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# Scaffolding mathematical modelling with a solution plan

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**Abstract**

In the study presented in this paper, we examined the possibility to scaffold mathematical modelling with strategies. The strategies were prompted using an instrument called "solution plan" as a scaffold. The effects of this step by step instrument on mathematical modelling competency and on self-reported strategies were tested using 9th grade students (N=91) at a German middle track school (Realschule) in a quasi-experimental design. Six classes were randomly assigned to the experimental group, in which students used the solution plan, or to the control group. The quantitative data analysis using ANOVAs reveals that (1) in the posttest the experimental group students reported more frequently about planning, rehearsal, elaboration and organizing strategies while solving modelling problems than the control group; (2) the "solution plan" as a scaffold supports the development of students' modelling competency, including its sub-competencies. The students who used the solution plan outperformed the other students in solving modelling problems concerning the topic "Pythagorean theorem".

**ZDM-Classification: B10, C30, C70**

**Keywords: Modelling Competency, Scaffolding instrument, Strategies, Teaching-Learning Environment**

# 1 Introduction

Mathematical modelling is a complex competency, which includes the ability to solve problems related to the real world (Blum et al. 2007). Modelling is of high importance for students' current and future life and is, for example, an important part of the NCTM principles and standards for school mathematics (National Council of Teachers of Mathematics 2000). Students from different countries all over the world are required to learn how to solve modelling problems. However, a number of empirical studies show that many students have an insufficient level of modelling competency by the end of lower secondary education (Blum 2011). Thus, research on mathematical modelling should be focused on instructional methods that can support the acquisition of mathematical modelling competency from primary through to secondary school. In several empirical studies it was found that support of students' strategy use is a promising approach to improve student learning in different domains (Collins et al. 1989) and that strategy use is connected to students' modelling performance (Stillman and Galbraith 1998). In this paper we propose a strategic instrument, called "solution plan", as the focus of an instructional method for improving students' mathematical modelling competency. We report here on the effects of scaffolding mathematical modelling, using this "solution plan" as a scaffold, on students' strategies and modelling competency, including its sub-competencies.

# 2 Strategies, scaffolding, and mathematical modelling

## 2.1 Cognitive and metacognitive strategies

### 2.1.1 Definition of strategies and effects of strategy use

Strategies can be defined as behaviors and thoughts that learners engage in and that are intended to influence their learning or problem solving process (Weinstein and Mayer 1986). Strategies are predominantly discussed in connection with metacognition and self-regulated learning. Cognitive learning strategies like organization, elaboration and rehearsal are applied for collecting, processing and memorizing information. Organization strategies help to link the information given in a text or a problem. Elaboration strategies connect the given information with prior knowledge. Rehearsal strategies focus on selecting the most important information from a text or a problem. Metacognitive strategies include, among others, the planning of the solution process, which is closely related to the learning process (Weinstein and Mayer 1986). One specific feature of strategies is that there is the possibility to improve them in short-term interventional settings (Weinstein et al. 2000). Thus, strategies have often been the main focus of interventional studies in different domains such as science or reading comprehension (Heinze et al. 2009; Leopold and Leutner 2012).

Strategies are also intensively discussed in the context of problem solving activities. Schoenfeld (1992) describes, for example, the problem solving strategies “special cases” and “exploring similar problems” and emphasizes the complexity involved in understanding these strategies and in applying them in an appropriate way. A collection of promising problem solving strategies can be found in Polya (1948). Similar to the learning strategies domain, self-regulation and metacognition are regarded as crucial for using problem solving strategies and for successful problem solving (Schoenfeld 1992).

A positive correlation between strategy use and performance has already been proven in different domains such as reading comprehension (Schneider and Artelt 2010). However, in mathematics a correlation between strategy use and performance varies between zero and medium effect sizes and this variation has not yet been satisfactorily explained (De Corte et al. 2000; Greer and Verschaffel 2007; Hembree 1992; Schoenfeld 1992; Schukajlow et al. 2009).

Use of strategies does not happen automatically but requires conscious regulation and a certain degree of willingness by an individual to work hard. De Corte (2007) emphasises the relevance of “adaptive expertise“ and of reaching a meta-level. Only after students have understood the structural benefit of a new strategy and are able to decide when a strategy might be useful, can the strategy be used flexibly in other situations and provide concrete benefit (Puntambekar and Hubscher 2005; Teong 2003). To date there has been limited investigation into the teaching of content-related strategies at school that enable students to independently conquer new fields of knowledge, deepen their understanding or solve complex tasks. In the following section, we will present findings from other studies about the effects of strategy programs on students’ learning in mathematics.

### 2.1.2 Effects of strategic instruments on students’ learning in mathematics

From the many existing cognitive and metacognitive strategic instruments and programs, we will focus the framework in this study on those instruments which provide approaches for solving mathematical tasks or which scaffold students through the solution process and its structure.

The provision of prepared “worked examples“ or “heuristic examples“, as sample solutions to be processed independently by students, seems to be a promising approach to expand students’ repertoire of strategies (Atkinson et al. 2000; Sweller et al. 1998; Kirschner et al. 2006). In mathematics, these supporting measures have been implemented successfully in the area of modelling for students in Year 8 (Zöttl et al. 2010). A combination of this approach with a “cognitive tutor”, as a feedback tool, has been found to positively influence the acquisition of strategies (Anderson et al. 1995; Salden et al. 2009).

This contrasts with the use of *“process worksheets”*, which only give an overview of the general solution process and its single steps (Van Merriënboer 1997). For the different phases of the solution process useful rules or hints are listed, but a complete exemplary solution for a comprehension of the entire solution process is not offered. In connection with the area of mathematics, two programs are relevant. The first program, “IMPROVE“[1] (Mevarech and Kramarski 1997), involved students undertaking a training program over two weeks (10 maths lessons). Students were provided with questions about the entire problem, about similar tasks already successfully completed, about possible strategies and about the solution process. The second program, called “CRIME“[2] (Teong 2003), provided students with a working plan for solving word problems and involved 4 training units of 60 minutes duration. In studies on the effectiveness of both programs it was found that experimental group students improved their performance in solving reality-related mathematical tasks significantly more than the control group students who were not trained about strategies. Likewise, Stillman and Galbraith (1998) reported a

[1] IMPROVE: *I*ntroducing the new concepts, *M*etacognitive questioning, *P*racticing, *R*eviewing and reducing difficulties, *O*btaining mastery, *V*erification, *E*nrichment

[2] CRIME: *C*areful reading, *R*ecall possible strategies, *I*mplement strategy, *M*onitor, *E*valuation

case study at the upper secondary level where a positive impact was shown for a conscious application of metacognitive elements in solving modelling tasks. Summarizing these findings, positive effects were found for computer based interventions or in a specific case study. However, the use and effects of a cognitive and metacognitive strategic instrument as scaffold in mathematics teaching and learning have not been investigated in an ecologically valid learning environment so far. The focus of the research in this study is the development and evaluation of a strategic instrument for learning, including both its implementation in the classroom and its contribution to the development of cognitive and metacognitive strategies.

## 2.2 Scaffolding as teaching approach

In its origin, scaffolding means adaptive support for children's learning by an adult (Wood et al. 1976). The scaffolding concept was developed in a way that the support should not build a random or spontaneous scaffold, but instead should be a systematic scaffold (like a framework or skeleton) for the individual learner (Smit et al. 2013). It does not aim to overcome short-term local difficulties, but instead aims to develop long-term competency. Smit et al (2013, p. 817) describe scaffolding as "a teacher's temporary support that helps pupils to perform a task they cannot complete by themselves and that is intended to bring pupils gradually to a state of competence in which they can complete a similar task independently". The scaffolding concept is prominent in recent teaching research and is studied in several areas of learning support. Of particular relevance here is the research on scaffolding instruments (or designed scaffolds) ranging from working plans on operation sheets or in textbooks to interactive programs that offer an orientation and framework for the solution process.

Puntambekar and Hübscher (2005) devised several requirements for tools for scaffolding to be used as strategy instruments. Firstly, there must be an accompanying *ongoing diagnosis* of the learning process (by the teacher or by students) and the tool must provide an *adaptive support* of the learning process. Secondly, there is "*fading*" and thus an increasing withdrawal of the tool with a resultant transfer of the responsibility to the student for his/her own advancement (see also Smit et al. 2013). There are many results of empirical research that show a positive impact of scaffolding on students' learning (see summary by Azevedo and Hadwin 2005). For example, studies have found positive effects of diagnosis, adaptive support and fading on self-regulation, strategy use and students' knowledge (Azevedo and Hadwin 2005; Hadwin and Winne 2001; Hadwin et al. 2005).

Cognitive structuring is another important supportive element in the teaching methods that provides a structure for thinking and acting and to stimulate teacher-student interactions (Tharp and Gallimore 1988). By means of cognitive structuring students adapt not only solutions, but also structures for solving or thinking, and they are empowered to become self-regulated problem-solvers. We assume that cognitive structuring is essential for diagnosis, adaptive support and fading and thus, it is a precondition of scaffolding and crucial for the implementation of scaffolding in the classroom. The solution plan is such a promising designed scaffold that offers students a cognitive structure for their solution process. Before we present the solution plan and its cognitive background, we describe the framework of the project within which the solution plan was developed.

## 2.3 Scaffolding Mathematical Modelling

### 2.3.1 The DISUM project

The current study is embedded in the research project DISUM[3], which investigated how teachers and students deal with mathematical modelling tasks and how the modelling competency of students can adequately be developed. As newer studies show (for an overview see Blum 2011), the development of modelling competency can occur differently in different content areas. In the present study, research questions are addressed in relation to the topic areas "Pythagorean theorem" and "linear functions". These topics were selected because of the important role they play in mathematics curricula in many countries. Each content area poses, within typical cognitive processes (compare to section 3), specific challenges for modelling proficiency.

[3] DISUM: **D**idaktische **I**nterventionsformen für einen **s**elbständigkeitsorientierten aufgabengesteuerten **U**nterricht am Beispiel **M**athematik. (In English: Didactical intervention modes for mathematics teaching oriented towards self-regulation and directed by tasks). The project has been in operation since 2002 and was 2005-2012 sponsored by the German Research Foundation (Deutsche Forschungs-Gemeinschaft). Directors: W. Blum, R. Messner (both University of Kassel), R. Pekrun (University of Munich)

In previous studies within the DISUM project we have shown the benefits of a student centered "operative-strategic" teaching method compared to a teacher centered "directive" teaching method regarding various affective variables and students' modelling competency (Blum and Leiss 2007b; Schukajlow et al. 2012). The operative-strategic teaching method is characterized by a systematic change between two phases of classroom instructions: individual work in groups and presentation of solutions and reflection about solution processes by the whole class. Most of the time, students work independently in groups of three or four according to a fixed cooperation script (individual-group-individual), aiming at individual solutions (for detailed information, see Schukajlow et al. 2012). Students' work is supported by the teacher with the help of strategy-oriented interventions. These interventions should be strongly geared towards the individual needs of the students and as adaptive as possible, orientated towards the ideal-typical seven-step modelling cycle according to Blum & Leiss (2007a) shown in Figure 1.

Although the operative-strategic teaching method proved superior concerning students' modelling competency development, the success of this design in its original form was, from a normative point of view, not yet satisfactory. Thus, other ways to improve this teaching method such as prompting students to develop multiple solutions while solving modelling problems (Schukajlow and Krug 2014; Schukajlow et al. in press) have been investigated. In particular, the role of metacognitive tools given directly to students (not only to the teacher) for a strategic support has not been investigated so far. In the study presented here we investigate the effects of scaffolding mathematical modelling within the operative-strategic learning environment by means of the "solution plan".

In the next section we will report on the conceptualization of the strategic element "solution plan".

### 2.3.2 Solution Plan as scaffolding instrument

According to the requirements discussed above, an effective strategic instrument for students should include aspects of accompanying diagnosis, adaptive support and fading (Puntambekar and Hubscher 2005). The attempt to implement these aspects in the form of a "process worksheet" – including a clear cognitive structure as an important element of a designed scaffold – is intended by the DISUM "solution plan" (see Figure 2). Through breaking down the entire solution process into four single steps of the modelling process, student difficulties ought to be detected and located more easily and tangibly. Thus, an autonomous *diagnosis* by the students should be possible, and, if necessary, the support noted in the "solution plan" may already provide sufficient help for students to overcome their difficulties. If not, the teacher may give further immediate and appropriate feedback. The solution plan is meant to help to locate students' difficulties within the solution process and improve the *adaptive support* provided by teacher. Application of *fading* can be easily done by providing an instrument on paper, as students only need to use the solution plan in case of difficulties and if no suitable strategy is available. As soon as students know the phases of solutions and understand the help given in the plan the instrument is no longer required. After students have internalised an appropriate solution process, they are expected to use strategies more frequently and to improve their modelling competency.

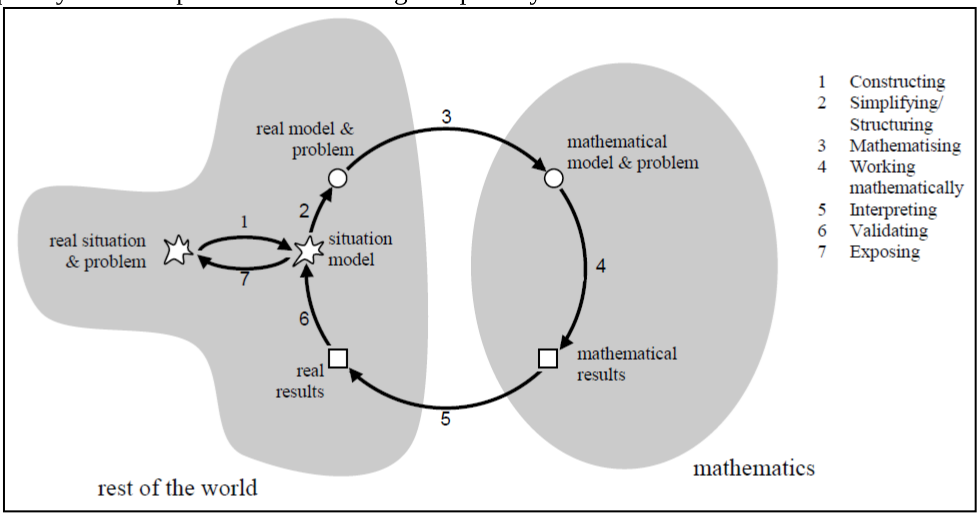


**Fig. 1** Seven-step modelling cycle (Blum and Leiss 2007b)

As indicated above, the acquisition of modelling competency varies with different areas of content. An intention of the current study was to develop an instrument at an intermediate level of generality that was task-based (for mathematical modelling) and across content areas, and therefore beneficial regarding its applicability in various topic areas. Due to this intermediate level of generality, students' autonomy and long-term acquisition of modelling competency can be facilitated, both of which are regarded as important goals of scaffolding.

For mathematical modelling considerable research already exists, e.g. about (sub-)competencies and ideal-typical solution processes (see Blum 2011). It is therefore reasonable to construct the strategic aid "solution plan (for modelling tasks)" based on an already existing modelling cycle. The seven-step cycle shown in Fig. 1 seems to be particularly suitable, as in previous years it has been used as an effective analysis instrument for researchers in several studies (see Schukajlow et al. 2012; Schukajlow and Krug 2014; Schukajlow et al. in press), as well as a helpful diagnosis instrument for teachers.

However, learners, especially those who have just begun to learn modelling and to control their working process, cannot easily distinguish steps 2/3 and steps 5/6/7. In order to reduce the complexity of representation and to create an instrument that is comprehensible by students, a simplified model which combines certain steps is reasonable.

This has led to the following four ideal-typical process phases as shown in the solution plan (see Fig. 2), which partly consist of elements that are also found in other descriptions of modelling processes (e.g. Staub and Reusser 1995; Verschaffel et al. 2000).

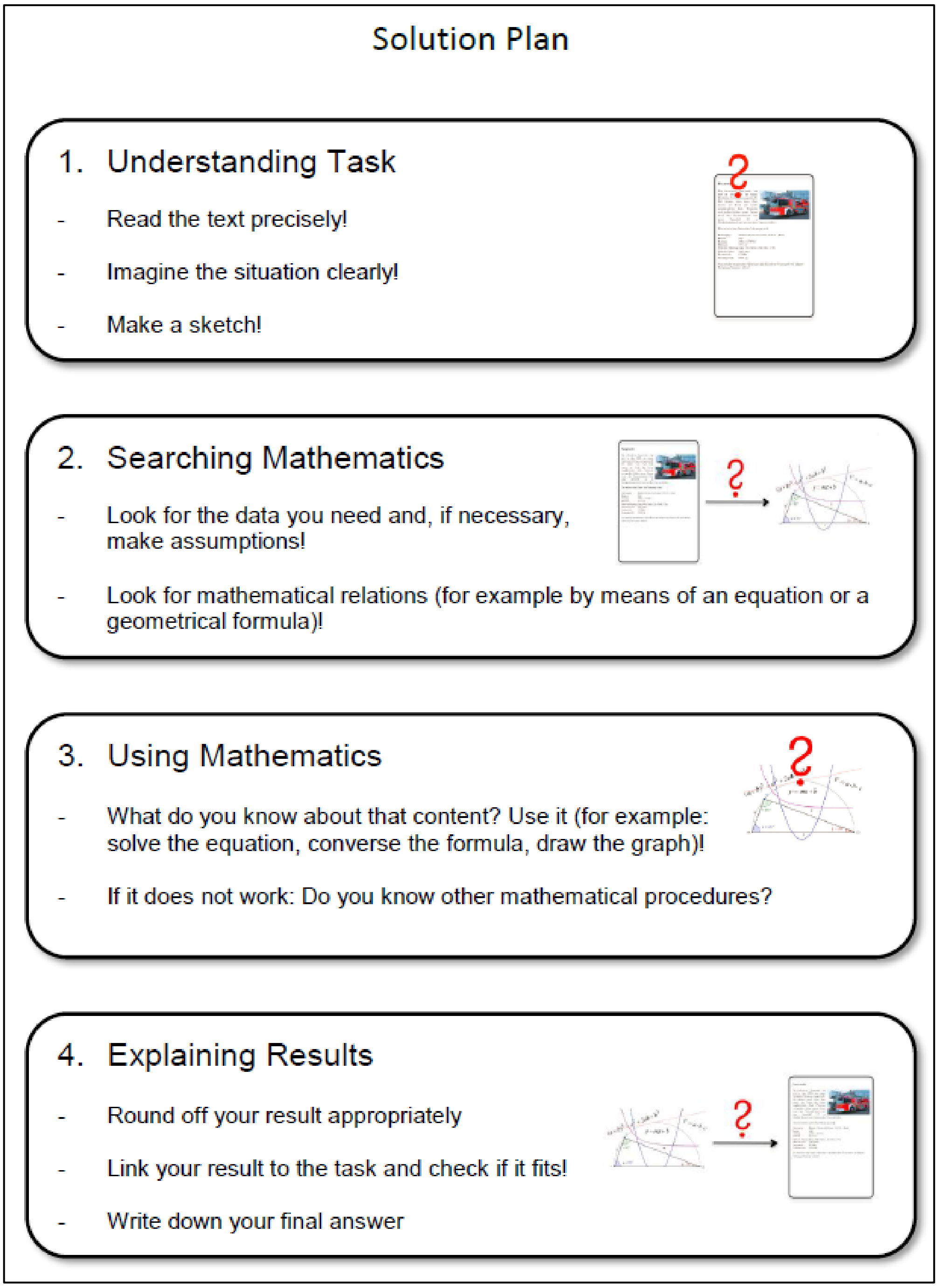


**Fig. 2** Solution plan

1. *Towards the situation model*: The given real-world situation and the inherent problem are captured and then visualised or sketched on paper (for the importance of situation model while solving modelling problems, see Leiss et al. 2010).
2. *From the situation model towards the mathematical model*: Relevant and redundant information are filtered, and given and required values are identified. If information is still lacking, then students make suitable assumptions about missing values. Known formulae and procedures fitting with the context are selected.
3. *From the mathematical model towards the mathematical result*: Mathematical processing by using equations or other mathematical entities leads to a mathematical result.
4. *From the mathematical result towards the real result*, including validation of the result and explanation of the solution: The solution is translated back into the real-world context, and simultaneously it ought to be assessed, based on real-life experience, to determine whether the result may be correct and whether it satisfies the real situation. Should any doubt arise, the student has to go back to one of the previous steps. When the result seems to make sense, the initial question is finally answered comprehensively.

According to the types of strategy, the solution plan as a whole is a metacognitive planning strategy. At the same time, in each of the mentioned steps, cognitive learning strategies (which are in this case also specific problem solving strategies) like rehearsal (e.g. the repeated reading in step 1), organisation (e.g. drawing a figure in step 1) and elaboration (e.g. making assumptions in step 3 or estimation and rounding in step 4) are stimulated.

Like a toolbox, the solution plan offers suitable kinds of help for each of its phases. It is also possible to use only selected single elements of help from this plan. An adequate use of the solution plan requires students to have a deep understanding of the suggested strategies. For applying strategies correctly, as described above in section 2, the teacher's support is indispensable.

In order to make the intended solution plan understandable and visually attractive for students, the partial steps are given as specific procedural instructions, supplemented by further hints and additional illustrations.

We use the task "Fire-brigade" from the DISUM project to demonstrate the ideal-typical four-step solution process as defined by the solution plan (see Fig. 3).

**Fire-brigade**

In 2004, the Munich fire-brigade got a new fire engine with a turn-ladder. Using the cage at the end of the ladder, the fire-brigade can rescue people from great heights. According to the official rules, while rescuing people, the truck has to maintain a distance of at least 12 metres from the burning house.

*Technical data of the engine*

| | |
|---|---|
| Engine model: | Daimler Chrysler AG Econic 18/28 LL - Diesel |
| Construction year: | 2004 |
| Power: | 205 kw (279 HP) |
| Cubic capacity: | 6374 cm³ |
| Dimensions of engine: | length 10 m  width 2.5 m  height 3.19 m |
| Dimensions of ladder: | length 30 m |
| Weight of unloaded engine: | 15540 kg |
| Total weight: | 18000 kg |

From what maximal height can the Munich fire-brigade rescue people with this fire engine?

**Fig. 3** Modelling problem "Fire-brigade" (Blum 2011)

*Step 1 – Understanding Task*

The first step is to understand the given problem situation. The problem solver has to construct a situation model that includes, at a minimum, a burning house and a fire engine with a ladder that extends from the back of the fire engine. Students have to find the maximal height from which people in the house can be rescued.

*Step 2 – Searching Mathematics*

The second step is to structure the situation by bringing certain variables into play and making assumptions based on these variables. In particular students must note the position of the fire engine (relative to the house) and they must define what "maximal height" means in order to simplify the situation leading to a real model of the situation. The maximal height depends on the position of the engine. The real model is then transformed into a mathematical model which consists of a right-angled triangle on top of a line segment representing the height of the engine.

*Step 3 – Using Mathematics*

The next step is working mathematically (using Pythagorean theorem, calculating etc.) which yields mathematical results dependent on the assumptions.

*Step 4 – Explaining results*

These results are interpreted in the real world context, resulting in a calculated height dependent on the position of the engine. A validation of these results may show that it is appropriate or necessary to repeat steps 2 and 3 of the solution plan a second time, for instance to take into account more factors such as the height of the fireman or a possible jumping of the person that has to be rescued from the burning house. The final step is to write down the final solution.

# 3 Research questions

In the light of the theoretical basis and of empirical results, our research questions have been formulated as follows:

1. Do students whose work on modelling problems was scaffolded with the “solution plan” use more elaboration, rehearsal, organizing and planning strategies than students who worked on the same problems without the “solution plan”?
2. Do students whose work on modelling problems was scaffolded with the “solution plan” outperform students who worked on the same tasks without the “solution plan” concerning their modelling competency? In order to specify that question the modelling competency is apportioned into the following categories:
    a. *Global modelling competency* that includes all sub-competencies of modelling including the technical part of working mathematically,
    b. Sub-competency *“Building a mathematical model”* which means to find an adequate situation model and real model (including making assumptions), as well as the correct translation into a mathematical model,
    c. Sub-competency “*Interpreting results”* which means writing down an appropriate answer to the initial problem.

Previous studies showed different developments in different content areas (Blum 2011). Because of that and for the purpose of verifying the demand on the instrument to be adaptable to different content areas, we will check whether or not achievement develops equally in both content areas “Pythagorean theorem” (PT) and “linear functions” (LF).

We assume that scaffolding modelling with the “solution plan” in the experimental group leads to a higher “Global modelling competency”, to higher achievement in the sub-competencies and a more frequent use of elaboration, rehearsal, organizing and planning strategies than in the control group (where students solve modelling problems without using the “solution plan”).

# 4 Method

## 4.1 Sample

Ninety-six German ninth graders from six classes of four middle track schools (Realschule) took part in the present study. The experimental group (EG) was composed of one class, selected at random, from each of three different schools. The other three classes formed the control group (CG). According to the teachers’ statements, all students had already been taught the mathematical content dealt with in our study.

Based on an initial mathematics achievement test (23 items testing various mathematical content areas and competencies, see Leiss et al. 2010), all classes were reduced to 16 students so that the average achievement in the six classes did not differ significantly. Five students missed parts of the experiment (lessons or tests) so they were excluded from the statistical analysis. Hence the final sample size is N=91; the mean age was 15.9 years (SD 0.61); approximately 60% were female.

## 4.2 Design

The experiment began for all six classes with a pretest which included an achievement test (see section 4.6.2) and a questionnaire concerning strategies (refer to Fig. 4). After the pretest, students of both the experimental and control groups were instructed using the operative-strategic teaching method (in fig. 4 “op-str.” in short; see section 2.3.1 for more details) and the same modelling tasks from both content areas (PT and LF). Following this the students performed a posttest.

In order to control for the teacher variable, the lessons for all six classes were given by a specially trained lecturer of Kassel University who is a highly experienced mathematics teacher and did not know the students before the lessons.

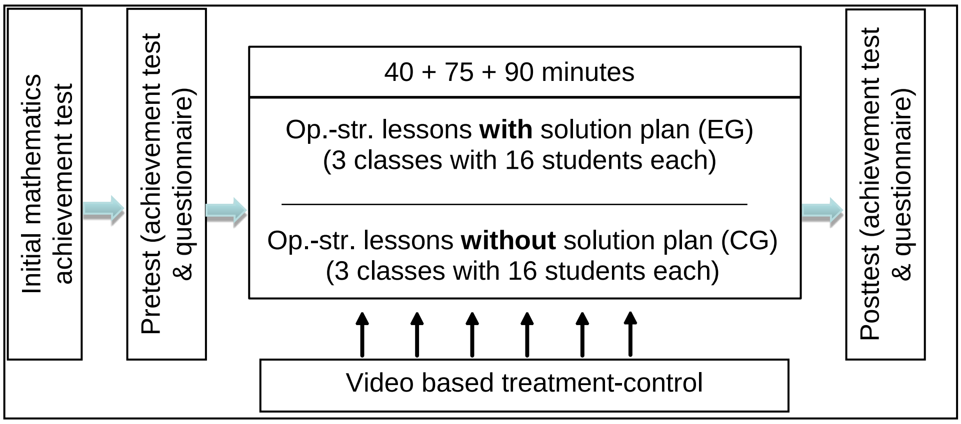


**Fig. 4** Design of the experiment

## 4.3 Treatment

During the teaching unit, five modelling tasks were treated in both groups: three tasks from the content area of Pythagorean theorem and two from the content area of linear functions ("Fire-Brigade", "Salt Mountain", "Sugar Loaf", "Horse-Riding Center" and "Filling up", see Blum and Leiss 2006, 2007a; Schukajlow et al. 2011; Schukajlow and Krug 2014). Students worked on the tasks and were requested to find individual solutions even though they had their working-group (3 or 2 classmates) as back up and, if necessary, the teacher's support. According to the operative-strategic teaching method the teacher was instructed to give only minimal-adaptive interventions and to promote students' autonomy in their solution process as far as possible.

In the three EG-classes, the strategic element "solution plan" was implemented as a designed scaffold. The solution plan was given to every student at the beginning of the teaching unit and was discussed in detail in the classroom. Before the students started to work with the solution plan by themselves, the teacher demonstrated its application (in a way of "modeling" according to Cognitive Apprenticeship, see Collins et al. 1989), using the task "Fire-Brigade" as an example. Every step of the solution plan was explained and demonstrated and, if something was unclear, questions of the students were discussed, in order to enable an autonomous *diagnosis* of difficulties in the solution processes by the students themselves, while solving other modelling problems. The teacher also involved the solution plan in his interventions ("What does the solution plan tell you"?) for stimulating each student's independent use of this scaffolding instrument and for improving *adaptive* support. At the beginning of the teaching unit the teacher was instructed to ask the students what phase of the solution process, according to the solution plan, they were currently using and whether they had used the strategies listed in the plan. The direct reference to the solution plan by teachers decreased during the teaching unit to support a *fading* process.

In the CG the teacher's interventions were made verbally and without a visual or schematic depiction corresponding to the solution plan.

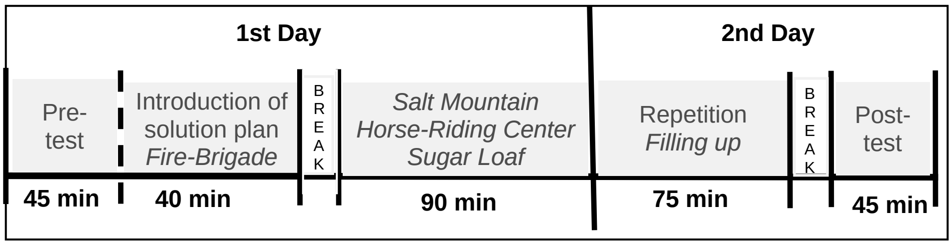


**Fig. 5** Lesson plan

Although the experimental lesson differed a lot from everyday school life (videotaping of the lessons, no homework, several tests), the teaching unit included some typical attributes of whole-class scaffolding (Smit et al. 2013): Teacher and students had opportunities for diagnosis and reflection on the solution steps and on the strategies used. Moreover, the solution plan facilitated a gradual transfer of the responsibility for the solution process towards the students, which complies with the attribute of the layering of scaffolds. In addition, the use of the solution plan was spread over the entire lesson (distributed nature), because on the one hand it applies to every step in the solution process and on the other hand it is (or can be) applied for all tasks.

## 4.4 Treatment fidelity

Several measures served to ensure a high degree of fidelity of the treatment. The instructor had more than 15 years of teaching experience and was familiar with the teaching of modelling problems, as well as the operative-strategic teaching method used in this study. During the study, at least one person from the research group was present in order to administer the tests and assessment scales and to observe the implementation of the treatment. All lessons were video recorded, and students' written work was collected; analysis of both materials showed that the treatment conditions were implemented accurately.

## 4.5 Instruments

### 4.5.1 Questionnaire on strategy use

Five-point Likert scales (1 = not at all true, 5 = completely true) were used to assess students' strategies before and after the experimental lessons. All scales had been used in other studies for measuring students' strategies (Pekrun et al. 2002; Rakoczy et al. 2005). Some items were modified in order to specify the mathematics-typical nature of strategies as it is explained by Schukajlow and Leiss (2011).
Reliablities of the scales (Cronbach's α) are at least on a satisfactory level (see Tab. 1). The instruction for all items was the following: "When I am working on a difficult word problem…"

**Table 1** Questionnaire on strategy use (see Schukajlow and Leiss 2011)

| Scale | Items | Example | Reference to steps in the solution plan | Cronbach's α Pretest / posttest |
|---|---|---|---|---|
| Elaboration | 4 | … I reason what I know already and how I can use this knowledge for finding a solution. | 2 | .74 / .80 |
| Rehearsal | 5 | ... I read some sentences once again. | 1 | .69 / .75 |
| Organizing | 7 | … I reason how the information from the task is connected | 1 | .79 / .83 |
| Planning | 4 | … I reason first, which solution steps are necessary. | whole plan | .62 / .74 |

### 4.5.2 Modelling achievement test

In the DISUM Project more than fifty modelling tasks were developed and deployed in different studies. Item parameters for these tasks were known from former scaling, so we could create a pre- and a posttest for the measurement of students' achievements (12 Items each, arranged in a rotation design between the test booklets; an example is given in Fig. 6) that are on a very similar level of difficulty.

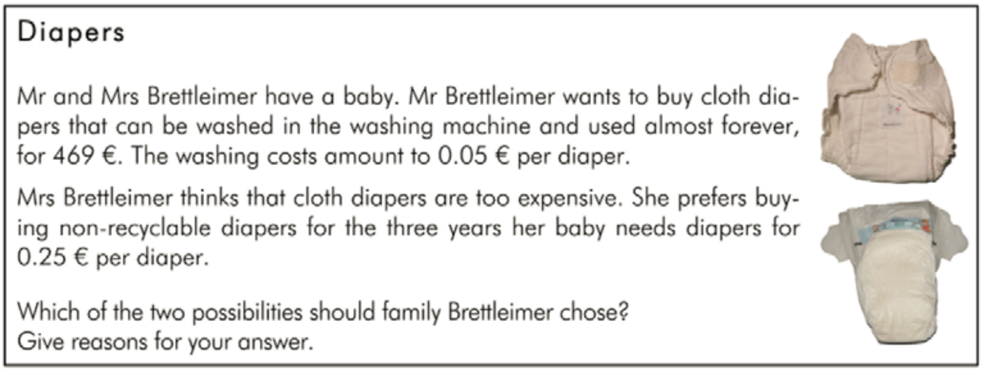


**Fig. 6** Test item "Diapers" (Leiss et al. 2010)

To examine the students' answers a high inference rating was used. The coding scheme consisted of two categories that referred to certain steps in the modelling process and of one category for the "Global modelling competency" (see Tab. 2). All categories were rated with 0 or 1 by two well-instructed researchers. First, the raters were instructed about the codes they should assign using students’ solutions to the same problems from another study. Second, they rated the solutions from the current study and the inter-rater reliability was calculated. Inter-rater reliability Cohen’s Kappa (Cohen 1960) is satisfactory for all cases (see Tab. 2).

**Table** 2 Categories of the coding scheme

| Category | Full credit (code 1) was given for… | Range of Cohen‘s Kappa |
|---|---|---|
| Global modelling competency | a correct mathematical model is created & calculations / mathematical manipulations are correct & results are adequately interpreted | .73 - .96 |
| Building a mathematical model | mathematical model and mathematical approach (un-manipulated equation or geometric formula) are correct | .84 - .96 |
| Interpreting results | the mathematical results are (whether they are correct or not) interpreted adequately in the given context | .76 - .96 |

Items selected for analysis were those which allowed for examination of the respective category. For example, we cannot expect a written interpretation of a multiple-choice item (category “Interpreting results”).

Using the Rasch model (Wu et al. 1998) students’ performance parameters (weighted likelihood estimator WLE, see Warm 1989) of pre- and posttest for all categories were estimated, although no student worked on one item twice. The WLEs characterize students’ performance using continuous scales. These parameters (transformed on the PISA-Level with a mean of 500 and a standard deviation of 100) are the basis for all following analyses.

As modelling competency may increase differently in the two content areas PT and LF, we conducted a supplementary analysis using a two-dimensional model for the categories “Global modelling competency” and “Building a mathematical model”. In this two-dimensional model we differentiate between both content dimensions (Tab. 3). For the last category this differentiated approach turned out to be redundant because the achievement developed very similarly in both content areas (see section 5.2.3). All test reliabilities were found to be satisfactory (Tab. 3).

**Table 3** Test dimensions, number of items in each dimension and test reliabilities

| Category | Test Dimension | Number of items | Reliability (EAP)* |
|---|---|---|---|
| Global modelling competency | whole test | 24 | .675 |
| | Just Pythagorean theorem | 12 | .615 |
| | Just linear functions | 10 | .600 |
| Building a mathematical model | whole test | 24 | .676 |
| | Just Pythagorean theorem | 12 | .622 |
| | Just linear functions | 10 | .603 |
| Interpreting results | whole test | 19 | .705 |

Note: * EAP reliability estimates can be interpreted in the same way as Cronbach’s alpha.

# 5 Results

## 5.1 Development of strategy use

The first research question of the present study was about the influence of scaffolding students’ learning with the solution plan on students’ strategy use. The preliminary analysis revealed that on average the self-reported use of strategies was slightly higher than the theoretical mean of the scales (which is 3.0) (see Tab. 4). Statistical analysis of students’ strategies at pretest using t-tests shows that students of the experimental and the control group did not differ in self-reported strategy use ($p > .10$).

**Table 4** Students‘ self-reported strategy use

| Scale | EG Pretest M (SD) | EG Posttest M (SD) | CG Pretest M (SD) | CG Posttest M (SD) |
|---|---|---|---|---|
| Elaboration | 3.85 (0.59) | 3.98 (0.71) | 3.91 (0.81) | 3.76 (0.85) |
| Rehearsal | 3.63 (0.62) | 3.70 (0.68) | 3.43 (0.78) | 3.28 (0.74) |
| Organizing | 3.19 (0.68) | 3.60 (0.76) | 3.21 (0.75) | 3.24 (0.78) |
| Planning | 3.03 (0.65) | 3.18 (0.70) | 2.93 (0.75) | 2.83 (0.76) |

Using repeated measures (ANOVA) the two groups were analyzed regarding the development of their self-reported strategy use. In all scales students of the EG outperform the CG-students; the attribution to one or the other condition (with or without solution plan) significantly accounts for variance explanation of strategy use in posttest. By means of the reported elaboration-strategies (time $F(1) = 0.024$, $\eta^2 < .001$, $p = .877$, time*design $F(1) = 4.933$, $\eta^2 = .053$, $p = .029$) one can state that EG-students noticed the strategies which were explicitly specified in step 2 of the solution plan and reported on a significantly higher (nearly a medium effect size) usage of them. Also a slight increase of rehearsal strategies, as are claimed in steps 1 and 4, can be ascribed to the use of the solution plan in the lesson (time $F(1) = 0.383$, $\eta^2 = .004$, $p = .537$, time*design $F(1) = 2.808$, $\eta^2 = .031$, $p = .097$). Slightly more than three percent of variance in the different development of rehearsal strategies in both groups was attributed to the use of the solution plan as scaffold. Organizing strategies presented in step 1 have “reached” the students as well. The analysis of variance shows the significant influence (with a medium effect size) of the application of the solution plan on the usage of these kinds of strategies (time $F(1) = 8.042$, $\eta^2 = .084$, $p = .006$, time*design $F(1) = 7.284$, $\eta^2 = .076$, $p = .008$). The effects of the solution plan on students’ planning strategy can be also confirmed (time $F(1) = 0.138$, $\eta^2 = .002$, $p = .711$, time*design $F(1) = 3.262$, $\eta^2 = .036$, $p = .074$).
Thus, as intended in our study, scaffolding students’ learning with the solution plan had a positive impact on the development of elaboration, rehearsal, organizing and planning strategies.

## 5.2 Development of modelling competency

The main research interest of our study was the development of the modelling competency of those students (EG) whose work on modelling tasks was scaffolded with the solution plan in comparison to those students (CG) who solved the same tasks without solution plan (see question 2 in section 3). For this purpose the students’ performance parameters (see mean M and standard deviation SD in Tab. 5) constitute the starting material for repeated measures (ANOVA), as presented in this section.

**Table** 5 **Means and standard deviations for students’ global modelling competency and sub-competencies**

| Category | Test Dimension | EG Pretest M (SD) | EG Posttest M (SD) | CG Pretest M (SD) | CG Posttest M (SD) |
|---|---|---|---|---|---|
| Global modelling competency | Whole test | 334 (90) | 382 (101) | 363 (95) | 393 (103) |
| | Pythagorean theorem | 422 (79) | 459 (104) | 466 (112) | 465 (97) |
| | Linear functions | 392 (95) | 440 (104) | 378 (92) | 456 (110) |
| Building a mathematical model | Whole test | 362 (100) | 412 (98) | 397 (89) | 420 (100) |
| | Pythagorean theorem | 435 (84) | 476 (106) | 483 (111) | 475 (93) |
| | Linear functions | 430 (99) | 475 (95) | 413 (95) | 490 (105) |
| Interpreting results | Whole test | 392 (83) | 478 (80) | 429 (74) | 474 (113) |

At the pretest, students of both groups did not differ in most of the categories ($p > .10$), as a comparison of the groups using t-tests shows. Just concerning the content area PT, the CG is significantly better than the EG (Global modelling competency $p = .025$, Building a mathematical model $p = .023$).

### 5.2.1 “Global modelling competency”

The analysis (repeated measures, ANOVA) shows that all students improved their pretest results in the posttest and that there is no significant difference between the experimental and the control group (time: $F(1, 88) = 16.972$, $\eta^2 = .162$, $p < .001$; time*design: $F(1, 88) = 0.926$, $\eta^2 = .01$, $p = .339$).
A more differentiated analysis of the “Global modelling competency” reveals that the content area has a significant influence (with medium effects size) on the development of students’ modelling competency (time*content: $F(1, 88) = 12.379$, $\eta^2 = .123$, $p = .001$; time*content*design: $F(1, 88) = 7.984$, $\eta^2 = 0.083$, $p = .006$). In the content area of PT, the “Global modelling competency” increased significantly more in the EG, whereas it did not change in the CG (time: $F(1, 88) = 2.28$, $\eta^2 = .025$, $p = .135$; time*design: $F(1, 88) = 4.921$, $\eta^2 = .053$, $p = .029$). In the content area of LF students of both groups achieved significantly higher results in the posttest. The use of the solution plan had no influence on the development of “Global modelling competency” (time: $F(1, 88) = 29.243$, $\eta^2 = 0.249$, $p < .001$; time*design: $F(1, 88) = 2.521$, $\eta^2 = 0.028$, $p = .116$). Thus, we can

confirm a positive influence of scaffolding on students' modelling competency in the topic area PT (with effect size between low and medium), but not in the topic area LF.
For the investigation of the benefits of the operative-strategic lesson with and without the solution plan as a scaffolding instrument (cf. section 4.1) the above-mentioned sub-competencies of mathematical modelling are analyzed in the next sections.

### 5.2.2 Sub-competency "Building a mathematical model"

Analysis of students' achievement concerning "Building a mathematical model" shows small and not statistically significant benefits for the students instructed in the EG (time: $F(1, 88) = 16.2$, $\eta^2 = .155$, $p < .001$; time*design: $F(1, 88) = 1.913$, $\eta^2 = .017$, $p = .17$). The development of this sub-competency differs in both content areas (time*content: $F(1, 88) = 11.503$, $\eta^2 = .117$, $p = .001$; time*design*content: $F(1, 88) = 10.319$, $\eta^2 = .106$, $p = .001$).
The positive influence of the solution plan on the sub-competency "Building a mathematical model" was found in the content area PT. A closer look reveals that students of the CG achieved similar scores in pre- and posttest, whereas students of the EG showed significantly better scores in the posttest, with effects of medium size (time: $F(1, 88) = 2.088$, $\eta^2 = .023$, $p = .152$; time*design: $F(1, 88) = 6.985$, $\eta^2 = .074$, $p = .01$). In the content area LF all students increased their scores strongly, but there are no significant differences between EG and CG (time: $F(1, 88) = 28.523$, $\eta^2 = .247$, $p < .001$; time*design: $F(1, 88) = 2.566$, $\eta^2 = .029$, $p = .114$).

### 5.2.3 Sub-competency "Interpreting results"

We found a difference between the experimental and control groups in the development of this sub-competency. First, students of the whole group increased their achievement scores in the posttest (time: $F(1, 88) = 43.937$, $\eta^2 = .333$, $p < .001$). The development of the competency "Interpreting results" was higher in the EG than in the CG. Thus, the use of the scaffolding instrument "solution plan" affected the development of this sub-competency significantly (time*design: $F(1, 88) = 4.962$, $\eta^2 = .053$, $p = .028$).
The analysis of the influence of the content area on "Interpreting results" showed no significant findings (time*content: $F(1, 88) = 0.196$, $\eta^2 = .002$, $p = .659$; time*content*design: $F(1, 88) = 1.475$, $\eta^2 = .016$, $p = .228$).

# 6 Summary and discussion

The strategic instrument "solution plan" is a scaffold designed specifically for students solving modelling tasks. By means of organizing, elaboration, rehearsal and planning strategies being systematically offered by the solution plan, a cognitive structuring of the solution process and hence an improvement of students' modelling competency were intended. Our two-day experiment with an experimental and a control group has shown significant improvement of performance of the whole population which indicates the effectiveness of operative-strategic teaching for the development of students' modelling competency and replicates the results of previous studies (Blum 2011; Schukajlow et al. 2009; Schukajlow et al. 2012). The possibility of class or group effects as a consequence of clumping of the sample should be mentioned here as a methodological limitation of this study and therefore applies to all analyses reported here.
The first research question was about the effects of scaffolding students' learning with the solution plan on students' strategy use. A significant increase in organizing, elaboration, rehearsal and planning strategies in the experimental group, where students' learning was scaffolded with the solution plan, was found. The effects ranged from middle to slightly under the middle size for most strategies. By introducing and repeating single steps from the solution plan, learners of this group (EG) have apparently internalised these steps. An extensive discussion of strategies and the continuing presence of the solution plan as a helpful scaffold for solving the tasks have obviously stimulated the students to include the strategies into their solution process for modelling tasks. Our results support findings of other studies that revealed that it is possible to improve students' strategy use in a relatively short time (Heinze et al. 2009; Weinstein et al. 2000). Furthermore, we extended the research on strategies and showed how the use of strategies can be improved in mathematics lessons. Scaffolding of learning using a cognitive structuring influenced students' strategies positively, as it promoted autonomous diagnosis of difficulties in problem solving by students, with the goal of improving teachers' adaptive support and to realize fading in the classroom. This result is in line with findings from other studies that have shown positive effects of scaffolding on strategies and self-regulation (Azevedo and Hadwin 2005; Hadwin and Winne 2001; Hadwin et al. 2005).

The extent to which performance in solving modelling tasks has been improved and differences between the EG and the CG, was the second research question of our study.
First of all, the analysis showed a significant impact of the topic area upon the "Global modelling competency" of students for both groups. Thus, theoretical considerations about the importance of content area for mathematical modelling competency (Blum 2011) were confirmed in our study empirically. A transfer of modelling competency between different content areas seems to be difficult to realize and treatments with a particular focus on such a transfer should be investigated in future studies.
In the topic area "linear functions" both groups developed strongly, but the differences in development between two groups were weak and not statistically significant. In contrast, in the area "Pythagorean theorem" only the group with the solution plan as scaffold showed higher modelling performance at the posttest. Almost identical results came out regarding the sub-competency "Building a mathematical model" for the two groups. Further, the solution plan effects the sub-competency "Interpreting mathematical results" positively. Students of the experimental group interpreted the mathematical results better while solving modelling problems in both content areas compared to students of the control group.
One explanation for these different developments in different content areas can be the strategic prompts given in the solution plan. Whereas a prompt such as "Make a sketch!" can be directly used by students when solving modelling tasks regarding the Pythagorean theorem, it cannot be used in modelling tasks regarding linear functions equally directly. A supplement of the solution plan, with new strategic prompts such as "Which quantities may vary and which may not" to support the identification of variables and the construction of equations, could perhaps improve the beneficial effects of the solution plan when solving problems from the content area linear functions. However, the weaknesses of such content specific prompts are that the solution instrument must not be topic-specific. If there were specific solution plans for different topic areas, the cognitive effort to identify the right topic and to choose the suitable plan could counterbalance the plan's benefits.
Future studies should also investigate which strategic prompts in the solution plan are particularly helpful and which are not, from students' point of view. The analysis of different ways of the use of the solution plan by different individuals is another interesting new study. The results of such analyses can help to better adapt the solution plan to the students' needs and to improve its power. Further, the scaffolding of mathematical learning with cognitive structures such as the solution plan is another important issue that should be addressed in future studies. The investigation of students' learning behavior and teacher-student interactions, with and without a solution plan, can provide important insights into the effects of such a designed scaffold on diagnosis, support and fading procedures (Puntambekar and Hübscher 2005).
Summarizing our results, we conclude that our initial assumptions were confirmed for students' strategy use and partly confirmed for their improvement in modelling competency. One explanation for the partly weak effects of scaffolding on students' modelling competency may be the low quality of strategy use while solving modelling problems. As we know from other studies (for example Leutner et al. 2007), the quality of strategy use is crucial for the effects on students' performance and these effects can be increased when strategy training is combined with self-regulation. The impact of such a combined treatment condition on modelling competency should be investigated in future studies. Another explanation of partly weak effects might be the short duration of the intervention.
We suggest further investigation into the effects of such a solution plan on the performance of students in other age groups and other content areas. There are other studies that indicate the positive impact of meta-cognitive elements on students' performance in early grades (Mevarech et al. 2010). Another important future research question may be whether the strategic prompts or the implementation of design scaffolds ought to be different for students of different grades, and equally interesting are long-term effects of the solution plan. As teacher support is crucial for the implementation of the solution plan in everyday mathematic classrooms, the investigation of teacher competencies needed for effective instruction with the solution plan and how such competencies can be improved are also very important open research questions.

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