Problem 2 *Show that if* -UBH *holds, then there is a nontame premouse.*

[This problem was contributed by J.R. Steel.]

Here UBH is the Unique Branches Hypothesis, the assertion that every iteration tree on V has at most one cofinal wellfounded branch. See section 5 of [MaSt94] for a discussion of the importance of UBH. A premouse is nontame if its sequence has extenders overlapping local Woodin cardinals; see e.g. [St∞], section x.

The real problem here is to improve the existing lower bound on the consistency strength of -UBH. The best lower bound currently known is infinitely many Woodin cardinals; this comes from [St$\infty\infty$], which has in sections 1 and 4 a discussion of previous work on the problem. Woodin (unpublished, unwritten) has shown that the existence of a nontrivial elementary $j\colon L(V_{\lambda+1}) \to L(V_{\lambda+1})$ outright implies -UBH. This also provides the best known upper bound on the consistency strength of -UBH.

I suspect that -UBH implies there are mice with superstrong cardinals. The problem above provides a reasonable target in the region where much of the relevant basic inner model theory may already exist. I suspect the "poor man's core model induction" methods of [St$\infty\infty$] have been pushed as far as they will go in that paper, and that to get a nontame mouse from -UBH one needs the full machinery of Woodin's core model induction. ([K00], [SchSt∞], and [Z00] describe and use this method.) Ketchersid's thesis [K00] contains the only core model induction to date which has gone far enough to produce a nontame mouse.

If [St$\infty\infty$] is mining strength from -UBH in the right way, then this problem looks similar to the problem of getting strength from the failure of square at a singular strong limit.

References

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