I ENJOY MAKING DRAWINGS! ENJOYMENT, KNOWLEDGE ABOUT DRAWINGS, USE OF DRAWINGS, AND STUDENTS' PERFORMANCE

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Positive emotions in mathematics have been found to be important for problem solving. In the present study, 196 ninth- and tenth-graders filled out a questionnaire about how much they enjoyed making drawings and took tests that measured their knowledge about drawings, intra-mathematical performance, and modelling performance. Because students had to calculate the length of one side of a right-angled triangle to solve the modelling problems, some students made drawings. We confirmed the expectation that enjoyment of drawing was positively related to the use of drawings to solve problems. However, enjoyment of drawing was not related to knowledge about drawings, intra-mathematical performance, or modelling performance. We suggest that more attention should be paid to emotions toward strategies in future studies.

INTRODUCTION

Emotions are important for learning and problem solving (Hannula, 2015; Pekrun, 2006). One positive emotion that students frequently experience in the classroom is enjoyment. Consequently, the enjoyment of mathematics was found to affect problem solving and even more specifically students' modelling performance and students' interest in mathematics (Schukajlow & Rakoczy, 2016).

However, we do not know much about emotions toward strategies. As the use of strategies has been found to be important for successful task processing (Hembree, 1992), more attention should be paid to the affective factors (e.g., emotions) that can affect students' use of strategies. In the present study, we aimed to take the first step toward clarifying the role of emotions in the use of strategies and investigated the relation between enjoyment of drawing and achievement-related factors.

THEORETICAL BACKGROUND

Enjoyment

According to the control-value theory of achievement emotions, enjoyment is an activating positive emotion (Pekrun, 2006). Students who enjoy an activity feel arousal when engaging in this activity and are expected to outperform students who do not feel enjoyment. Research on emotions has revealed that enjoyment of problem solving arises in students if they have high control appraisals (the confidence to master the problem) and high value appraisals (they value the problem-solving activity) (Buff, 2014).

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Similar to other affective constructs, enjoyment can refer to different objects. The object of emotions varies from general (enjoyment of life) to specific (enjoyment of solving a problem). An analysis of the structure of the emotion of enjoyment regarding different objects with 513 students in Grades 5 to 10 confirmed that the enjoyment of strategies could be statistically separated from the enjoyment of life, school, and problem solving (Goetz, Hall, Frenzel, & Pekrun, 2006).

Self-generated drawing and modelling

Weinstein and Mayer (1986) defined strategies as behaviors and thoughts that learners engage in and that are intended to influence their learning and problem solving. Strategic knowledge makes up part of the static knowledge component of metacognition as opposed to the dynamic process component of metacognition, which includes strategy use (the application of a strategy during problem solving).

Modelling problems are problems with a connection to reality, and their solutions require demanding transfer processes between reality and mathematics (Niss, Blum, & Galbraith, 2007). Modelling performance can be clearly distinguished from students' performance in solving problems without a connection to reality, called intra-mathematical performance. An important goal of research in mathematical modelling is to clarify factors that are related to modelling performance and can be addressed in interventional studies (Schukajlow, Kaiser, & Stillman, 2018). One of these factors is students' strategies, such as making a drawing.

Self-generated drawing has been identified as an important strategy for problem solving (Hembree, 1992). It can be defined as the process and the product of generating an illustration that corresponds to the objects and relations described in a problem (Rellensmann, Schukajlow, & Leopold, 2017). Students' strategic knowledge about drawing comprises students' knowledge about the characteristics of a drawing that fits a given problem. In a prior study, we confirmed the importance of strategic knowledge for solving modelling problems (Rellensmann et al., 2017). Apart from strategic knowledge about drawing, students' strategy use was found to predict students' performance in problem solving. Students who spontaneously applied a drawing strategy were found to demonstrate higher performance than students who did not apply a strategy (Hembree, 1992). However, this finding was not always confirmed when students were asked to construct a drawing (De Bock, Verschaffel, Janssens, Van Dooren, & Claes, 2003). Students who were asked to make drawings for geometric word problems were found to perform the same or even worse than students who were not asked to make drawings. Thus, it is important to identify the factors that influence the spontaneous use of drawings. One reason why students do not always spontaneously generate drawings might be their affective perceptions of this strategy such as how much they enjoy making a drawing to solve a problem.

Enjoyment of drawing and achievement-related outcomes

To the best of our knowledge, the relations between enjoyment of strategy use and achievement-related outcomes has yet to be investigated. However, research has found that affective factors that are related to enjoyment might be important for the spontaneous use of the drawing strategy. In a correlational study, Uesaka and Manalo (2017) found that students made drawings less often when they perceived that making a drawing would cost them a lot of effort and when they were not confident they could perform this strategy well. Given that in the control-value theory of achievement emotions, high efficacy beliefs were suggested to accompany students' enjoyment, and the two constructs were found to be positively related (Buff, 2014), we expected that enjoyment of drawing would also be positively related to the use of drawings to solve modelling problems.

The relation between enjoyment of drawing and intra-mathematical performance cannot be clearly derived from prior research. As intra-mathematical performance is an important part of modelling (Niss et al., 2007) and enjoyment is positively related to modelling (Schukajlow & Rakoczy, 2016), enjoyment of drawing might be positively related to intra-mathematical performance.

Further, students who enjoy applying strategies might apply this strategy more often and might thereby gain deeper knowledge of this strategy. Thus, a positive relation between enjoyment of drawing and students' knowledge about drawings could be expected.

As enjoyment of strategies was found to be related to enjoyment in mathematics (Goetz et al., 2006) and enjoyment in mathematics was found to affect students' modelling (Schukajlow & Rakoczy, 2016), we expected a positive relation between enjoyment of drawing and modelling. We expected that this positive relation would hold even after we controlled for strategic knowledge about drawings and intra-mathematical performance.

RESEARCH QUESTIONS

RQ: 1. Is enjoyment of self-generated drawing related to the use of drawings, intra-mathematical performance, strategic knowledge about drawings, and modelling performance? We expected a positive correlation between enjoyment and learning outcomes.

RQ: 2. Is enjoyment of self-generated drawing related to the use of drawings and modelling performance after strategic knowledge about drawing and intra-mathematical performance are controlled for? We expected that the relation between enjoyment and modelling would remain positive even after the strategic factor and the achievement factor were controlled for.

METHOD

Sample and design of the study

Two hundred twenty German ninth- and tenth-graders from 10 classes in middle- and high-track schools (German Gesamtschule and Gymnasium) participated in the present study (mean age about 15 years, 109 female students). The study consisted of two test sessions on two different days. On the first day of the study, students filled out a questionnaire on enjoyment of drawing, age, and other constructs. They also worked on a test on strategic knowledge about drawing. On the second day, at least 2 weeks after the first day, they took a modelling test and an intra-mathematical test. Because each student worked on the tests on two different days, some data were missing (3% on the first day and 8% on the second day). In the presented analyses, we analyzed students with complete data only. This means that we excluded students from the analysis if they missed the first or second test session, and we performed the analysis on 192 students. Students were not instructed to make a drawing while working on the modelling problems because we were interested in their natural use of the drawing strategy. Their spontaneous use of drawings was coded by analyzing students' solutions to the modelling problems.

Measures

The scale for the assessment of enjoyment of self-generated drawing was adapted from the enjoyment Likert scale from the Achievement Emotions Questionnaire (Pekrun, Goetz, Frenzel, Barchfeld, & Perry, 2011) by changing the scope of the items from problem solving to the drawing strategy while solving word problems. It consisted of three items that were rated on a Likert scale ranging from 1 (*not at all true*) to 5 (*completely true*). One sample item was: "I enjoy making a drawing when solving a difficult word problem." The reliability (Cronbach's α) was .766.

We developed and validated the strategic knowledge about drawing scale in a prior study (Rellensmann, Schukajlow, & Leopold, under review). It comprised eight real-world problems. Each problem was followed by an item that included situational drawings and an item that included mathematical drawings (see Figure 1). On the 16-item test, students were asked to evaluate the utility of the drawings provided for each problem by comparing the drawings with regard to their helpfulness in solving the presented problem. Each item comprised three drawings (a correct and complete drawing, a correct but incomplete drawing, and an incorrect drawing) that students rated on a scale ranging from 1 (*not helpful at all*) to 5 (*very helpful*). The scores for each item ranged from 0 to 3. The number of points depended on the sequence of drawings concerning their usefulness for solving the task. If students identified that the correct and complete drawing was more helpful than the correct but incomplete drawing and that the correct and incomplete drawing was more helpful than the incorrect drawing, they were given 3 points (test reliability .76). If a student suggested the reverse order of the drawings, he or she received 0 points.

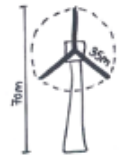
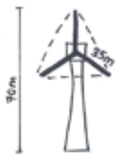
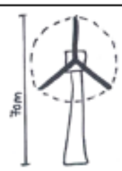
The modelling performance test comprised eight problems that all required students to identify a right-angled triangle and could be solved by applying the Pythagoras theorem (see examples of the tasks by Rellensmann et al., 2017). Students' solutions were scored by two raters on a scale ranging from 2 (correct problem solution) to 0 (incorrect solution or a missing solution). The interrater reliability (Cohen's κ) was > .81. The test reliability of the modelling performance test was .772.

Wind

The rotor blade of a 70 m high wind turbine has a length of 35 m and a so-called wind area of about 3,800 m². The wind area is the area that is swept by a rotor blade during a complete rotation. Can the information on the swept area be correct?

You do not have to solve the task!

First compare these three drawings with one another. Evaluate how useful each drawing is for solving the task.

Drawings	Not helpful at all	Not helpful	Rather not helpful	Rather helpful	Helpful	Very helpful
	☐	☐	☐	☐	☐	☐
	☐	☐	☐	☐	☐	☐
	☐	☐	☐	☐	☐	☐

Next compare these three drawings with one another. Evaluate how useful each drawing is for solving the task.

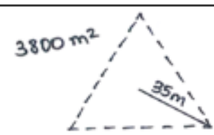
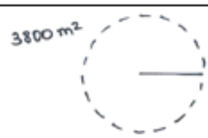
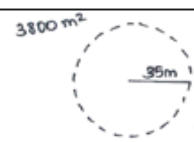
Drawings	Not helpful at all	Not helpful	Rather not helpful	Rather helpful	Helpful	Very helpful
	☐	☐	☐	☐	☐	☐
	☐	☐	☐	☐	☐	☐
	☐	☐	☐	☐	☐	☐

Figure 1: An item from the knowledge about drawing scale (Rellensmann, Schukajlow, & Leopold, under review)

Students' use of drawings was assessed via the number of drawings they constructed while solving the eight problems on the modelling test. If students constructed a drawing for a modelling problem, they received a score of 1; if they did not make a drawing, they were given a score of 0. The test reliability (Cronbach's α) was .866.

Intra-mathematical performance was assessed with the test that included 10 items on applying the Pythagorean theorem or solving quadratic equations (e.g., x^2 = 3.82 –

2.52). Students received 1 point for the correct solution and 0 points for an incorrect or missing solution. The test reliability was .760.

RESULTS

Our first research question referred to the relation between enjoyment of self-generated drawing and achievement-related outcomes. By computing Pearson correlations, we found a positive relation between enjoyment of drawing and the use of drawings to solve modelling problems (see Table 1). This result confirmed our expectation that students who enjoy making drawings would construct more drawings to solve the problems than students who do not enjoy this strategy.

		UD	SKD	IM	MOD
Enjoyment of self-generated drawing	*r*	.159*	.09	-.016	.046

Note. * *p:* two-tailed, $p < .01$, UD: Use of drawings, SKD: strategic knowledge about drawing, IM: intra-mathematical performance, MOD: Modelling performance.

Table 1: Correlations between enjoyment and the achievement-related measures

However, we did not find relations between enjoyment and the other achievement-related factors. Enjoyment of self-generated drawing was not correlated with strategic knowledge about drawings, intra-mathematical performance, or modelling performance. We found that students with high and low enjoyment of drawing had similar knowledge about drawings and that they did not differ in their performance in solving intra-mathematical and modelling problems.

The second research question was aimed at investigating the relation between enjoyment and modelling performance while controlling for important factors that have been found to influence modelling performance in prior studies (knowledge about drawings and intra-mathematical performance). Partial Pearson correlations did not support our expectations because we did not find a positive relation between enjoyment of drawing and modelling after we controlled for knowledge about drawing and intra-mathematical performance ($r = .050$, $p = .488$).

DISCUSSION

The goal of our study was to explore the role of enjoyment toward strategies in students' learning outcomes. We found that enjoyment of drawing was positively related to the spontaneous use of the drawing strategy. This result confirmed the importance of affective factors for students' strategic behavior. Apart from the costs of strategy use (Uesaka & Manalo, 2017), students' emotions might influence their decision to apply a strategy to solve a problem. In future studies, the role of other emotions and more broadly the role of other affective factors for strategy use should be explored. A practical implication of this finding might be that improving students' enjoyment of strategy use might increase the frequency with which they use these strategies.

However, students' enjoyment of self-generated drawing was not related to students' knowledge about drawings. We could not confirm our expectation that students who enjoy a strategy and thus apply this strategy more often gain deeper knowledge of this strategy. One reason for this finding might be that teachers do not often discuss when a student has applied a strategy appropriately. In the case of drawings, teachers should not only present the correct solution, but they should also discuss why a drawing is helpful and how a student can make a helpful drawing (Rellensmann et al., 2017).

Enjoyment was not related to intra-mathematical performance. For some tasks, students with high or low mathematical knowledge can be confident about making a drawing, and they can value making a drawing, whereas for other tasks, they might feel less confident about making a drawing and might not value this strategy. Because of the similarity between students' control and value appraisals assumed in the control-value theory (Pekrun, 2006), enjoyment did not differ for students on different levels of intra-mathematical knowledge. However, the importance of control and value appraisals regarding strategies for students' emotions toward these strategies had not been investigated until now and should be addressed in future research.

Given that enjoyment of strategies and enjoyment of mathematics were previously found to be related (Goetz et al., 2006) and enjoyment of mathematics was found to affect modelling (Schukajlow & Rakoczy, 2016), we expected a positive relation between enjoyment of drawing and modelling. However, we could not confirm this expectation. Other factors that are related to the strategies (e.g., knowledge about a strategy or the quality of the strategy use) were previously found to be more important to modelling performance than enjoyment (Rellensmann et al., 2017). Further, in our study, enjoyment was not related to strategic knowledge about drawings. However, it might be different for other emotions. One of our previous analyses revealed that anxiety about drawing was related to modelling performance.

In the present study, we focused on making drawings because this strategy was found to be helpful for solving modelling problems with a spatial structure of the mathematical model (Rellensmann et al., 2017). However, from other studies on modelling, we know that students also use other cognitive (e.g., highlighting, forward/ backward strategy) and metacognitive (e.g., planning, monitoring) strategies (Stillman & Galbraith, 1998). It will be interesting to determine whether emotions influence the spontaneous use of these strategies.

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