Arch. Math. 77 (2001) 215–221
0003-889X/01/030215-07 $ 2.90/0

Sonderdruck aus
Archiv der Mathematik

Semi-stability and base change

By

URS T. HARTL

Abstract. Let X and Y be regular strictly semi-stable varieties over a discrete valuation ring R and let $R \subsetneqq \widetilde{R}$ be a finite ramified extension of discrete valuation rings. We explicitly give desingularization procedures for the base change $X \times_R \widetilde{R}$ and for the product $X \times_R Y$.

0. Introduction. The concept of semi-stable curves was extremely fruitful for the development of arithmetic algebraic geometry. It plays a central role in analyzing the degeneration of smooth curves and so is an important tool in passing from curves over the field of fractions K of a discrete valuation ring R to curves over the residue field k of R. Beyond the one dimensional case the notion of semi-stability can be generalized to higher dimensions; cf. [1]. There again semi-stable varieties possess the mildest sort of singularities one may hope for in the degeneration of smooth varieties. But whereas every smooth curve over K has a semi-stable model over R after allowing a finite extension of discrete valuation rings, the analogous statement in higher dimension is known only when the residue characteristic (k) is zero; cf. [3]. In general this is still a conjecture. On the other hand A. J. de Jong has shown in [1, Theorem 6.5] that any variety X over a complete discrete valuation ring admits an alteration $X' \longrightarrow X$, i.e. a proper surjective generically finite morphism, by a regular strictly semi-stable variety X' after a finite extension of discrete valuation rings. This is less than the desired general existence of semi-stable models but nevertheless it is valuable for plenty of applications as de Jong has demonstrated.

To further investigate this useful concept of semi-stability is the aim of this article. We explicitly give desingularization procedures for the product of two regular strictly semi-stable varieties over R and for the base change of a regular strictly semi-stable variety by a ramified extension of discrete valuation rings. They allow to recover regularity, whenever given regular strictly semi-stable varieties are subject to product or base change operations. For instance they can be used in the investigation of the rigid analytic Picard functor; cf. [2]. There one faces the problem of extending line bundles from the generic fiber $X_K \times_K Y_K$ of a product $X \times_R Y$ of two regular strictly semi-stable formal schemes to $X \times_R Y$. For this purpose one needs a regular model of $X_K \times_K Y_K$. Our desingularization procedures yield such a model.

Mathematics Subject Classification (1991): 14E15, 14B07.

1. Definition and Examples. Let R be a discrete valuation ring with field of fractions K, uniformizer π and residue field k and let $S = \operatorname{Spec} R$. Let further $\widehat{R}$ be the π-adic completion of R.

Let us recall the definition of a semi-stable S-variety (cf. A. J. de Jong [1, 2.16]).

Definition 1.1. An *S-variety* is an irreducible, reduced and separated scheme X, flat and of finite type over S. We denote by $X_K := X \times_S \operatorname{Spec} K$ the generic fiber of X and by $X_0 := X \times_S \operatorname{Spec} k$ the special fiber. Let X_0^σ for $\sigma \in N$ be the irreducible components of the special fiber X_0 of X. For every $\emptyset \neq M \subseteqq N$ we define

$$X_0^M := \bigcap_{\sigma \in M} X_0^\sigma$$

as the scheme-theoretic intersection.

The S-variety X is called *regular strictly semi-stable* if

(a) X_K is smooth over K,
(b) X_0 is geometrically reduced,
(c) X_0^σ is a Cartier divisor on X for all $\sigma \in N$ and
(d) X_0^M is smooth over k for all $M \subseteqq N$ and equidimensional of dimension $\dim X - \# M$.

X is called *regular semi-stable over S* if étale locally X is regular strictly semi-stable.

Remark 1.1.1. Any regular, (strictly) semi-stable S-variety is in fact *regular*. This is a consequence of condition (c); cf. Proposition 1.3. Its special fiber is a divisor with normal crossings.

Remark 1.1.2. If X is proper over S, condition (a) is a consequence of the other three conditions, since then every closed point of X_K specializes to a point of X_0; cf. Proposition 1.3.

Example 1.1.3. Let X be a *semi-stable curve over S*, i.e. X is flat and proper over S and all geometric fibers of X are reduced connected curves having at most ordinary double points as singularities. Assume further that the generic fiber X_K is smooth over K. If we repeatedly blow up all the points in which X is not regular we obtain a desingularization $\widetilde{X} \longrightarrow X$ of X. Since X is a semi-stable curve, all irreducible components of its special fiber X_0 and all its singular points are defined over a separable field extension of k. Therefore the special fiber $\widetilde{X}_0$ of $\widetilde{X}$ is geometrically reduced and has at most ordinary double points as singularities. Since $\widetilde{X}$ is regular, all irreducible components of $\widetilde{X}_0$ are Cartier divisors. So we see that $\widetilde{X}$ is a regular semi-stable S-variety. If all irreducible components of the special fiber $\widetilde{X}_0$ are smooth then $\widetilde{X}$ is even regular strictly semi-stable. On the other hand if there are components of $\widetilde{X}_0$ which intersect themselves, one can make X regular strictly semi-stable only after an extension of discrete valuation rings $R \subseteqq R'$ of ramification index 2. Then blowing up all the non-smooth points of the special fiber of $\widetilde{X} \otimes_R R'$ yields a regular strictly semi-stable variety over R'. The base extension is necessary to avoid the appearance of non-reduced components.

Example 1.1.4. In [4, Theorem 4.6] K. Künnemann has shown that every abelian variety A_K over K which has semi-abelian reduction and is polarized by an ample line bundle admits a regular strictly semi-stable model over R after a finite extension of discrete valuation rings.

E x a m p l e 1.1.5. In the general case A. J. de Jong has shown in [1, Theorem 6.5] that every S-variety X over the spectrum S of a complete discrete valuation ring admits an alteration $X' \longrightarrow X$, i.e. a proper surjective generically finite morphism, by a regular strictly semi-stable S-variety X' after a finite extension of complete discrete valuation rings.

R e m a r k 1.1.6. If we further assume that R is a complete discrete valuation ring, we can do rigid geometry over K. In this context we can analogously define the notion of a *regular, (strictly) semi-stable formal model* of a rigid analytic space (cf. [2]) by imposing the same conditions (b), (c) and (d) on the special fiber of the formal model. In this case however the smoothness of the rigid space is always a consequence of the other three conditions, since every (closed) point of the rigid space specializes.

E x a m p l e 1.1.7. Let R be a complete discrete valuation ring. G. A. Mustafin [5] has given an example for a rigid analytic space over K that has a regular strictly semi-stable formal model. It is the *rigid analytic Hopf variety* which is defined analogously to its counterpart in complex algebraic geometry. Namely let X_K be the quotient of the pointed affine space $\mathbb{A}^n_K - \{0\}$ by the action of the group $\varGamma := g^{\mathbb{Z}} \subseteqq \mathrm{GL}_n(K)$ with

$$g = \begin{pmatrix} \pi^e u_1 & & 0 \\ & \ddots & \\ 0 & & \pi^e u_n \end{pmatrix}, \quad u_i \in R^{\times}, e \geqq 2\,.$$

According to [5, Theorem a] X_K is a proper smooth connected rigid analytic space over K with regular strictly semi-stable formal model X over R. The special fiber X_0 of X has the following shape. Let Y_0 be the blowing up of $\mathbb{P}^n_k$ in the origin. Then X_0 is a closed chain of e copies of Y_0 where two consecutive copies are glued by identifying the $\mathbb{P}^{n-1}_k$ at infinity in one copy with the exceptional divisor of the blowing-up in the next copy. X_K is not algebraizable.

We next want to give a characterization of regular strictly semi-stable S-varieties. We start with the following local description.

Lemma 1.2. *Let X be a regular strictly semi-stable S-variety and let $x \in X_0$ be a point lying on the irreducible components $X_0^{i_1}, \dots, X_0^{i_s}$ and not on the other components of X_0. Then there exists an open affine neighborhood* Spec *A of x such that the completion of A with respect to the ideal I corresponding to the closed subscheme $X_0^{\{i_1,\dots,i_s\}}$ is of the form*

$$\widehat{A^I} \cong C\,[\![\xi_{i_1}, \dots, \xi_{i_s}]\!]/(\xi_{i_1} \cdots \xi_{i_s} - \pi)$$

for a formally smooth $\widehat{R}$-algebra C complete and separated with respect to the ideal πC, with $A/I = C/\pi C$. The ξ_σ correspond to generators of the ideal associated to the Cartier divisor X_0^σ for $\sigma \in \{i_1, \dots, i_s\}$.

P r o o f. (cf. [1, 2.16]) Let Spec A be an open neighborhood of x such that the Cartier divisor X_0^σ is principal on Spec A, generated by $\xi_\sigma \in A$ for all $\sigma \in \{i_1, \dots, i_s\}$ and such that $C_0 := A/I$ is an integral domain where $I := (\xi_{i_1}, \dots, \xi_{i_s})$. By (d) the k-algebra C_0 is smooth and can therefore be lifted to a smooth R-algebra of finite type. Let C be the completion of that R-algebra with respect to the π-adic topology. It is formally smooth over $\widehat{R}$ with $C/\pi C = C_0$ and therefore an integral domain. Due to its smoothness we can lift the identity map

$$C[\![\xi_{i_1}, \dots, \xi_{i_s}]\!]/(\xi_{i_1}, \dots, \xi_{i_s}, \pi) = C_0 = A/I$$

to a surjective morphism

$$C[\![\xi_{i_1}, \dots, \xi_{i_s}]\!] \longrightarrow \widehat{A^I}.$$

There is an equation $\xi_{i_1} \cdots \xi_{i_s} = u \cdot \pi$ on A. After replacing ξ_{i_1} by $u \cdot \xi_{i_1}$, we may assume $u = 1$ and hence we obtain a morphism

$$C\,[\![\xi_{i_1}, \dots, \xi_{i_s}]\!]/(\xi_{i_1} \cdots \xi_{i_s} - \pi) \longrightarrow\!\!\!\!\rightarrow \widehat{A^I}.$$

This epimorphism is an isomorphism since its source is an integral domain of the same dimension $s + \dim C_0$ as its target; use condition (d). $\square$

This description leads to

Proposition 1.3. *An S-variety X is regular strictly semi-stable if and only if*

(a) *the generic fiber X_K is smooth over K and*
(b) *every closed point $x \in X_0$ of the special fiber admits an open neighborhood which for some $s \in \mathbb{N}$ is smooth over the scheme*

$$\operatorname{Spec} R[\xi_{i_1}, \dots, \xi_{i_s}]/(\xi_{i_1} \cdots \xi_{i_s} - \pi)\,.$$

If X is proper over S condition (a) *is a consequence of condition* (b) .

P r o o f. The sufficiency of conditions (a) and (b) is obvious since in the given neighborhood of x the irreducible components of X_0 are exactly the $\mathrm{V}(\xi_\sigma)$ for $\sigma \in \{i_1, \dots, i_s\}$.

We now demonstrate the necessity of (a) and (b). Take a point $x \in X_0$. Say it lies on the irreducible components $X_{i_1}, \dots, X_{i_s}$ of X_0 and not on any other component. Let $\operatorname{Spec} A$ be the open neighborhood of x and ξ_σ for $\sigma \in \{i_1, \dots, i_s\}$ the generator of the principal Cartier divisor X_σ on $\operatorname{Spec} A$ from Lemma 1.2. Since the identity $\xi_{i_1} \cdots \xi_{i_s} = \pi$ holds in A we obtain a morphism

$$f : \operatorname{Spec} A \longrightarrow \operatorname{Spec} R[\xi_{i_1}, \dots, \xi_{i_s}]/(\xi_{i_1} \cdots \xi_{i_s} - \pi)\,.$$

It is flat in x because of Lemma 1.2. The fiber containing x is $X^{\{i_1,\dots,i_s\}} \longrightarrow \operatorname{Spec} k$ and thus is smooth. So f is smooth in x and therefore there exists an open neighborhood of x on which f is smooth. $\square$

2. Desingularization of products of semi-stable *S*-varieties. We now want to describe a desingularization procedure for the product of two regular strictly semi-stable S-varieties.

Proposition 2.1. *Let X and Y be two regular strictly semi-stable S-varieties. Let X_0^σ for $\sigma \in M$, respectively Y_0^τ for $\tau \in N$ be the irreducible components of their special fibers. Then successively blowing up all the closed subschemes $X_0^\sigma \times_k Y_0^\tau$ of $X \times_R Y$ leads to a desingularization $X \times_R Y =: V^0 \longleftarrow V^1 \longleftarrow \dots \longleftarrow V^r$, where V^r is a regular strictly semi-stable S-variety with $X_K \times_K Y_K = V_K^r$.*

P r o o f. We pick a closed point of $X_0 \times_k Y_0$ and investigate the mentioned blowing-ups in a neighborhood of this point. By Proposition 1.3 the point has an open neighborhood U which is smooth over

$$T := \operatorname{Spec} R[\xi_1, \dots, \xi_m, \zeta_1, \dots, \zeta_n]/(\xi_1 \cdots \xi_m - \zeta_1 \cdots \zeta_n\,, \quad \xi_1 \cdots \xi_m - \pi)$$

for some n and m. So in this neighborhood each subscheme $X_0^\sigma \times_k Y_0^\tau$ is given as $\mathrm{V}(\xi_i, \zeta_j)$ for suitable i and j. This means that the blowing-ups restricted to U can all be defined as blowing-ups of T in the irreducible components $\mathrm{V}(\xi_i, \zeta_j)$ of the special fiber. Due to the flatness of U over T, the corresponding blowing-up of $X \times_R Y$ restricted to U is obtained by base change.

We now claim that at each step every closed point of the special fiber of V^ρ for $0 \leqq \rho \leqq r$ has an open neighborhood which is smooth over an S-variety

$$T := \operatorname{Spec} R[\xi_1, \ldots, \xi_m, \ \zeta_1, \ldots, \zeta_n]/(\xi_\mu \cdots \xi_m - \zeta_\nu \cdots \zeta_n, \ \xi_1 \cdots \xi_m \zeta_1 \cdots \zeta_{\nu-1} - \pi)$$

for some $1 \leqq \mu \leqq m$ and $1 \leqq \nu \leqq n$. This is true at the beginning for $\mu = \nu = 1$.

By the above at each step the blowing-up of V^ρ is given by blowing up an irreducible component of the special fiber of T. These components are

$$\mathrm{V}(\xi_i) \ \text{ for } i < \mu\,, \quad \mathrm{V}(\zeta_j) \ \text{ for } j < \nu \quad \text{and} \quad \mathrm{V}(\xi_i, \zeta_j) \ \text{ for } i \geqq \mu \text{ and } j \geqq \nu\,.$$

The first two types are Cartier divisors, therefore the blowing-up in those is an isomorphism. So we can without loss of generality assume that $\mathrm{V}(\xi_\mu, \zeta_\nu)$ is blown up. Thereby we obtain two charts:

- $\xi_\mu = \zeta_\nu \cdot \xi'_\mu$: The relations are equivalent to

$$\begin{aligned} \xi'_\mu \cdots \xi_m \, - \, \zeta_{\nu+1} \cdots \zeta_n \qquad &= 0 \quad \text{and} \\ \xi_1 \cdots \xi'_\mu \cdots \xi_m \, \cdot \, \zeta_1 \cdots \zeta_\nu \, - \, \pi &= 0\,. \end{aligned}$$

- $\zeta_\nu = \xi_\mu \cdot \zeta'_\nu$: The relations are equivalent to

$$\begin{aligned} \xi_{\mu+1} \cdots \xi_m \, - \, \zeta'_\nu \cdots \zeta_n \qquad &= 0 \quad \text{and} \\ \xi_1 \cdots \xi_m \, \cdot \, \zeta_1 \cdots \zeta_{\nu-1} \, - \, \pi &= 0\,. \end{aligned}$$

So we see, that after the blowing-up each point of the special fiber of $V^{\rho+1}$ indeed has a neighborhood which is smooth over an S-variety T of the specified type. Thereby either μ or ν was increased by one. This proves the claim.

At each step the number of irreducible components of T_0 which are not Cartier divisors on T is lowered. The remaining ones will be blown up in the sequel. If all components are Cartier divisors, then either $\mu = m$ or $\nu = n$ and T is regular. The open subscheme of V^r above T is then regular strictly semi-stable. $\square$

We now want to describe the case of a ramified base extension. So let $R \subsetneqq \widetilde{R}$ be a finite extension of discrete valuation rings and let $\widetilde{S} = \operatorname{Spec} \widetilde{R}$. We denote by $\widetilde{K}, \widetilde{k}$ and $\widetilde{\pi}$ respectively the field of fractions, the residue field and a uniformizer of $\widetilde{R}$. Let e be the ramification index, i.e. we have $\pi = \widetilde{u}\,\widetilde{\pi}^e$ for a unit $\widetilde{u} \in \widetilde{R}$.

Proposition 2.2. *Let X be a regular strictly semi-stable S-variety with irreducible components X_0^σ for $\sigma \in M$ of its special fiber. Let $R \subsetneqq \widetilde{R}$ be a ramified ring extension as above. We assume that all the $X_0^\sigma \times_k \widetilde{k}$ are still irreducible. We perform the following blowing-ups of $X \times_R \widetilde{R}$:*

a) *We blow up all the irreducible components $X_0^\sigma \times_k \widetilde{k}$ of $X_0 \times_k \widetilde{k}$, each one maybe several times until it becomes a Cartier divisor.*
b) *In this process new irreducible components of the special fiber will appear. We also blow up all these, maybe several times, until they too become Cartier divisors.*

This leads to a desingularization $X \times_R \widetilde{R} =: V^0 \longleftarrow V^1 \longleftarrow \ldots \longleftarrow V^r$*, where* V^r *is a regular strictly semi-stable* $\widetilde{S}$*-variety with* $X_K \times_K \widetilde{K} = V^r_{\widetilde{K}}$ *.*

R e m a r k 2.2.1. The hypothesis on the irreducibility of the $X_0^\sigma \times_k \widetilde{k}$ is not necessary. If we drop it, only the description of the blowing-ups in b) becomes slightly more complicated (cf. the proof).On the other hand if R is henselian, there exists an intermediate extension $R \subseteqq R' \subseteqq \widetilde{R}$, which is étale over R and whose residue field k' is the separable closure of k in $\widetilde{k}$. Let X be a regular strictly semi-stable S-variety. Then $X' := X \times_R R'$ is still regular strictly semi-stable over Spec R'. Moreover all the irreducible components of $X'_0 := X' \times_{R'} k'$ remain irreducible in $X'_0 \times_{k'} \widetilde{k}$.

P r o o f. We pick a closed point of $X_0 \times_k \widetilde{k}$ and investigate the mentioned blowing-ups in a neighborhood of that point. By Proposition 1.3 the point has an open neighborhood U which is smooth over

$$T := \operatorname{Spec} \widetilde{R}[\xi_1, \ldots, \xi_m]/(\xi_1 \cdots \xi_m - \widetilde{\pi}^e)$$

for some m. So in this neighborhood each subscheme $X_0^\sigma \times_k \widetilde{k}$ is given as $\mathrm{V}(\xi_i, \widetilde{\pi})$ for a suitable i. This means that all the blowing-ups in a) can be defined as blowing-ups of T in the irreducible components $\mathrm{V}(\xi_i, \widetilde{\pi})$ of the special fiber. Due to the flatness of U over T, the corresponding blowing-up of $X \times_R \widetilde{R}$ restricted to U is obtained by base change.

We now claim that at each step every closed point of the special fiber of V^ρ for $0 \leqq \rho \leqq r$ has an open neighborhood which is smooth over an S-variety

$$T := \operatorname{Spec} \widetilde{R}\,[\xi_0, \ldots, \xi_m]/\left(\xi_\mu \cdots \xi_m - \prod_{i=0}^{\mu-1} \xi_i^{e_i}\ ,\ \ \xi_0 \cdots \xi_{\mu-1} - \widetilde{\pi}\right)$$

for integers $1 \leqq \mu \leqq m$ and $0 \leqq e_i \leqq e$. This is true at the beginning for $\mu = 1$, $e_0 = e$ and $\xi_0 = \widetilde{\pi}$.

By the above at each step the blowing-up of V^ρ is given by blowing up an irreducible component of the special fiber of T. These components are

$$\mathrm{V}(\xi_i) \ \text{ for } 0 \leqq i < \mu \ \text{ if } e_i = 0 \ \text{ and } \ \mathrm{V}(\xi_i, \xi_j) \quad \text{for } 0 \leqq i < \mu \leqq j \quad \text{if } e_i > 0\,.$$

Components of the first type are Cartier divisors, therefore the blowing-up in those is an isomorphism. So we can without loss of generality assume $e_0 \geqq 1$ and that $\mathrm{V}(\xi_0, \xi_\mu)$ is blown up. Thereby we obtain two charts:

- $\xi_0 = \xi_\mu \cdot \xi_0'$: The relations are equivalent to

$$\begin{aligned} \xi_{\mu+1} \cdots \xi_m - \xi_0'^{\,e_0} \cdot \prod_{i=1}^{\mu-1} \xi_i^{e_i} \cdot \xi_\mu^{e_0-1} &= 0 \quad \text{and} \\ \xi_0' \cdot \xi_1 \cdots \xi_\mu - \widetilde{\pi} &= 0\,. \end{aligned}$$

- $\xi_\mu = \xi_0 \cdot \xi_\mu'$: The relations are equivalent to

$$\begin{aligned} \xi_\mu' \cdot \xi_{\mu+1} \cdots \xi_m - \xi_0^{e_0-1} \cdot \prod_{i=1}^{\mu-1} \xi_i^{e_i} &= 0 \quad \text{and} \\ \xi_0 \cdots \xi_{\mu-1} - \widetilde{\pi} &= 0\,. \end{aligned}$$

We see, that after the blowing-up indeed each point of the special fiber of $V^{\rho+1}$ has a neighborhood which is smooth over an S-variety T of the specified type. The blown up component

$\mathrm{V}(\xi_0, \xi_\mu)$ has as strict transform in the second chart $\mathrm{V}(\xi_0)$ if $e_0 = 1$ respectively $\mathrm{V}(\xi_0, \xi'_\mu)$ if $e_0 > 1$. It is a Cartier divisor only if $e_0 = 1$. Otherwise it has to be blown up further.

The following new components appear. In case $e_0 = 1$ there is only one component, $\mathrm{V}(\xi_0, \xi_\mu)$. In case $e_0 > 1$ there are $\mathrm{V}(\xi_0, \xi_\mu, \xi_j)$ for each $\mu < j \leqq m$. They are the new irreducible components mentioned under b) and they too have to be blown up in the sequel. These blowing-ups can also be defined in T. If we drop the irreducibility assumption (cf. Remark 2.2.1), then they may not be irreducible, but they still correspond to irreducible components of T_0. This proves the claim.

The scheme T is regular if and only if each irreducible component of its special fiber is a Cartier divisor. Blowing up an irreducible component which is not a Cartier divisor lowers the number $e(m-\mu) + \sum_{i=0}^{\mu-1} e_i$. This shows that the procedure will stop after finitely many blowing-ups at a regular strictly semi-stable $\widetilde{S}$-variety V^r. $\square$

Remark 2.2.2. The procedures of Propositions 2.1 and 2.2 are compatible with passing to the henselization R^h of R, i.e. if we consider a desingularization of $V^0 \times_R R^h$ according to Propositions 2.1 or 2.2 then this desingularization can be obtained as a desingularization of V^0 with subsequent base change to R^h. The same is true for passing to the completion of R or for a base extension to a discrete valuation ring with ramification index 1 and purely inseparable residue field extension.

Remark 2.2.3. In the context of formal and rigid geometry (cf. Remark 1.1.6) the above procedure can also be applied to obtain desingularizations of a product of two regular strictly semi-stable formal schemes and of the base change of a regular strictly semi-stable formal scheme by a ramified ring extension. This is used for instance in [2] in the investigation of the rigid analytic Picard functor.

Acknowledgements. The author would like to thank the referee for his helpful comments. Further he would like to thank Prof. Lütkebohmert for several valuable discussions. The author gratefully acknowledges support received in the form of DFG grant LU 224/4-1.

References

[1] A. J. DE JONG, Smoothness, semi-stability and alterations. Publ. Math. IHES **83**, 51–93 (1996).
[2] U. HARTL and W. LÜTKEBOHMERT, On rigid-analytic Picard varieties. J. Reine Angew. Math. **528** (2000).
[3] G. KEMPF, F. KNUDSEN, D. MUMFORD and B. SAINT-DONAT, Toroidal embeddings I. LNM **339**, Berlin-Heidelberg-New York 1973.
[4] K. KÜNNEMANN, Projective regular models for abelian varieties, semistable reduction, and the height pairing. Duke Math. J. **95**, 161–212 (1998).
[5] G. A. MUSTAFIN, p-adic Hopf varieties. Functional Anal. Appl. **11**, 234–235 (1977).

Eingegangen am 22. 12. 1999

Anschrift des Autors:

Urs T. Hartl
Abteilung Reine Mathematik
Universität Ulm
D-89069 Ulm