Problem 7 *Suppose that Ω is measurable and that there is no inner model with a Woodin cardinal. Let K denote Steel's core model.*

(a) Suppose that $\kappa < \Omega$ is measurable and $2^\kappa = \lambda > \kappa^+$. How large should κ be in K ? Is then $o(\kappa) \geq \lambda$ in K ?

(b) Suppose that $\kappa < \Omega$ is a strong limit cardinal with $cf(\kappa) = \delta$, $\omega_1 < \delta < \kappa$, and that $2^\kappa \geq \lambda > \kappa^+$, where λ is not the successor of a cardinal of cofinality less than κ. Is then $o(\kappa) \geq \lambda + \delta$ in K ?

[This problem was contributed by M. Gitik]

The answers to both questions are affirmative assuming that $0^\P$ doesn't exist, see [Gi93] and [GiMi96]. It would be very interesting to extend this to bigger core models. It is likely that this can be done up to Schindler's core model for almost linear iterations [Sch02]. But further up iteration trees come essentially into play and it is unclear what kind of indiscernibles should correspond to them.

The following, if it is true, can help much:

Question. Suppose that Ω is measurable and that there is no inner model with a Woodin cardinal. Let K denote Steel's core model.

Let $\kappa < \Omega$ be measurable and let $j\colon V \to M$ be the elementary embedding by a normal measure over κ. Should $j \restriction K$ be always a linear iteration? Or can it be rearranged as a linear iteration?

By [Sch?], $j \restriction K$ is an iteration of K. The concept of the "linearity" of an iteration has to be made precise before attacking this question.

Another possibility, probably a bit more interesting, would be to find a place where the answer to problem # 7 will be negative. This may suggest a new forcing for dealing with with old questions on cardinal arithmetic.

References

[Gi93] Gitik, M., *On measurable cardinals violating the continuum hypothesis*, Ann. Pure Appl. Logic **63** (1993), pp. 227-240.

[GiMi96] Gitik, M., Mitchell, W., *Indiscernible sequences for extenders and the singular cardinal hypothesis*, Ann. Pure Appl. Logic **82** (1996), pp. 273-316.

[Sch02] Schindler, R., *The core model for almost linear iterations*, Ann. Pure Appl. Logic, to appear.

[Sch?] Schindler, R., *Iterations of the core model*, submitted.