that were not missing information). Before and after the teaching unit, students completed a questionnaire about their interest and a performance test. The teaching unit consisted of five 45-minute long lessons that were presented across three sessions. The first and second sessions consisted of two 45-minute long lessons each, with no break between the lessons. After the first and second sessions, students filled out questionnaires about the number of solutions they developed in the previous lessons and their enjoyment and boredom during the teaching unit (cf. Figure 3).

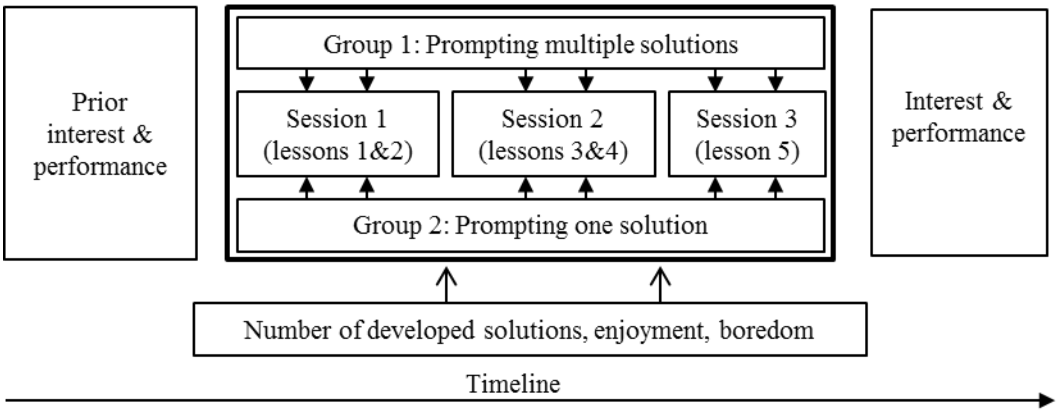


Figure 3. Overview of the study design.

Each group was instructed by one of four mathematics teachers (25 to 54 years of age; two female). One teacher instructed six groups, and the other three teachers instructed two groups each. In order to balance the impact of teachers' personality on students' learning between the two conditions, each teacher instructed the same number of multiple-solution and one-solution groups. The teaching manuals included modelling problems, solution spaces, and lesson plans for each instructional condition and were discussed before the study began.

Both treatment programs were based on the same student-centered teaching method whose effectiveness was established in previous studies in comparison with teacher-centered instruction for enjoyment, interest, and performance (Schukajlow et al., 2012). In the multiple-solution condition, the modification "Find two possible solutions. Write down both solution methods" was added to the problems in the first and second sections of the teaching unit. As people in the real world have to find one solution only, the two problems that were missing information and that were given in the last lesson were not modified and were similar to the "Half-timbered house" problem. In the one-solution condition, students solved similar tasks with the only modifications being that all important information was given in the problem and one solution had to be developed. In the sample problem "Half-timbered house," the modified sentence was "In such a construction, wooden beams forming *square* areas are used."

The fidelity of the treatment was ensured by using the following procedures. The teachers had experience in teaching modelling problems and were instructed for one day about the teaching methods that they should apply in the present study. All lessons were observed and videotaped by research group members, and students' solutions were collected. Analyses of videos and solutions confirmed that (1) the teachers provided the students with the previewed tasks, which requested that the students construct multiple solutions (multiple-solution condition) or one solution (one-solution condition); (2) the same previewed methodical order in both conditions (goals of teaching unit, group work, presentation and discussion of solution/solutions, summarizing the key points of the lesson) was applied in both conditions (see more detailed by Schukajlow & Krug, 2014b); (3) the time available for instruction did not differ between experimental conditions.

3.3 Measures

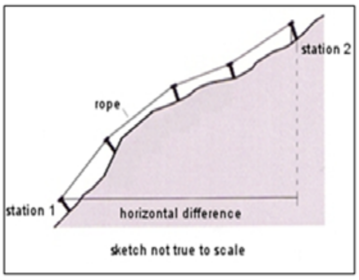
3.3.1 Interest and performance tests

Interest was measured with a well-evaluated 5-point Likert scale ranging from 1 (not at all true) to 5 (completely true) (Frenzel et al., 2012). The three items were (a) “I am interested in mathematics,” (b) “I like to read books and solve brain teasers related to mathematics,” and (c) “Doing mathematics is one of my favorite activities.” The reliabilities (Cronbach’s alpha) were .80 and .74 at pre- and posttest, respectively (Schukajlow & Krug, 2014b).

The performance test consisted of modelling and intra-mathematical problems on the topic Pythagoras’ Theorem. Students’ correct solutions were coded 1 and wrong solutions 0. The time allocated for each test booklet was one hour. The item difficulties of the two mathematics scales can be approximated by a two-dimensional Rasch model (Bond & Fox, 2001). This model allowed us to construct parallel test versions (with no item overlap) for each scale (here, performance in solving modelling or intra-mathematical problems) at two measurement points (pre- and posttest) so that students’ achievement was measured on an interval scale and could be compared using parametric statistical procedures. As each student solved similar but not identical items at pretest and posttest, students’ performance could be measured accurately, and memorization effects were minimized. The ConQuest software (Wu, Adams, & Wilson, 1998) was used to scale students’ performance data. Weighted likelihood estimator (WLE) parameters (Warm, 1989) were estimated for each student and characterized students’ performance using continuous scales. In the current study, we were interested in the students’ performance in modelling and thus used 18 modelling problems from the test (Schukajlow et al., 2015). The EAP/PV test reliability for modelling was .66. A sample item from the modelling test is presented in Figure 4.

Cable car

The rope of the Ristis cable car has to be replaced. 1 meter of the rope costs 8 €. How much does a new rope cost approximately? Write down your solution method.



Name:	“Ristis” Cable Car
Station 1:	1600 meters above sea level
Station 2:	1897 meters above sea level
Horizontal difference:	869 meters
Weight capacity:	132 x 3 persons
Hauling capacity:	1200 persons per hour
Speed:	1.5 meters per second
Time of travel:	10 minutes

Figure 4. A sample problem from the modelling performance tests (Schukajlow et al., 2015).

3.3.2 Scales for number of solutions developed, enjoyment, and boredom

Number of solutions developed, enjoyment, and boredom were assessed using 4- or 5-point scales. After the second lesson, students indicated the number of solutions they developed for the first and second problems and after the fourth lesson for the number of solutions they developed for the third and fourth problems. One sample item was “While solving the ‘Half-timbered house’ problem, I developed (0 = no solution; 1 = one solution; 2 = two solutions, 3 = more than two solutions) today.” In order to simplify the interpretation of this scale, the last two categories were aggregated into the category “two or more solutions.”

Students' enjoyment and boredom were assessed on a scale ranging from from 1 (not at all true) to 5 (completely true). Each scale included three items (see Table 1) and was adapted from the well-evaluated Achievement Emotions Questionnaire, AEQ (Pekrun et al., 2011). The Cronbach's alpha reliabilities were .84 and .82 for enjoyment and .85 and .86 for boredom. Enjoyment and boredom measured two times during the treatment were aggregated over time into a mean value for enjoyment and a mean value for boredom, respectively. The same procedure was also applied for the number of solutions developed.

Table 1. Items used in the study to assess enjoyment and boredom

Scale	Item
Enjoyment	I enjoyed task processing. I was happy during task processing. Task processing was great fun for me.
Boredom	Task processing was boring. I got so bored during task processing that I had problems staying alert. I did not want to continue my work because it was so boring.

3.4 Data analysis

We tested the hypothesized model with regard to interest and performance using the number of solutions developed, enjoyment, and boredom as intervening variables. All regression analyses employed dummy codes for the treatment factor (0 = one solution; 1 = multiple solutions).

To test the hypotheses, two path models (one for interest and one for performance) with 14 free parameters and 144 subjects were used. The ratio of subjects to parameters was 11 (144/14), which was above the critical value of 5 for obtaining solid results (Kline, 2005).

3.4.1 Clustering of the data

To increase the external validity of the current study, the students were instructed in groups of 11–14 students from the same mathematics class rather than individually. To examine the degree of dependence within the groups (n = 12) for interest and performance at pretest, we calculated the intraclass correlation coefficient (ICC) using the statistical program Mplus (Muthén & Muthén, 1998 - 2012). The ICC was very low for interest (.03) and low for performance (.137). As we were not interested in context effects other than the manipulated teaching method, we calculated fit statistics and assessed the effects using maximum-likelihood estimations with adjusted standard errors (MLR) using the type = complex analysis in Mplus. This statistical method takes into account the dependence of observations for parameter estimates and goodness-of-fit model testing (Muthén & Muthén, 1998 - 2012).

3.4.2 Missing values

In the present study, the percentage of missing values ranged from 0.7% for enjoyment and boredom to 9% for performance. The missing values in the current study were estimated using the maximum likelihood algorithm (FIML) implemented in Mplus. This algorithm uses all of the information from the covariance matrices to estimate the missing values.

4 Results

4.1 Analysis of model fit for path models

The calculation of model fit values and path estimators was based on the correlation matrix of the variables presented in Table 2. The analysis of the correlations showed that all the values were in the expected direction (e.g., interest measures were significantly correlated with each other, and the correlation between enjoyment and boredom was negative).

Table 2. Means, standard deviations, and correlations between all variables

								Multiple solutions		One solution	
Variable	1	2	3	4	5	6	7	M	SD	M	SD
1. Prior performance	–							-0.79	1.02	-1.11	1.18
2. Prior interest	.06	–						2.55	0.92	2.53	0.99
3. Number of solutions	.15	-.05	–					1.55	0.38	1.16	0.34
4. Enjoyment	.07	.32*	.22*	–				3.38	0.94	3.29	0.88
5. Boredom	-.11	-.03	-.18	-.52*	–			1.94	0.89	1.95	0.82
6. Performance at posttest	.52*	.01	-.03	.21*	-.16	–		-0.26	1.03	-0.27	1.13
7. Interest at posttest	.17	.62*	.11	.42*	-.06	.04	–	2.85	0.94	2.85	0.74

* $p < .05$, two-tailed.

The goodness-of-fit values show whether the data provide a good fit to the hypothesized model and the regression estimates can be used to analyze the effects. The comparative fit index (CFI) and the standardized root mean squared residual (SRMR) are most adequate for sample sizes that are smaller than 250 (Hu & Bentler, 1999). We applied the combination of cutoff values of CFI > .95 and SRMR < .09 to examine the goodness of the fit of the model. In addition, we also calculated the chi-square goodness of fit. Both path models fit the data well according to all fit indices (see Table 3).

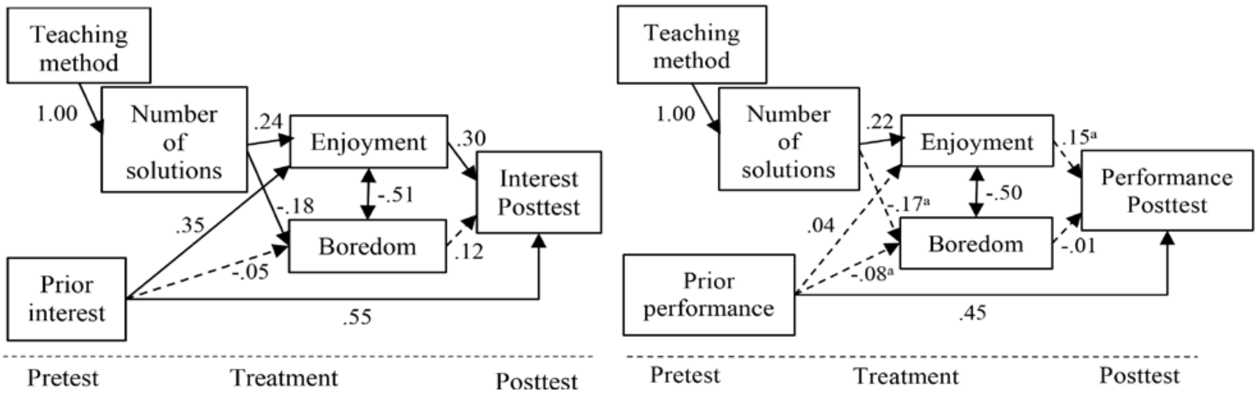
Table 3. Fit Values for the Path Models for Interest (Model I) and Performance (Model P)

	Model I	Model P
R^2	.45	.29
χ^2	10.380	6.473
df	5	5
p	> .05	> .05
CFI	.953	.985
SRMR	.040	.037

Note. R^2 = variance explained at posttest; *p* = two-tailed; CFI = comparative fit index; SRMR = standardized root mean squared residual.

4.2 Tests of Hypotheses

In this section, we present the results of the estimates calculated for the hypothesized path models. Estimates for the path models for interest and performance are presented in the Appendix and are graphically illustrated in Figure 5.



Note: p = one-tailed; Significant paths ($p < .05$) are illustrated with solid lines and nonsignificant paths with broken lines; $^{a}p < .10$. Because treatment conditions represented a binary factor (one-solution condition vs. multiple-solutions condition), the StdY values were used in Mplus to calculate the standardized estimates. Thus, β coefficients may be interpreted as the predicted change in (residualized) criterion measures (in standard deviation units) when the treatment changes from 0 (one solution) to 1 (multiple solutions). According to the hypothesized mediation model, we can see that if the prior interest increases one standard deviation (SD_{pre_i}), the interest at posttest directly changes by $\beta_i * SD_{post\_i} = .55 * SD_{post\_i}$.

Figure 5. Hypothesized path models that tested the indirect effects of prompting multiple solutions and prior interest and performance on interest and performance at posttest through enjoyment and boredom.

4.2.1 Effects of prior interest and performance on enjoyment and boredom

Positive effects of prior interest and performance on enjoyment and negative effects of prior interest and performance on boredom were expected (Hypothesis 1). This hypothesis was partly confirmed for enjoyment, as prior interest positively affected enjoyment during the teaching unit ($\beta = 0.35$, $p < .05$), and we found no effects of prior performance on enjoyment ($\beta = 0.04$, $p = .29$). Further, prior interest did not influence boredom during mathematics lessons ($\beta = -0.05$, $p = .31$), and the negative effect of prior performance on students' boredom was not quite significant and weak ($\beta = -0.08$, $p = .085$). Hence, students' prior interest was found to be important for their enjoyment during the teaching unit, and prior performance may be one of the factors that influences students' boredom during mathematics lessons.

4.2.2 Effects of enjoyment and boredom on interest and performance

We hypothesized positive effects of enjoyment and negative effects of boredom on interest and performance at posttest (Hypothesis 2). This hypothesis was confirmed for enjoyment but not for boredom. Enjoyment during the teaching unit improved students' interest ($\beta = 0.30$, $p < .05$) and almost significantly ($\beta = 0.15$, $p = .066$) enhanced performance at posttest. Students who enjoyed their mathematics lessons reported higher interest and showed better results at posttest. Students' boredom, however, did not influence their interest or performance at posttest (interest: $\beta = 0.12$, $p = .16$; performance: $\beta = -0.01$, $p = .45$).

4.2.3 Effects of prior interest and performance on interest and performance at posttest via enjoyment and boredom

We analyzed the roles of enjoyment and boredom as intervening factors between prior interest and interest at posttest and between prior performance and performance at posttest (Hypothesis 3). As hypothesized, prior interest indirectly positively affected interest at posttest with enjoyment at posttest as an intervening variable ($\beta = 0.10$, $p < .05$). Contrary to our expectations, no indirect effects of prior performance on students' performance at posttest through students' enjoyment were found ($\beta = 0.005$, $p = .30$). Prior interest did not indirectly affect interest at posttest, and prior performance did not indirectly affect performance at posttest via boredom as an intervening variable (interest: $\beta = -0.01$, $p = .35$; performance: $\beta = 0.001$, $p = .45$). Students who were more interested in mathematics before the teaching unit improved their interest in mathematics because they experienced enjoyment during the mathematics lessons. Total effects of prior interest and performance on interest and performance at posttest were also found (interest: $\beta = 0.65$, $p < .05$; performance: $\beta = 0.46$, $p < .05$).

4.2.4 Effects of number of solution and treatment on enjoyment and boredom

We tested the effect of prompting students to develop multiple solutions on enjoyment and boredom (Hypothesis 4). As expected, the treatment enhanced the number of solutions ($\beta = 1.00$, $p < .05$) (Schukajlow & Krug, 2014b), the number of solutions positively affected enjoyment ($\beta = 0.24$, $p < .05$, Model I; $\beta = 0.22$, $p < .05$, Model P), and the treatment improved enjoyment indirectly via the number of solutions ($\beta = 0.23$, $p < .05$, Model I; $\beta = 0.22$, $p < .05$, Model P). The opposite effects were expected and found for boredom. The number of solutions negatively affected boredom ($\beta = -0.18$, $p < .05$ Model I; $\beta = -0.17$, $p < .05$, Model P), and the treatment decreased boredom indirectly ($\beta = -0.18$, $p < .05$, Model I; $\beta = -0.17$, $p < .05$, Model I) via the number of solutions. Prompting students to find multiple solutions for modelling problems that were missing information enhanced enjoyment and impeded boredom via the number of solutions developed.

4.2.5 Effects of number of solutions and treatment on interest and performance via enjoyment and boredom

The fifth hypothesis stated that enjoyment and boredom would be intervening factors between the number of solutions and the treatment on the one hand and interest and performance at posttest on the other hand (Hypothesis 5). We found positive indirect effects of the number of solutions and the treatment on interest at posttest mediated via enjoyment ($\beta = 0.07$, $p < .05$). However, the effect of the number of solutions on performance via enjoyment was not quite significant ($\beta = 0.03$, $p = .089$), and the indirect effect of the treatment on performance via the number of solutions and enjoyment was not significant ($\beta = 0.03$, $p = .10$). Moreover, boredom did not mediate the effects of the number of solutions or the treatment on interest ($\beta = -0.02$, $p = .13$) or performance ($\beta = 0.002$, $p = .447$). Students who were prompted to find multiple solutions and developed more solutions were more interested in mathematics after the teaching unit if they enjoyed mathematics classes.

5 Discussion

On the basis on Pekrun's (2006) control-value theory of achievement emotions, we hypothesized and tested a model linking teaching methods for modelling problems, motivational orientations, and performance to students' emotions.

5.1 Effects of prior interest and performance on enjoyment and boredom

As suggested by Ainley & Hidi (2014) or Carmichael et al. (2009), students' interest at pretest predicted enjoyment during teaching units. This result is in line with studies that found a positive impact of motivational beliefs on positive emotions (Ahmed et al., 2010; Winberg et al., 2014). Positive effects of prior academic achievement on enjoyment that were deduced from cross-sectional (Schukajlow & Krug, 2014a) and longitudinal studies (Ahmed et al., 2013; Hannula et al., 2014) were not confirmed in the current study. One reason for this result can be the teaching method we used. The student-centered, cooperative learning setting could arouse a feeling of involvement in student-student interactions and therefore increase enjoyment and decrease boredom (Martínez-Sierra & García González, 2014) for students at different performance levels.

Neither prior interest nor prior performance predicted boredom during the teaching unit. A lack of interest can induce negative emotions such as anger or anxiety (Pekrun, Hall, et al., 2014), but it did not increase boredom. Our results on the effects of performance on boredom in early secondary school are partly in line with previous findings at the university level. No effects of performance on boredom were also found in one of four measures by college students (Pekrun, Hall, et al., 2014).

5.2 Effects of enjoyment and boredom on interest and performance

The effects of positive emotional experiences on motivation-related measures (Buff et al., 2011; Mega et al., 2014) were confirmed in our study for students' enjoyment and interest. One practical implication is the importance of supporting students' enjoyment during problem solving for interest development. Combining this result with the result of a positive influence of prior interest on enjoyment reported in a previous section, a strong positive feedback loop is suggested for interest and enjoyment.

However, boredom during mathematics lessons did not affect students' interest in the current study. Research on interest has indicated that interest development can be accompanied not only by positive but sometimes also by negative affect (Ainley & Hidi, 2014; Hidi & Harackiewicz, 2000). Hence, the impact of boredom on interest would not be negative but neutral.

Similar effects were also found for the impact of emotions on performance. Enjoyment had positive effects on performance, similar to most cross-sectional and longitudinal studies on this issue (Ahmed et al., 2013; Goetz et al., 2007; Mega et al., 2014; Schukajlow & Krug, 2014a). Thus, triggering students' enjoyment during mathematics classes improves both interest and performance. Students' performance at posttest, however, was not predicted by boredom. We could not confirm the negative effect of boredom on students' school grades (Ahmed et al., 2013) with our measure of students' performance in our study. One possible explanation for the stronger negative impact of boredom on grades may be that grades are influenced not only by performance but also by students' activities during mathematics lessons. If students are bored during mathematics lessons, they might not participate in classroom discussions, and as a result receive worse school grades. This supposition needs to be tested in future studies.

5.3 Effects of prior interest and performance on interest and performance at posttest via enjoyment and boredom

Enjoyment was found to be a mediator between prior interest and interest at posttest. Similar results were also found for students' experience of competency but not for autonomy (Schukajlow & Krug, 2014b). However, prior performance did not have effects on students' performance via enjoyment. Students' prior interest or performance also did not affect interest or performance at posttest via boredom. These results indicate that students with different performance levels benefit from the teaching unit to the same extent.

5.4 Effects of multiple solutions on enjoyment and boredom

One of the aims of our study was to investigate how enjoyment and boredom could be influenced in the mathematical classroom. We found that the number of solutions developed positively influenced enjoyment and negatively influenced boredom. Moreover, prompting students to find multiple solutions for modelling problem was found to be an effective way to improve students' enjoyment and decrease their boredom. These results are in line with the control-value theory of achievement emotions. The development of individual solutions, based on personal assumptions about missing information, improved the controllability and value of problem-solving activities. The enhancement of value and control appraisals then triggered students' emotions (Buff, 2014; Pekrun, Elliot, & Maier, 2006). The positive effects of prompting students to find multiple solutions on students' monitoring activities and self-regulation but not on value appraisals at posttest (Schukajlow & Krug, 2012, 2013a) support the view that control appraisals are more important when prompting students to find multiple solutions for modelling problems with missing information. This hypothesis needs to be confirmed in future studies.

5.5 Effects of multiple solutions on interest and performance via enjoyment and boredom

Students' enjoyment was found to be a mediator between the treatment and interest at posttest. These findings expanded our knowledge about how prompting students to find multiple solutions improves interest. Apart from experiences of competence (Schukajlow & Krug, 2014b), enjoyment may be a valuable factor that transmits the effects of instructional settings on interest. Whereas experiences of competence refer more strongly to cognitive appraisals (understanding of learning materials), enjoyment is more strongly connected to emotional appraisals (feelings during learning). As cognitive and emotional components are important for interest development (Hidi & Renninger, 2006; Krapp, 2005), triggering competency and enjoyment can improve interest in mathematics.

No effects of enjoyment were found for students' performance. Other results were found for the experience of competency, which was identified as an intervening variable between the treatment and performance (Schukajlow et al., 2015). This finding indicates that emotions are more important for motivational than for achievement measures. Because there were no effects of boredom during mathematics lessons on interest at posttest, we also did not find indirect effects of the treatment and the number of solutions on interest and performance at posttest via boredom.

5.5 General discussion

The treatment conditions that were compared in the current study included cognitively demanding real-world problems with the only difference being whether or not students were prompted to construct multiple solutions. As cognitively demanding problems are expected to promote positive emotions and to impede negative emotions (Pekrun, 2006), the effects of the treatment on emotions might be stronger if the problems differed more significantly. Moreover, boredom as an intervening variable between treatment and interest or performance might play an important role if students worked on boring algorithmic problems in one of the conditions. These possibilities should be addressed in future studies. Further research should also clarify whether the power of emotions in the learning process stems from actually experiencing a specific emotion or whether it is the realization that one tends to experience a certain emotion in the classroom (Barret et al., 2001).

In the present study, we tested the hypothesized mediation model in the domain of mathematics. In future studies, the model should be tested in other domains in order to analyze the extent to which our findings can be generalized beyond the domain of mathematics. Further, the roles of emotions in self-efficacy beliefs and other motivational measures are important issues that should be explored in future studies. The integration of emotional theories into the models of self-regulation, such as the self-teaching model proposed by Share (1999), is a promising research question.

6 Strengths and limitations

The roles of enjoyment and boredom as dependent and intervening variables for interest and performance were tested using path analyses. The paths in the hypothesized models should be causally interpreted with caution. The validity of the analysis of path models strongly depends on the times at which the data were collected and on evidence from previous research about the possibilities of directed effects such as the influence of enjoyment on students' interest at posttest. As we assessed variables before, during, and after the teaching unit, the data collected in our study could be ordered along a timeline. Using these data, it was possible to determine the direction of the effects (e.g., from enjoyment during the teaching unit to students' interest after the teaching unit) and thus also to test the hypothesized path model. However, even though we assigned an order to the number of solutions and emotions in the path model by presenting a causal chain, both of these variables were assessed during the treatment. Our assumptions about the implied causal structure came from the control-value theory and empirical considerations from our previous study. This is a strong limitation of the present study. Experimental studies are needed to collect stronger evidence for the direction and power of the tested effects.

The path models were derived from the control-value theory of achievement emotions (Pekrun, 2006). The results of previous empirical studies supported the hypothesized paths between the applied teaching method, the number of solutions developed, enjoyment, boredom, and prior and final interest and performance. However, this path model may be incomplete as other intervening variables such as anticipated feedback or goal orientation (Pekrun, Cusack, et al., 2014) could effect the number of solutions, enjoyment, boredom, and performance at posttest.

Finally, we would like to note other important limitations of the current study. We investigated the effects of an intervention across a relatively short time period because more studies of short duration in mathematics education were recently requested (Stylianides & Stylianides, 2014). The stability of effects on interest and performance is another important issue for future research as long-term effects can differ from short-term effects. Further, the results of our study should be replicated using a larger sample.

7 Summary

In the current study, we analyzed the power of enjoyment and boredom using cognitively demanding modelling problems. In the framework of the hypothesized model, enjoyment was identified as a powerful emotion that is important for interest and performance. Boredom was less important than enjoyment, confirming the view that positive affect has a greater weight not only on motivation (Mega et al., 2014) but also on performance. Further, we extended our knowledge of the effects of prompting students to find multiple solutions for modelling problems with missing information. In addition to effects on self-regulation, monitoring, preferences for finding multiple solutions, interest, and performance (Schukajlow & Krug, 2012, 2013a, 2013b, 2014a, 2014b), this teaching method triggered enjoyment and reduced boredom via the number of solutions developed. Summarizing these findings, we argue that achievement emotions are powerful variables that can be influenced in instructional settings and in turn influence motivation and performance.

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8 References

Ahmed, W., van der Werf, G., Kuyper, H., & Minnaert, A. (2013). Emotions, self-regulated learning, and achievement in mathematics: A growth curve analysis. *Journal of Educational Psychology, 105*(1), 150-161.

Ahmed, W., van der Werf, G., Minnaert, A., & Kuyper, H. (2010). Students' daily emotions in the classroom: Intra-individual variability and appraisal correlates. *British Journal of Educational Psychology, 80*(4), 583-597.

Ainley, M. (2007). Being and feeling interested: Transient state, mood and disposition. In P. A. Schutz & R. Pekrun (Eds.), *Emotion in education*. Burlington. USA: Academic Press.

Ainley, M., & Hidi, S. (2014). Interest and enjoyment. In R. Pekrun & L. Linnenbrink-Garcia (Eds.), *International handbook of emotions in education* (pp. 205-227). New York: Routledge.

Bond, T. G., & Fox, C. M. (2001). *Applying the Rasch model: Fundamental measurement in the human sciences*. Mahwah, NJ: Lawrence Erlbaum.

Brown, M., Brown, P., & Bibby, T. (2008). “I would rather die”: reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education, 10*(1), 3-18.

Buff, A. (2014). Enjoyment of learning and its personal antecedents: Testing the change–change assumption of the control-value theory of achievement emotions. *Learning and Individual Differences, 31*(0), 21-29.

Buff, A., Reusser, K., Rakoczy, K., & Pauli, C. (2011). Activating positive affective experiences in the classroom: “Nice to have” or something more? *Learning and Instruction, 21*(3), 452-466.

Carmichael, C. S., Callingham, R., Watson, J. M., & Hay, I. (2009). Factors influencing the development of middle school students’ interest in statistical literacy. *Statistics Education Research Journal, 8*(1), 62-81.

Goetz, T., Frenzel, A. C., Pekrun, R., Hall, N. C., & Lüdtke, O. (2007). Between- and within-domain relations of students’ academic emotions. *Journal of Educational Psychology, 99*, 715-733.

Goetz, T., Pekrun, R., Hall, N., & Haag, L. (2006). Academic emotions from a social-cognitive perspective: Antecedents and domain specificity of students' affect in the context of Latin instruction. *British Journal of Educational Psychology, 76*(2), 289-308.

Goldin, G. A. (2014). Perspectives on emotion in mathematical engagement, learning, and problem solving. In R. Pekrun & L. Linnenbrink-Garcia (Eds.), *Handbook of Emotions and Education* (pp. 391-414). New York: Routledge.

Hannula, M. S., Bofah, E., Tuohilampi, L., & Metsämuuronen, J. (2014). A longitudinal analysis of the relationship between mathematics-related affect and achievement in Finland. In C. Nicol, S. Oesterle, P. Liljedahl, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 3, pp. 249-256). Vancouver, Canada: PME.

Hidi, S., & Harackiewicz, J. M. (2000). Motivating the academically unmotivated: A critical issue for the 21st century. *Review of Educational Research and Evaluation, 70*, 151-179.

Hidi, S., & Renninger, K. A. (2006). The four phase model of interest development. *Educational Psychologist, 41*(2), 111–127.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., . . . Stigler, J. (2003). *Teaching mathematics in seven countries. Results from the TIMSS 1999 video study*. Washington, DC: NCES.

Kline, R. B. (2005). *Principles and practice of structural equation modeling*. New York, NY: Guilford Press.

Krapp, A. (2005). Basic needs and the development of interest and intrinsic motivational orientations. *Learning and Instruction, 15*, 381-395.

Krapp, A. (2006). Interesse. In D. H. Rost (Ed.), *Handwörterbuch Pädagogische Psychologie* (pp. 280-290). Weinheim: Beltz.

Levav-Waynberg, A., & Leikin, R. (2012). The role of multiple solution tasks in developing knowledge and creativity in geometry. *Journal of Mathematical Behavior, 31*, 73- 90.

Martínez-Sierra, G., & García González, M. d. S. (2014). High school students' emotional experiences in mathematics classes. *Research in Mathematics Education, 16*(3), 234-250.

Mega, C., Ronconi, L., & De Beni, R. (2014). What makes a good student? How emotions, self-regulated learning, and motivation contribute to academic achievement. *Journal of Educational Psychology, 106*(1), 121.

Muthén, L. K., & Muthén, B. O. (1998 - 2012). *Mplus user’s guide* (5th ed ed.). Los Angeles, CA: Muthe´n & Muthe´n.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Niss, M., Blum, W., & Galbraith, P. L. (2007). Introduction. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and Applications in Mathematics Education: the 14th ICMI Study* (pp. 1-32). New York: Springer.

Pekrun, R. (2006). The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review, 18*, 315-341.

Pekrun, R., Cusack, A., Murayama, K., Elliot, A. J., & Thomas, K. (2014). The power of anticipated feedback: Effects on students' achievement goals and achievement emotions. *Learning and Instruction, 29*, 115-124.

Pekrun, R., Elliot, A. J., & Maier, M. A. (2006). Achievement goals and discrete achievement emotions: A theoretical model and prospective test. *Journal of Educational Psychology, 98*(3), 583.

Pekrun, R., Goetz, T., Daniels, L. M., Stupnisky, R. H., & Perry, R. P. (2010). Boredom in achievement settings: Exploring control-value antecedents and performance outcomes of a neglected emotion. *Journal of Educational Psychology, 102*(3), 531 - 549.

Pekrun, R., Goetz, T., Frenzel, A. C., Barchfeld, P., & Perry, R. P. (2011). Measuring emotions in students' learning and performance: The Achievement Emotions Questionnaire (AEQ). *Contemporary educational psychology, 36*, 36-48.

Pekrun, R., Hall, N. C., Goetz, T., & Perry, R. P. (2014). Boredom and academic achievement: Testing a model of reciprocal causation. *Journal of Educational Psychology, 106*(3), 696.

Pekrun, R., & Linnenbrink-Garcia, L. (2014). Introduction to emotions in education. In R. Pekrun & L. Linnenbrink-Garcia (Eds.), International handbook of emotions in education (pp. 1-10). New York: Taylor & Francis.

Rittle-Johnson, B., & Star, J. R. (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving. *Journal of Educational Psychology, 101*(3), 529-544.

Schukajlow, S. (2016). Is boredom important for students' performance? In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 1273-1279). Prag, Szech Republic: Charles University in Prague, Faculty of Education and ERME.

Schukajlow, S., & Krug, A. (2012). Effects of treating multiple solutions on students' self-regulation, self-efficacy and value. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 59-66). Taipei, Taiwan: PME.

Schukajlow, S., & Krug, A. (2013a). Planning, monitoring and multiple solutions while solving modelling problems. In A. M. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 177-184). Kiel, Germany: PME.

Schukajlow, S., & Krug, A. (2013b). Uncertainty orientation, preferences for solving tasks with multiple solutions and modelling. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eight Congress of the European Society for Research in Mathematics Education* (pp. 1429-1438). Ankara, Turkey: Middle East Technical University.

Schukajlow, S., & Krug, A. (2014a). Are interest and enjoyment important for students' performance? In C. Nicol, S. Oesterle, P. Liljedahl, & D. Allan (Eds.), Proceedings of the Joint Meeting of PME 38 and PME-NA 36 (Vol. 5, pp. 129-136). Vancouver, Canada: PME.

Schukajlow, S., & Krug, A. (2014b). Do multiple solutions matter? Prompting multiple solutions, interest, competence, and autonomy. *Journal for Research in Mathematics Education, 45*(4), 497-533.

Schukajlow, S., Krug, A., & Rakoczy, K. (2015). Effects of prompting multiple solutions for modelling problems on students' performance. *Educational Studies in Mathematics, 89*(3), 393-417.

Schukajlow, S., Leiss, D., Pekrun, R., Blum, W., Müller, M., & Messner, R. (2012). Teaching methods for modelling problems and students' task-specific enjoyment, value, interest and self-efficacy expectations. *Educational Studies in Mathematics, 79*(2), 215-237.

Stylianides, A. J., & Stylianides, G. J. (2014). Impacting positively on students' mathematical problem solving beliefs: An instructional intervention of short duration. *The Journal of Mathematical Behavior*, *33*, 8-29.

Tulis, M., & Ainley, M. (2011). Interest, enjoyment and pride after failure experiences? Predictors of students' state-emotions after success and failure during learning in mathematics. *Educational Psychology: An International Journal of Experimental Educational Psychology, 31*(7), 779-807.

Vogel-Walcutt, J. J., Fiorella, L., Carper, T., & Schatz, S. (2012). The definition, assessment, and mitigation of state boredom within educational settings: a comprehensive review. *Educational Psychology Review, 24*, 89-111.

Warm, T. A. (1989). Weighted likelihood estimation of ability in item response theory. *Psychometrika, 54*, 427-450.

Wigfield, A., Battle, A., Keller, L. B., & Eccles, J. S. (2002). Sex differences in motivation, self-concept, career aspiration, and career choice: implications for cognitive development. In R. De Lisi & A. McGillicuddy-De Lisi (Eds.), *The development of sex differences in cognition* (pp. 93-124). Westport, CT, US: Ablex Publishing.

Winberg, T. M., Hellgren, J., & Palm, T. (2014). Stimulating positive emotional experiences in mathematics learning: influence of situational and personal factors. *European Journal of Psychology of Education, 29*(4), 673-691.

Wu, M. L., Adams, R. J., & Wilson, M. R. (1998). *ACER conquest*. Melbourne: The Australian Council for Educational Research.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in Mathematics Education: An Introduction. *Educational Studies in Mathematics, 63*(2), 113-122.