Problem 9 *Let n be an integer. Suppose that V is closed under $M_n^\#$, but there is no inner model with $n+1$ Woodin cardinals. Is K Σ^1_{n+3} correct ?*

[This problem was contributed by R. Schindler]

Let X be a set of ordinals, and let $n \in \omega$. We let $M_n^\#(X)$ denote the least X-premouse which is not n-small and which is such that all countable premice which embed into it are $\omega_1 + 1$ iterable "above the preimage of X." (In particular, $M_0^\#(X) = X^\#$.) V is closed under $M_n^\#$ if and only if $M_n^\#(X)$ exists for all sets X of ordinals. If V is closed under $M_n^\#$ and there is no inner model with $n+1$ Woodin cardinals then the core model K exists.

We say that a transitive model M is Σ^1_n correct if $M \prec_{\Sigma^1_n} V$. Every inner model of set theory is Σ^1_2 correct. Jensen showed that if $\mathbb{R}$ is closed under sharps and there is no inner model with a measurable cardinal then K is Σ^1_3 correct. Steel and Welch showed that if $\mathbb{R}$ is closed under sharps and there is no inner model with a strong cardinal then K is Σ^1_3 correct (cf. [StWe98]). Steel showed that if there are two measurable cardinals and there is no inner model with a Woodin cardinal then K is Σ^1_3 correct (cf. [St96]).

Woodin has shown that every sufficiently iterable inner model with $2n$ Woodin cardinals is Σ^1_{2n+2} correct (cf. [St]).

References

[St] Steel, J., *Projectively well-ordered inner models*, Ann. Pure Appl. Logic.

[St96] Steel, J., *The core model iterability problem*, LNL #8, 1996.

[StWe98] Steel, J., and Welch, P., Σ^1_3 *absoluteness and the second uniform indiscernible*, Israel Journal of Mathematics **104** (1998), pp. 157-190.