STABILITY OF A σ_P-UNITAL CONTINUOUS FIELD ALGEBRA

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ABSTRACT. We study the question of whether stability is preserved under the operation of forming a continuous field algebra. This is not necessarily true when the base space is infinite dimensional. However, it is always true when the base space is an n-cube or an n-torus, and when the continuous field algebra is σ_P-unital. Specifically, we prove the following:

Theorem 0.1. *Let $\mathcal{A}$ be a σ_P-unital separable maximal full algebra of operator fields with base space either an n-cube $X = [0,1]^n$ or an n-torus $X = \mathbb{T}^n$ and fibre algebras $\{\mathcal{A}_x\}_{x \in X}$. If $\mathcal{A}_x$ is stable for all $x \in X$ then $\mathcal{A}$ is a stable C^*-algebra.*

We also show that, under the same hypotheses, the corona factorization property is also preserved under the formation of continuous field algebras.

Theorem 0.2. *Let $\mathcal{A}$ be a σ_P-unital separable maximal full algebra of operator fields with base space either an n-cube $X = [0,1]^n$ or an n-torus $X = \mathbb{T}^n$ and fibre algebras $\{\mathcal{A}_x\}_{x \in X}$. If $\mathcal{A}_x$ has the corona factorization property for all $x \in X$ then $\mathcal{A}$ also has the corona factorization property.*

1. INTRODUCTION

Let $\mathcal{K} = \mathcal{K}(H)$ be the C^*-algebra of compact operators on a separable infinite dimensional Hilbert space H. A C^*-algebra algebra $\mathcal{A}$ is said to be *stable* if $\mathcal{A} \otimes \mathcal{K} \cong \mathcal{A}$. Stability for C^*-algebras is interesting from the point of view of the structure of C^*-algebras. For example, a unital separable simple C^*-algebra is purely infinite if and only if every nonzero hereditary subalgebra contains a stable subalgebra ([17] Proposition 5.2). Stability is also interesting for other reasons. Among other things, the K_0 group of a C^*-algebra $\mathcal{C}$ is built up from the (equivalence classes of) projections in the stabilization $\mathcal{C} \otimes \mathcal{K}$ (moving up matrices allows us to define addition on (not necessarily orthogonal) projections - thus giving us a semigroup structure on $K_0(\mathcal{C})$ etc.).

It is interesting to study the permanence properties of stability - i.e., preservation of stability under various operations. The study of this question has lead to insights into the structure theory and K-theory (especially extension theory) of C^*-algebras.

Firstly, Hjelmborg and Rørdam have shown that stability is preserved under a large number of operations[15]. The C^*-inductive limit of a sequence of separable stable C^*-algebras is stable ([15] Corollary 4.1). If $\mathcal{B} \subseteq \mathcal{A}$ is an inclusion of separable C^*-algebras such that $\mathcal{B}$ contains an approximate unit for $\mathcal{A}$, then if $\mathcal{B}$ is stable then $\mathcal{A}$ is stable ([15] Proposition 4.4). Crossed products of separable stable C^*-algebras by discrete groups are stable ([15] Corollary 4.5). If $\mathcal{A}$ is a separable stable C^*-algebra, then every ideal of $\mathcal{A}$ and every quotient of $\mathcal{A}$ is stable ([17] Corollary 2.3 (ii)).

On the other hand, stability is not closed under other quite natural operations. For example, the extension of a separable stable C^*-algebra by a separable stable C^*-algebra need not give a stable extension algebra. Specifically, in [18], Rørdam constructed an exact sequence of the form

$$0 \to C(X) \otimes \mathcal{K} \to \mathcal{E} \to \mathcal{K} \to 0$$

such that the extension algebra $\mathcal{E}$ is not a stable C^*-algebra. We note that in this example, $X = \prod_{i=1}^{\infty} S^2$ is an infinite Cartesian product of spheres (an infinite dimensional space; actually, one can achieve a counterexample with an infinite Cartesian product of circles (the infinite torus) but spheres are technically easier to deal with). We also note that this example is also interesting from the point of view of extension theory since it gives an example of an extension of $C(X) \otimes \mathcal{K}$ by $\mathcal{K}$ that fails the generalized Pimsner-Popa-Voiculescu Theorem; i.e., it gives an example of a homogeneous extension of $C(X) \otimes \mathcal{K}$ by $\mathcal{K}$ which is not an absorbing extension (see [14] and [6]).

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In still another direction, Rørdam gave an example showing that stability is not a stable property for C^*-algebras [16]. More precisely, we note that if $\mathcal{A}$ is a stable C^*-algebra then for every integer $n \geq 1$, $\mathbb{M}_n(\mathcal{A})$ is also stable. Rørdam showed that the opposite need not hold. In particular, in [16], Rørdam gave an example of a simple σ_P-unital AH-algebra $\mathcal{A}$ with cancellation of projections such that $\mathcal{A}$ is not stable but $\mathbb{M}_2(\mathcal{A})$ is stable. Hence, stability is not a stable property; and the construction of Rørdam actually gives examples of new C^*-algebras with new and rather difficult (though interesting) phenomenon.

Another question in the same spirit which has not been studied is whether stability is preserved under the formation of continuous field algebras. In other words, if all the fibres in a continuous field algebra are stable, is the continuous field algebra itself necessarily stable? The answer to this question is necessarily no in general (in particular, no for certain infinite dimensional base spaces). In [6] section 3, there is an example of a maximal full algebra $\mathcal{A}$ of operator fields with base space X being the countably infinite Cartesian product of 2-spheres and fibres being the compact operators such that $\mathcal{A}$ is not stable. (In slightly more detail, $\mathcal{A}$ has the form $\overline{P(C(X) \otimes \mathcal{K})P}$ where P is a ("infinite rank") projection in the multiplier algebra $\mathcal{M}(C(X) \otimes \mathcal{K})$ such that P never contains a trivial vector bundle over X. Also, once more, one can use the infinite torus (infinite Cartesian product of circles) instead of an infinite Cartesian product of spheres.) The construction in [6] depends on the fact that X is an infinite dimensional topological space.

Hence, the natural question to ask is the following:

Question 1.1. *Let $\mathcal{A}$ be a separable maximal full algebra of operator fields with finite dimensional base space and stable fibre algebras. Then is $\mathcal{A}$ a stable C^*-algebra?*

We answer the above question for the case where the base space is either an n-cube or an n-torus and where $\mathcal{A}$ is assumed to be σ_P-unital.

Theorem 1.2. *Let $\mathcal{A}$ be a separable σ_P-unital maximal full algebra of operator fields with base space either an n-cube or an n-torus. Then if every fibre algebra of $\mathcal{A}$ is stable then $\mathcal{A}$ is stable.*

We next study the related question of whether the corona factorization property is preserved under the formation of continuous field algebras.

Definition 1.1. *Let $\mathcal{A}$ be a separable stable C^*-algebra. Then $\mathcal{A}$ is said to have the* corona factorization property *if every norm full projection in $\mathcal{M}(\mathcal{A})$ is Murray-von Neumann equivalent to $1_{\mathcal{M}(\mathcal{A})}$.*

By a *norm-full* element of $\mathcal{M}(\mathcal{A})$, we mean an element that is not contained in any proper ideal of $\mathcal{M}(\mathcal{A})$ (in the literature, one often uses "full" instead of "norm-full").

Many "nice" C^*-algebras have the corona factorization property. For example, if $\mathcal{A}$ is a separable stable simple C^*-algebra with real rank zero, stable rank one and weak unperforation, then $\mathcal{A}$ has the corona factorization property. If $\mathcal{A}$ is a simple separable stable purely infinite C^*-algebra then $\mathcal{A}$ has the corona factorization. If $\mathcal{A}$ is a separable stable type I C^*-algebra with finite decomposition rank (in particular, if $\mathcal{A}$ has the form $C(X) \otimes \mathcal{K}$ for a finite dimensional compact metric space X) then $\mathcal{A}$ has the corona factorization property (see [6] and [12]).

The corona factorization property first arose in studies of extension theory (specifically, the theory of absorbing extensions). Among other things, it says that $\mathcal{A}$ has lots of absorbing extensions and leads to interesting uniqueness theorems as well as a clean characterization of KK-theory (see [12] and [6]). These aspects have recently been useful in the classification of purely infinite C^*-algebras with a unique (nontrivial) ideal, as well as the classification of certain C^*-algebras coming from dynamical systems (see [7] and [12] and the references therein).

The corona factorization property also gives interesting information about the structure and stability of C^*-algebras. Among other things, it rules out the two types of pathological counterexamples constructed by Rørdam, which we have already mentioned above. Firstly, we have the following (see [8] and [12]):

Theorem 1.3. *Let $\mathcal{B}$ be a separable stable C^*-algebra with the corona factorization property. Then every extension of $\mathcal{B}$, by a separable stable C^*-algebra, has a stable extension algebra.*

Hence, if the ideal in an exact sequence (extension) has the corona factorization property, then the type of pathological example constructed by Rørdam in [18] (and mentioned above already) cannot possibly occur. Moreover, under appropriate hypotheses, we actually have a converse (see [8] and [12]):

Theorem 1.4. *Let $\mathcal{B}$ be a separable simple stable C^*-algebra with real rank zero and cancellation. Then the following are equivalent:*

(a) *$\mathcal{B}$ has the corona factorization property.*

(b) *Every extension of $\mathcal{B}$, by a separable stable C^*-algebra, gives a stable extension algebra.*

Next, the corona factorization property also rules out the second type of counterexample mentioned above (and constructed by Rørdam in [16]).

Theorem 1.5. *Let $\mathcal{B}$ be a separable stable C^*-algebra. Then the following are equivalent:*

(1) *$\mathcal{B}$ has the corona factorization property.*
(2) *Let $\mathcal{D}$ be a full hereditary subalgebra of $\mathcal{B}$. Suppose that there exists an integer $n \geq 1$ such that $\mathbb{M}_n(\mathcal{D})$ is stable. Then $\mathcal{D}$ must be stable.*

Hence, the corona factorization property is exactly equivalent to stability being a stable property for full hereditary subalgebras.

In light of these results, it is interesting to ask whether or not the corona factorization property is preserved by the formation of continuous fields. Once more, there is a counterexample in the case where the base space is infinite dimensional. In particular, the same counterexample (as that for stability) from the last section of [6] (the one with base space space being the infinite Cartesion product of spheres and fibre algebras being the compact operators) is also a counterexample for the preservation of corona factorization under continuous field algebras. More precisely, if X is the infinite Cartesian product of spheres then $C(X) \otimes \mathcal{K}$ is a continuous field algebra with each fibre being isomorphic to the algebra $\mathcal{K}$ of compact operators, but $C(X) \otimes \mathcal{K}$ does not have the corona factorization property. (We note that the algebra $\mathcal{K}$ of compact operators has the corona factorization property (see [7] and [12]). Also, once more, we can replace X by an infinite Cartesian product of circles.)

However, we have a result when we assume that the base space (of the continuous field algebra) is a certain type of finite dimensional topological space. Using our theorem for the case of stability, we have a result under the same hypotheses.

Theorem 1.6. *Let $\mathcal{A}$ be a separable σ_P-unital maximal full algebra of operator fields with base space being either an n-cube or an n-torus. Then if every fibre algebra of $\mathcal{A}$ is a stable C^*-algebra with the corona factorization property, then $\mathcal{A}$ is a stable C^*-algebra with the corona factorization property.*

We end this introduction with an example: Let $\mathbb{H}_3^{\mathbb{Z}}$ be the discrete Heisenberg group and let $C^*(\mathbb{H}_3^{\mathbb{Z}})$ be the universal C^*-algebra of $\mathbb{H}_3^{\mathbb{Z}}$. $C^*(\mathbb{H}_3^{\mathbb{Z}})$ is a unital maximal full algebra of operator fields with base space being the circle and fibre algebras being the rotation algebras (above a rational point, there is a rational rotation algebra; above an irrational point, there is an irrational rotation algebra; see, for example, [10]). Then the stabilization $C^*(\mathbb{H}_3^{\mathbb{Z}}) \otimes \mathcal{K}$ is a maximal full algebra of operator fields with base space the being the circle and fibre algebras being the stabilizations of the rotation algebras. By [7] and [6], the stabilization of a rotation algebra has the corona factorization property. Hence, by Theorem 1.6, $C^*(\mathbb{H}_3^{\mathbb{Z}}) \otimes \mathcal{K}$ has the corona factorization property.

2. Stability

For the convenience of the reader, we first recall the definition and some basic facts about maximal full algebras of operator fields. Let X be a locally compact Hausdorff space, and for each $x \in X$, let a C^*-algebra $\mathcal{A}_x$ be given. A function f on X such that $f(x) \in \mathcal{A}_x$ for all $x \in X$ is called an *operator field.*

Definition 2.1. *A* full algebra of operator fields *on X (with values in $\{\mathcal{A}_x\}_{x \in X}$) is a $*$-algebra $\mathcal{A}$ of operator fields on X such that*

(a) *the function $x \mapsto \|f(x)\|$ is continuous on X and vanishes at infinity for all $f \in \mathcal{A}$;*
(b) *for each $x \in X$, the set $\{f(x) : f \in \mathcal{A}\}$ is dense in $\mathcal{A}_x$; and*
(c) *$\mathcal{A}$ is complete in the norm $\|f\| =_{df} sup\{\|f(x)\| : x \in X\}$.*

A full algebra of operator fields is said to be maximal *if it is not properly contained in any full algebra of operator fields with the same base space and fibre algebras.*

We note that it follows, from the above definition, that a full algebra of operator fields (and hence, a maximal full algebra of operator fields) is always a C^*-algebra.

The following proposition which characterizes maximal full algebras of operator fields is from [5] Lemma 1.7 and the corollary of Theorem 1.4 (see also [9] Proposition 1):

Proposition 2.1. *Let $\mathcal{A}$ be a full algebra of operator fields with base space X and fibre algebras $\{\mathcal{A}_x\}_{x \in X}$. Then the following statements are equivalent:*

(a) *$\mathcal{A}$ is maximal.*

(b) *for every* $x, y \in X$ *such that* $x \neq y$ *and for every* $a_1 \in \mathcal{A}_x$ *and every* $a_2 \in \mathcal{A}_y$*, there exists* $f \in \mathcal{A}$ *such that* $f(x) = a_1$ *and* $f(y) = a_2$*;*
(c) *for any complex-valued bounded continuous function* h *on* X *and for every* $f \in \mathcal{A}$*, the operator field* $x \mapsto h(x)f(x)$ *is an element of* $\mathcal{A}$*.*
(d) *if* g *is an operator field vanishing at infinity such that the function* $x \mapsto \|f(x) - g(x)\|$ *is continuous for all* $f \in \mathcal{A}$*, then* $g \in \mathcal{A}$*.*

We will need the following perturbation lemma for partial isometries which follows from standard spectral theory arguments (see, for example, the proof of [20] Lemma 5.1.6):

Lemma 2.2. *For every* $\epsilon > 0$*, there exists* $\delta > 0$ *such that the following holds: Let* $\mathcal{B}$ *be a* C^**-algebra and let* $a \in \mathcal{B}$*. Suppose that* $\|a\| \leq 1$ *and* a *is within* δ *of* aa^*a*. Then there is a partial isometry* $v \in \mathcal{B}$ *such that*

(a) *the initial and range projections of* v *are contained in the initial and range projections of* a *respectively;*
(b) v *is within* ϵ *of* a*; and*
(c) *if* $\pi : \mathcal{B} \to \mathcal{C}$ *is a* $*$*-homomorphism such that* $\pi(a)$ *is a partial isometry then* $\pi(v) = \pi(a)$*.*

We will also need the following perturbation lemma for unitaries which also follows from standard spectral theory arguments (see, for example, the proof of [20] Proposition 4.2.4):

Lemma 2.3. *Let* $\mathcal{C}$ *be a unital* C^**-algebra and* u *a unitary in* $\mathcal{C}$ *such that* $\|u - 1_{\mathcal{C}}\| < \delta < 1/4$*. Then there exists a norm-continuous path* $\{u_t\}_{t \in [0,1]}$ *of unitaries in* $\mathcal{C}$ *such that* $\|u_t - 1_{\mathcal{C}}\| < \delta$ *for all* $t \in [0, 1]$*,* $u_0 = u$ *and* $u_1 = 1_{\mathcal{C}}$*.*

Finally, we need the following perturbation lemma for projections (with precise estimates) which can be found in, say, [11] Lemma 2.5.1.

Lemma 2.4. *Let* $\mathcal{C}$ *be a unital* C^**-algebra and let* p, q *be projections in* $\mathcal{C}$ *such that* $\|p - q\| < 1$*. Then there is a unitary* $u \in \mathcal{C}$ *such that* $p = uqu^*$ *and* $\|u - 1_{\mathcal{C}}\| \leq \sqrt{2}\|p - q\|$*.*

Recall that a C^*-algebra $\mathcal{A}$ is said to be *σ_P-unital* if it has a countable approximate unit consisting of projections. The following characterization of stability for σ_P-unital C^*-algebras is due to Hjelmborg and Rørdam (see [15] Theorem 3.3):

Theorem 2.5. *Let* $\mathcal{A}$ *be a* σ_P*-unital* C^**-algebra. Then* $\mathcal{A}$ *is stable if and only if for each projection* $p \in \mathcal{A}$ *there is a projection* $q \in \mathcal{A}$ *such that* p *is orthogonal to* q *and* p *is Murray-von Neumann equivalent to* q*.*

We first prove our result about stability in the case where the base space (of the maximal full algebra of operator fields) is the unit interval:

Theorem 2.6. *Let* $\mathcal{A}$ *be a separable* σ_P*-unital maximal full algebra of operator fields over the base space* $[0, 1]$ *(the unit interval) and fibre algebras* $\{\mathcal{A}_x\}_{x \in [0,1]}$*. If* $\mathcal{A}_x$ *is stable for all* $x \in [0, 1]$ *then* $\mathcal{A}$ *is stable.*

Proof. Towards applying Theorem 2.5, let p be a projection in $\mathcal{A}$. We will construct a projection in $\mathcal{A}$ that is both orthogonal and Murray-von Neumann equivalent to p.

For each $x \in X$, $\mathcal{A}_x$ is a stable and σ_P-unital C^*-algebra. Hence, for each x, let v_x be a partial isometry in $\mathcal{A}_x$ with initial projection $p(x)$ and range projection orthogonal to $p(x)$. Now take $\epsilon > 0$ to be a positive real number strictly less than $1/100$. Apply Lemma 2.2 to ϵ to get a positive real number $\delta > 0$. We may also assume that $\delta < 1/100$.

Now by Proposition 2.1, for each $x \in X$, let $y \mapsto v_x(y)$ be an operator field in $\mathcal{A}$ such that $v_x(x) = v_x$. We may assume that $\|v_x(y)\| \leq 1$ for all $x \in X$. Since for each $x \in X$,

i. $y \mapsto \|v_x(y)^* v_x(y) - p(y)\|$,
ii. $y \mapsto \|p(y)v_x(y)\|$, and
iii. $y \mapsto \|v_x(y)v_x(y)^* v_x(y) - v_x(y)\|$

are all continuous real valued functions on X, we can find an open neighbourhood O_x (say an open interval in $[0, 1]$) of x such that for all $y \in \overline{O_x}$ (the closure of O_x),

i'. $\|v_x(y)^* v_x(y) - p(y)\| < \delta$,
ii'. $\|p(y)v_x(y)\| < \delta$, and
iii'. $\|v_x(y)v_x(y)^* v_x(y) - v_x(y)\| < \delta$.

Let us collectively denote the above conditions by "(*)".

Now $\{O_x : x \in X\}$ is an open cover of the unit interval $[0,1]$. Hence, since $[0,1]$ is compact, let $\{x_1, x_2, ..., x_n\}$ be a finite subset of $[0,1]$ such that $\{O_{x_i} : 1 \leq i \leq n\}$ covers $[0,1]$. For simplicity and contracting intervals if necessary, we assume that $n = 2$, $O_{x_1} = [0, b')$, and $O_{x_2} = (a, 1]$ where $0 < a < b' < 1$. (The proof for multiple intervals is an iteration of the argument for two intervals.)

For $i = 1, 2$, consider the restricted continuous field algebras $\mathcal{A}|_{\overline{O_{x_i}}}$. ($\mathcal{A}|_{\overline{O_{x_i}}}$ is a maximal full algebra of operator fields with base space $\overline{O_{x_i}}$. It consists of the restrictions, to $\overline{O_{x_i}}$, of operator fields from $\mathcal{A}$.) The restricted operator fields $v_{x_i}|_{O_{x_i}}$ and $p|_{O_{x_i}}$ satisfy all the properties of v_{x_i} and p in (*). Hence, by Lemma 2.2, if w_i is the partial isometry in the polar decomposition of $v_{x_i}|_{\overline{O_{x_i}}}$ then $w_i \in \mathcal{A}|_{\overline{O_{x_i}}}$ and w_i is within ϵ of $v_{x_i}|_{\overline{O_{x_i}}}$. Hence, $(w_i)^* w_i$ is within 2ϵ of $(v_{x_i}|_{\overline{O_{x_i}}})^* v_{x_i}|_{\overline{O_{x_i}}}$. Hence, by (*) i'., we must have that

$$(**) \qquad \|(w_i)^* w_i - p|_{\overline{O_{x_i}}}\| < 2\epsilon + \delta < 3/50.$$

Thus, $(w_i)^* w_i$ is Murray-von Neumann equivalent to $p|_{\overline{O_{x_i}}}$ in $\mathcal{A}|_{\overline{O_{x_i}}}$. But from (*) ii'., $\|p|_{\overline{O_{x_i}}} w_i\| < \epsilon + \delta < 1/50$. So $\|p|_{\overline{O_{x_i}}} w_i (w_i)^*\| < 1/50$. Hence,

$$(***) \qquad \|(1 - p|_{\overline{O_{x_i}}}) w_i (w_i)^* (1 - p|_{\overline{O_{x_i}}}) - w_i (w_i)^*\| < 3/50.$$

Also, a computation shows that

$$(****) \qquad \|(1 - p|_{\overline{O_{x_i}}}) w_i (w_i)^* (1 - p|_{\overline{O_{x_i}}}) - ((1 - p|_{\overline{O_{x_i}}}) w_i (w_i)^* (1 - p|_{\overline{O_{x_i}}}))^2\| < 2/25.$$

From (***), (****) and [20] Lemma 5.1.6, there is a projection $p_i \in (1 - p|_{\overline{O_{x_i}}}) \mathcal{A}|_{\overline{O_{x_i}}} (1 - p|_{\overline{O_{x_i}}})$ such that p_i is within $11/50$ of $w_i (w_i)^*$. From this, (**) and [20] Proposition 5.2.6, p_i is Murray-von Neumann equivalent to $p|_{\overline{O_{x_i}}}$ in $\mathcal{A}|_{\overline{O_{x_i}}}$. Hence, p_i is a projection in $\mathcal{A}|_{\overline{O_{x_i}}}$ which is orthogonal to $p|_{\overline{O_{x_i}}}$ but also Murray-von Neumann equivalent to $p|_{\overline{O_{x_i}}}$ for $i = 1, 2$.

Lift p_i to a norm one element in $\mathcal{A}$. For simplicity, we also denote this lifting by "p_i". Note though that p_i need not be a projection in $\mathcal{A}$ (since p_i need not be a projection outside of the $\overline{O_{x_i}}$). But this will not affect us much. Also, let v_i be an element in $\mathcal{A}$ with norm less than or equal to one such that $v_i|_{\overline{O_i}}$ is a partial isometry in $\mathcal{A}|_{\overline{O_i}}$ with inital projection $p|_{\overline{O_i}}$ and range projection p_i. We now construct by recursion a sequence $\{v_{1,k}\}_{k=1}^{\infty}$ of partially defined operator fields (i.e., only defined on a subset of $[0,1]$) and a sequence $\{y_k\}_{k=1}^{\infty}$ of strictly increasing points in the nonempty open interval (a, b') such that

(a) $\{y_k\}_{k=1}^{\infty}$ converges to a point in (a, b');
(b) for each k, $v_{1,k}$ is defined as an operator field and a partial isometry on $[0, y_k]$; moreover, for each $x \in [0, y_k]$, $v_{1,k}$ has initial projection $p(x)$ and range projection orthogonal to $p(x)$;
(c) $v_{1,1} = v_1|_{[0,y_1]}$ and for $k' \leq k$, the restriction of $v_{1,k}$ to $[0, y_{k'}]$ is the same as $v_{1,k'}$; and
(d) for each $k \geq 3$ and for all $y \in [y_{k-1}, y_k]$, $v_{1,k}(y)$ is within $1/2^k$ of $v_2(y)$.

We collectively denote the above conditions by "(+)".

Let b_0 be a point in $[0,1]$ such that $a < b_0 < b'$. We will do our construction on $(a, b_0]$. The construction will be by recursion on k.

Basis steps $k = 1$, $k = 2$ and $k = 3$. Firstly, take y_1 to be any point in (a, b_0) and take $v_{1,1} = v_1|_{[0,y_1]}$.

We now construct $v_{1,2}$ and y_2. Let $p_{1,1}(y_1)$ be the projection in $\mathcal{A}_{y_1}$ that is given by $p_{1,1}(y_1) =_{df} v_{1,1}(y_1)^* v_{1,1}(y_1)$. By [17] Corollary 2.3 (iii), since $\mathcal{A}_{y_1}$ is stable, so is $(1 - p(y_1)) \mathcal{A}_{y_1} (1 - p(y_1))$. Hence, there must be a path $\{u_t\}_{t \in [0,1]}$ of unitaries in $(1 - p(y_1)) \mathcal{A}_{y_1} (1 - p(y_1))$ such that $u_0 = 1 - p(y_1)$ and $u_1 p_{1,1}(y_1) (u_1)^* = p_2(y_1)$. Now consider the norm-continuous path $\{u_t v_{1,1}(y_1)\}_{t \in [0,1]}$ of partial isometries in $\mathcal{A}_{y_1}$. Note that $u_0 v_{1,1}(y_1) = v_{1,1}(y_1)$ and $u_1 v_{1,1}(y_1)$ has initial projection $p(y_1)$ and range projection $p_2(y_1)$ which is also the case for the partial isometry $v_2(y_1)$. Hence, $u_1 v_{1,1}(y_1) v_2(y_1)^*$ is a partial isometry with both initial and range projection being $p_2(y_1)$. In other words, $u_1 v_{1,1}(y_1) v_2(y_1)^*$ is a unitary in $p_2(y_1) \mathcal{A}_{y_1} p_2(y_1)$. Hence, by the same argument as that of [20] Theorem 4.2.9, $w =_{df} diag(u_1 v_{1,1}(y_1) v_2(y_1)^*, v_2(y_1) v_{1,1}(y_1)^* (u_1)^*)$, is homotopic to $p_2(y_1) \oplus p_2(y_1)$ in $\mathbb{M}_2(p_2(y_1) \mathcal{A}_{y_1} p_2(y_1))$. Hence, let $\{w_t\}_{t \in [0,1]}$ be a path of unitaries in $\mathbb{M}_2(p_2(y_1) \mathcal{A}_{y_1} p_2(y_1))$ such that $w_0 = w$ and $w_1 = p_2(y_1) \oplus p_2(y_1)$. Hence, let $\{a_t\}_{[0,1]}$ be a norm continuous path of partial isometries in $\mathcal{A}_{y_1}$ given by

i. $a_t = u_{2t} v_{1,1}(y_1)$ for $t \in [0, 1/2]$, and
ii. $a_t = w_{2t-1} diag(v_2(y_1), 0)$ for $t \in [1/2, 1]$.

Hence, $\{a_t\}_{t \in [0,1]}$ is a norm continuous path of partial isometries in $\mathcal{A}_{y_1}$ such that

(a) $a_0 = v_{1,1}(y_1)$,

(b) $a_1 = v_2(y_1)$, and
(c) a_t has initial projection $p(y_1)$ and range projection orthogonal to $p(y_1)$ for all $t \in [0,1]$.

Next, construct a continuous field algebra $\mathcal{A}' = C^*([0,1], \{C[0,1] \otimes \mathcal{A}_x\}_{x \in [0,1]}, \mathcal{F})$ with base space $[0,1]$ and fibre algebras $\{C[0,1] \otimes \mathcal{A}_x\}_{x \in [0,1]}$. The continuity structure $\mathcal{F}$ is given by $\mathcal{F} =_{df} \{h \otimes f : h \in C[0,1] \text{ and } f \in \mathcal{A}\}$. It is not hard to check that $\mathcal{F}$ satisfies the axioms of a continuity structure (see [5] 1.1) and we take $\mathcal{A}'$ to be the corresponding maximal full algebra of operator fields.

By the definition of maximal full algebra of operator fields, there must be an element $g \in \mathcal{A}'$ such that $\|g\| \leq 1$ and $g(y_1) = \{a_t\}_{t \in [0,1]}$. Replacing $g(.)$ by $(1-p)g(.)p$ if necessary, we may assume that $(1-p)g = g$ and $gp = g$. (Here, 1 is the unit adjoined to $\mathcal{A}$. Also, we are identifying each element $x \mapsto f(x)$ in $\mathcal{A}$ with the element $x \mapsto 1_{C[0,1]} \otimes f(x)$ in $\mathcal{A}'$.)

In Lemma 2.2, put $\epsilon = 1/100$ to get a positive real number δ_1 (δ_1 is the δ in Lemma 2.2). We may assume that $0 < \delta_1 < 1/100$. Now the functions $x \mapsto \|g(x)g(x)^*g(x) - g(x)\|$, $x \mapsto \|g(x)^*g(x) - p(x)\|$ and $x \mapsto \|p(x)g(x)\|$ are all continuous real-valued functions on $[0,1]$. (Here again, we are identifying each element $x \mapsto f(x)$ in $\mathcal{A}$ with the element $x \mapsto 1_{C[0,1]} \otimes f(x)$ in $\mathcal{A}'$. Also, the operator field $x \mapsto g(x)(1)$ is an element of $\mathcal{A}$. Hence, the function $x \mapsto \|g(x)(1) - v_2(x)\|$ is also a continuous function on $[0,1]$.

Hence, let y_2 be a point in $[0,1]$ with $y_1 < y_2 < b_0$ such that

(1) $\|g(x)g(x)^*g(x) - g(x)\| < \delta_1$ for all $x \in [y_1, y_2]$,
(2) $\|g(x)^*g(x) - p(x)\| < 1/100$ for all $x \in [y_1, y_2]$,
(3) $\|g(x)(1) - v_2(x)\| < 1/100$ for all $x \in [y_1, y_2]$.

Now restricting all the operator fields to $[y_1, y_2]$, the above inequalities are also true for $g|_{[y_1,y_2]}$ and $v_2|_{[y_1,y_2]}$ in $\mathcal{A}'|_{[y_1,y_2]}$ and $\mathcal{A}|_{[y_1,y_2]}$. Hence, apply Lemma 2.2 to $g|_{[y_1,y_2]}$ to get a partial isometry b in $\mathcal{A}'|_{[y_1,y_2]}$. Moreover, we have the following:

(a) b is within $1/100$ of $g|_{[y_1,y_2]}$. Hence, $b(x)(1)$ is within $1/50$ of $v_2(x)$ for all $x \in [y_1, y_2]$.
(b) $b(y_1) = \{a_t\}_{t \in [0,1]}$.
(c) $\|b(x)^*b(x) - p(x)\| < 3/100$ for all $x \in [y_1, y_2]$.

Since the initial projection of $b(x)$ is contained in $p(x)$ for all $x \in [y_1, y_2]$, (c) implies that the initial projection of $b(x)$ is actually $p(x)$ for all $x \in [y_1, y_2]$. Also, since we assumed that $(1-p)g = g$, the range projection of $b(x)$ is orthogonal to $p(x)$ for all $x \in [y_1, y_2]$. Hence, we can take $v_{1,2} \in \mathcal{A}|_{[0,y_2]}$ to be defined as follows:

(1) $v_{1,2}(x) = v_{1,1}(x)$ for $x \in [0, y_1]$ and
(2) $v_{1,2}(x) = b(x)((x - y_1)/(y_2 - y_1))$ for $x \in [y_1, y_2]$.

$v_{1,2}$ and y_2 satisfy the conditions in (+) for $k = 2$.

Now we do the basis step $k = 3$; i.e., we construct $v_{1,3}$ and y_3 (and this will complete the basis step). Parts of this step are similar to the basis step $k = 2$ but we now additionally require that $\|v_{1,3}(x) - v_2(x)\| < 1/8$ for all $x \in [y_2, y_3]$ (condition (d) of (+))). Since $v_{1,2}(y_2) = b(y_2)(1)$, $\|v_{1,2}(y_2) - v_2(y_2)\| < 1/50$. Hence, $\|v_{1,2}(y_2)(v_{1,2}(y_2))^* - p_2(y_2)\| < 1/25$. (Recall that $p_2(y_2) = v_2(y_2)(v_2(y_2))^*$.) Hence, by Lemma 2.4, let u' be a unitary in $\mathbb{C}(1 - p(y_2)) + (1 - p(y_2))\mathcal{A}_{y_2}(1 - p(y_2))$ such that $\|u' - (1 - p(y_2))\| < \sqrt{2}/25 < 2/25$ and $u'v_{1,2}(y_2)(v_{1,2}(y_2))^*(u')^* = p_2(y_2)$. By Lemma 2.3, let $\{u'_t\}_{t \in [0,1]}$ be a norm continuous path of unitaries in $\mathbb{C}(1 - p(y_2)) + (1 - p(y_2))\mathcal{A}_{y_2}(1 - p(y_2))$ such that $u'_0 = 1 - p(y_2)$ and $u'_1 = u'$ and $\|u'_t - (1 - p(y_2))\| < 2/25$ for $t \in [0,1]$. Note that for all $t \in [0,1]$, $\|u'_t v_{1,2}(y_2) - v_2(y_2)\| \leq \|u'_t v_{1,2}(y_2) - v_{1,2}(y_2)\| + \|v_{1,2}(y_2) - v_2(y_2)\| < 2/25 + 1/50 = 5/50 = 1/10$ for all $t \in [0,1]$.

Note that both $u'v_{1,2}(y_2)$ and $v_2(y_2)$ are partial isometries in $\mathcal{A}_{y_2}$ with initial projection $p(y_2)$ and range projection $p_2(y_2)$. Moreover, $\|u'v_{1,2}(y_2) - v_2(y_2)\| < 1/10$. Hence, $w' =_{df} u'v_{1,2}(y_2)(v_2(y_2))^*$ is a unitary in $p_2(y_2)\mathcal{A}_{y_2}p_2(y_2)$ such that $\|w' - p_2(y_2)\| < 1/10$. Hence, by Lemma 2.3, let $\{w'_t\}_{t \in [0,1]}$ be a norm-continuous path of unitaries in $p_2(y_2)\mathcal{A}_{y_2}p_2(y_2)$ such that $w'_0 = w'$, $w'_1 = p_2(y_2)$ and $\|w'_t - p_2(y_2)\| < 1/10$ for all $t \in [0,1]$. (In particular, note that $\|w'_t v_2(y_2) - v_2(y_2)\| < 1/10$ for all $t \in [0,1]$.) Hence, let $\{c_t\}_{t \in [0,1]}$ be the norm-continuous path of partial isometries in $\mathcal{A}_{y_2}$ given by

i. $c_t = u'_{2t} v_{1,2}(y_2)$ for $t \in [0, 1/2]$, and
ii. $c_t =_{df} w'_{2t-1} v_2(y_2)$ for $t \in [1/2, 1]$.

Hence, $\{c_t\}_{t \in [0,1]}$ is a norm-continuous path of partial isometries in $\mathcal{A}_{y_2}$ such that

(a) $c_0 = v_{1,2}(y_2)$,
(b) $c_1 = v_2(y_2)$,

(c) c_t is a partial isometry with initial projection $p(y_2)$ and range projection orthogonal to $p(y_2)$ for all $t \in [0,1]$, and
(d) $\|c_t - v_2(y_2)\| < 1/10$ for all $t \in [0,1]$.

As in the basis step $k=2$, we move up to the maximal full algebra of operator fields $\mathcal{A}' = C^*([0,1], \{C[0,1] \otimes \mathcal{A}_x\}_x \in [0,1], \mathcal{F})$ (same $\mathcal{A}'$ as before). Since $\mathcal{A}'$ is a maximal full algebra of operator fields, let $h \in \mathcal{A}'$ be such that $\|h\| \leq 1$ and $h(y_2) = \{c_t\}_{t \in [0,1]}$. Replacing h by $(1-p)hp$ if necessary, we may assume that $(1-p)h = h$ and $hp = h$.

Put $\epsilon = 1/(100^2) = 1/10000$ into Lemma 2.2 to get a positive real number δ_2. (δ_2 is the δ in Lemma 2.2.) We may assume that $0 < \delta_2 < 1/10000$. Now the functions $x \mapsto \|h(x)h(x)^*h(x) - h(x)\|$, $x \mapsto \|h(x)^*h(x) - p(x)\|$, $x \mapsto \|p(x)h(x)\|$, $x \mapsto \|h(x)(1) - v_2(x)\|$ and $x \mapsto \|h(x) - v_2(x)\|$ are all continuous functions on $[0,1]$. (Here, once more, whenever necessary, we identify an operator field $x \mapsto f(x)$ in $\mathcal{A}$ with the operator field $x \mapsto 1_{C[0,1]} \otimes f(x)$ in $\mathcal{A}'$.)

Hence, let y_3 be a point in $[0,1]$ with $y_2 < y_3 < b_0$ such that

(1) $\|h(x)h(x)^*h(x) - h(x)\| < \delta_2$ for all $x \in [y_2, y_3]$,
(2) $\|h(x)^*h(x) - p(x)\| < 1/10000$ for all $x \in [y_2, y_3]$,
(3) $\|h(x) - v_2(x)\| < 1/10$ for all $x \in [y_2, y_3]$, and
(4) $\|h(x)(1) - v_2(x)\| < 1/10000$ for all $x \in [y_2, y_3]$.

Now restricting all the operator fields to $[y_2, y_3]$, the above inequalities are also true for $h|_{[y_2,y_3]}$ and $v_2|_{[y_2,y_3]}$ in $\mathcal{A}'|_{[y_2,y_3]}$ and $\mathcal{A}|_{[y_2,y_3]}$. Hence, apply Lemma 2.2 to $h|_{[y_2,y_3]}$ to get a partial isometry d in $\mathcal{A}'|_{[y_2,y_3]}$. Moreover, we have the following:

(a) d is within $1/10000$ of $h|_{[y_2,y_3]}$. Hence, $d(x)(1)$ is within $1/5000$ of $v_2(x)$ for all $x \in [y_2, y_3]$.
(b) $d(y_2) = \{c_t\}_{t \in [0,1]}$.
(c) $d(x)$ is within $1001/10000 < 1/8$ of $v_2(x)$ for all $x \in [y_2, y_3]$.
(d) $\|d(x)^*d(x) - p(x)\| < 3/10000$ for all $x \in [y_2, y_3]$.

Since the initial projection of $d(x)$ is contained in $p(x)$ for $x \in [y_2, y_3]$, (d) implies that the initial projection of $d(x)$ is actually $p(x)$ for $x \in [y_2, y_3]$. Also, since we assumed that $(1-p)h = h$, the range projection of $d(x)$ is orthogonal to $p(x)$ for all $x \in [y_2, y_3]$. Hence, we can take $v_{1,3}$ to be defined as follows:

(1) $v_{1,3}(x) = v_{1,2}(x)$ for $x \in [0, y_2]$.
(2) $v_{1,3}(x) = d(x)((x - y_2)/(y_3 - y_2))$ for $x \in [y_2, y_3]$.

$v_{1,3}$ and y_3 satisfy the conditions in (+).

For the induction step, moving from k to $k+1$, the proof is similar to the basis step $k = 3$, but (to construct $v_{1,k+1}$) we replace $v_{1,2}$ by $v_{1,k}$; replace y_2 by y_k; and replace $1/100^2 = 1/10000$ by $1/100^3 = 1/1000000$. Everything else is exactly the same (modulo minor modifications).

Hence, we have constructed sequences $\{v_{1,k}\}_{k=1}^{\infty}$ and $\{y_k\}_{k=1}^{\infty}$ satisfying the conditions of (+). Since $\{y_k\}_{k=1}^{\infty}$ is a bounded increasing sequence, it converges to, say, y_∞ in $[0, b_0]$. We now define an operator field V as follows:

(a) $V(x) = v_{1,1}(x)$ for $x \in [0, y_1]$.
(b) $V(x) = v_{1,k}(x)$ for $x \in [y_{k-1}, y_k]$.
(c) $V(x) = v_2(x)$ for $x \in [y_\infty, 1]$.

Using Proposition 2.1, one can check that V is an element of $\mathcal{A}$. Hence, V is a partial isometry in $\mathcal{A}$ with initial projection p and range projection orthogonal to p. Since p is an arbitrary projection in $\mathcal{A}$, it follows, by Theorem 2.5, that $\mathcal{A}$ is a stable C^*-algebra. □

To move from intervals to arbitrary n-cubes, we need a theorem due to Lee. For a C^*-algebra $\mathcal{C}$, let $Prim(\mathcal{C})$ denote the primitive ideal space of $\mathcal{C}$, equipped with the hull-kernel topology. If $\mathcal{A}$ is a maximal full algebra of operator fields with base space X and with fibre algebras $\{\mathcal{A}_x\}_{x \in X}$, then for $x \in X$ and $P_x \in Prim(\mathcal{A}_x)$, $\tilde{P}_x =_{df} \{a \in \mathcal{A} : a(x) \in P_x\}$ is an element of $Prim(\mathcal{A})$. Moreover, by [19] Lemma 1.1, every element of $Prim(\mathcal{A})$ is of this form. The following theorem of Lee is [9] Theorem 4.

Theorem 2.7. *Let $\mathcal{A}$ be a separable maximal full algebra of operator fields with base space X and fibre algebras $\{\mathcal{A}_x\}_{x \in X}$. Then the mapping $\tilde{P}_x \mapsto x$ is a continuous open surjection from $Prim(\mathcal{A})$ onto X. Conversely, if $\mathcal{A}$ is a separable C^*-algebra such that there is a continuous open surjection π from $Prim(\mathcal{A})$ onto some Hausdorff topological space X, then X is locally compact and $\mathcal{A}$ is $*$-isomorphic to a maximal full algebra of operator fields with base space X and fibre algebras $\{\mathcal{A}/I_x\}_{x \in X}$, where $I_x =_{df} \bigcap \pi^{-1}(x)$.*

We are now in the position to prove the theorem for arbitrary finite-dimensional cubes.

Theorem 2.8. *Let $\mathcal{A}$ be a separable σ_P-unital maximal full algebra of operator fields with base space the n-cube $[0,1]^n$ and fibre algebras $\{\mathcal{A}_x\}_{x\in X}$. If $\mathcal{A}_x$ is stable for all $x \in [0,1]^n$ then $\mathcal{A}$ is stable.*

Proof. We prove the theorem by induction on n. The basis step $n = 1$ is Theorem 2.6.

So suppose that the theorem is true for n. We now try to prove it for $n + 1$. So suppose that $\mathcal{A}$ is a maximal full algebra of operator fields with base space $[0,1]^{n+1}$ and fibre algebras $\{\mathcal{A}_x\}_{x\in[0,1]^{n+1}}$. Let $\pi : Prim(\mathcal{A}) \to [0,1]^{n+1}$ be the continuous open surjection given by Theorem 2.7. Hence, by Theorem 2.7, for each x, $\mathcal{A}_x \cong \mathcal{A}/I_x$ where $I_x =_{df} \bigcap \pi^{-1}(x)$.

Now consider the map $\rho : [0,1]^{n+1} \to [0,1]^n$ which is the natural projection onto the first n coordinates. Hence, ρ is a continuous open surjection. Hence, the composition $\rho \circ \pi : Prim(\mathcal{A}) \to [0,1]^n$ is a continuous open surjection. Hence, by Theorem 2.7, $\mathcal{A}$ can be realized as a maximal full algebra of operator fields with base space $[0,1]^n$ and fibre algebras, say, $\{\mathcal{B}_t\}_{t\in[0,1]^n}$.

For each $t \in [0,1]^n$, the fibre algebra $\mathcal{B}_t$ can be realized as a maximal full algebra of operator fields with base space $\rho^{-1}(t) = \{t\} \times [0,1]$ and fibre algebras $\{\mathcal{A}_x\}_{x\in\{t\}\times[0,1]}$. (By Lee's Theorem, $\mathcal{B}_t$ is isomorphic to $\mathcal{A}/I$ where $I = \bigcap(\rho\circ\pi)^{-1}(t)$. Using this fact, one can construct the natural continuous open surjection from $Prim(\mathcal{B}_t)$ onto $\{t\} \times [0,1]$). Since the base space of this continuous field decomposition (of $\mathcal{B}_t$) is the interval $[0,1]$ and since the fibre algebras are stable, it follows, by Theorem 2.6, that $\mathcal{B}_t$ is a stable C^*-algebra.

Since $\mathcal{A}$ can be realized as a maximal full algebra of operator fields over $[0,1]^n$ with stable fibre algebras $\mathcal{B}_t$, it follows, by the induction hypothesis, that $\mathcal{A}$ must be a stable C^*-algebra. □

We note that the arguments for Theorem 2.8 (and Theorem 2.6) would also work if we replaced the n-cube by the n-torus (and replaced the interval by the circle). Hence, we also have the following theorem.

Theorem 2.9. *Let $\mathcal{A}$ be a separable σ_P-unital maximal full algebra of operator fields with base space the n-torus $\mathbb{T}^n$ and fibre algebras $\{\mathcal{A}_x\}_{x\in\mathbb{T}^n}$. If $\mathcal{A}_x$ is stable for all $x \in \mathbb{T}^n$ then $\mathcal{A}$ is stable.*

3. The corona factorization property

In this section, we show that the corona factorization property is also preserved under the operator of forming continuous field algebras when the base space is an n-cube or an n-torus.

Towards this, we need the following theorem which is [2] Theorem 4.23:

Theorem 3.1. *Let $\mathcal{B}$ be a separable stable C^*-algebra and let P be a projection in $\mathcal{M}(\mathcal{B})$ (the multiplier algebra of $\mathcal{B}$). Then $\overline{P\mathcal{B}P}$ is a stable full hereditary subalgebra of $\mathcal{B}$ if and only if P is Murray-von Neumann equivalent to $1_{\mathcal{M}(\mathcal{B})}$.*

We also need a lemma about the multiplier algebra of a continuous field algebra, which is [9] Lemma 2.

Lemma 3.2. *Let $\mathcal{A}$ be a maximal full algebra of operator fields with base space X and fibre algebras $\{\mathcal{A}_x\}_{x\in X}$. Let $\mathcal{M}$ be the set of all functions m on X such that $m(x) \in \mathcal{M}(\mathcal{A}_x)$ for all $x \in X$ (here $\mathcal{M}(\mathcal{A}_x)$ is the multiplier algebra of $\mathcal{A}_x$) and such that the operator fields $x \mapsto m(x)a(x)$ and $x \mapsto a(x)m(x)$ are in $\mathcal{A}$ for all $a \in \mathcal{A}$. Then*

- (a) *for any $m \in \mathcal{M}$, the function $x \mapsto \|m(x)\|$ is a bounded function on X, and $\mathcal{M}$ is a C^*-algebra under the pointwise operations and supremum norm; and*
- (b) *$\mathcal{M}$ is $*$-isomorphic to $\mathcal{M}(\mathcal{A})$, the multiplier algebra of $\mathcal{A}$.*

Theorem 3.3. *Let $\mathcal{A}$ be a separable σ_P-unital maximal full algebra of operator fields with base space either an n-cube $X = [0,1]^n$ or an n-torus $X = \mathbb{T}^n$ and with fibre algebras $\{\mathcal{A}_x\}_{x\in X}$. Suppose that for every $x \in X$, $\mathcal{A}_x$ is a stable C^*-algebra with the corona factorization property. Then $\mathcal{A}$ is a stable C^*-algebra with the corona factorization property.*

Proof. That $\mathcal{A}$ is a stable C^*-algebra follows from Theorem 2.8 and Theorem 2.9.

Now suppose that P is a norm-full projection in the multiplier algebra $\mathcal{M}(\mathcal{A})$. Then $\overline{P\mathcal{A}P}$ is a full hereditary subalgebra of $\mathcal{A}$.

Now by Lemma 3.2, for every $x \in X$, $P(x)$ is a full projection in $\mathcal{M}(\mathcal{A}_x)$. Hence, since $\mathcal{A}_x$ has the corona factorization property (by hypothesis), $P(x)$ is Murray-von Neumann equivalent to $1_{\mathcal{M}(\mathcal{A}_x)}$. Hence by Theorem 3.1, for every $x \in X$, $\overline{P(x)\mathcal{A}_xP(x)}$ is a stable full hereditary subalgebra of $\mathcal{A}_x$.

By Theorem 2.7, let $\pi : Prim(\mathcal{A}) \to X$ be the continuous open surjection for the continuous field decomposition of $\mathcal{A}$. By [13] Proposition 4.1.10, the map $\gamma : Prim(\mathcal{A}) \to Prim(\overline{P\mathcal{A}P}) : J \mapsto J \cap \overline{P\mathcal{A}P}$ is a homeomorphism. Hence, $\pi \circ (\gamma^{-1})$ is a continuous open surjection. Hence, by Theorem 2.7, $\overline{P\mathcal{A}P}$ is a maximal full algebra of operator fields with base space either $X = [0,1]^n$ or $X = \mathbb{T}^n$ and fibre algebras $\{\overline{P(x)\mathcal{A}_x P(x)}\}_{x \in X}$ which are stable. Hence, by Theorem 2.8 and Theorem 2.9, $\overline{P\mathcal{A}P}$ is a stable C^*-algebra. Hence, $\overline{P\mathcal{A}P}$ is a stable full hereditary subalgebra of $\mathcal{A}$. Hence, by Theorem 3.1, P must be Murray-von Neumann equivalent to $1_{\mathcal{M}(\mathcal{A})}$. Since P is an arbitrary full projection in $\mathcal{M}(\mathcal{A})$, $\mathcal{A}$ must have the corona factorization property. □

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